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Extended MULTIMOORA method based on Shannon entropy weight for materials selection

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Abstract Selection of appropriate material is a crucial step in engineering design and manufacturing process. Without a systematic technique, many useful engineering materials may be ignored for selection. The category of multiple attribute decision-making (MADM) methods is an effective set of structured techniques. Having uncomplicated assumptions and mathematics, the MULTIMOORA method as an MADM approach can be effectively utilized for materials selection. In this paper, we developed an extension of MULTIMOORA method based on Shannon entropy concept to tackle materials selection process. The entropy concept was considered to assign relative importance to decision-making attributes. The proposed model consists of two scenarios named the weighted and entropy-weighted MULTIMOORA methods. In the first scenario, subjective weight was considered in the formulation of the approach like most of conventional MADM methods. The general form of entropy weight that is a combination of subjective and objective weighting factors was employed for the second scenario. We examined two popular practical examples concerning materials selection to show the application of the suggested approach and to reveal the effect of entropy weights. Our results were compared with the earlier studies.

Keywords Multiple attribute decision making · MULTIMOORA · Shannon entropy · Materials selection

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Introduction

More than 40,000 practical metallic alloys and a same number of nonmetallic materials like polymers, ceramics, and composites are utilized in various industries (Farag 2002). Because of the considerable number, dissimilar production techniques, and different properties of engineering materials, the selection process of materials can be regarded as a complex undertaking for an engineer or designer. If the process takes place unsystematically, many significant materials may be neglected. Therefore, a structured mathematical approach is needed for materials selection.

MADM methods can be used as effective systematic tools for materials selection. Each MADM technique has specific assumptions and principles. A number of MADM methods have been utilized in the materials selection process by earlier researchers, like the technique for order preference by similarity to ideal solution (TOPSIS) (Bakhoum and Brown 2013; Das 2012; Huang et al. 2011; Jee and Kang 2000), analytic hierarchy approach (AHP) (Chauhan and Vaish 2013; Dweiri and Al-Oqla 2006), compromise ranking also known as vlse kriterijumska optimizacija kompromisno resenje (VIKOR) (Jahan and Edwards 2013b; Liu et al. 2013), diverse versions of elimination and choice expressing the reality (ELECTRE) also recognized as outranking method (Anojkumar et al. 2014; Chatterjee et al. 2009; Shanian and Savadogo 2009), preference ranking organization method for enrichment evaluation (PROMETHEE)

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(Jiao et al. 2011; Peng and Xiao 2013), graph theory and matrix approach (Rao 2006), gray relational analysis (Chan and Tong 2007; Zhao et al. 2012), various preference ranking-based techniques (Chatterjee and Chakraborty 2012; Maity et al. 2012), preference selection index (Maniya and Bhatt 2010), utility additive (UTA) (Athawale et al. 2011), weighted property index (Findik and Turan 2012), linear assignment (Jahan et al. 2010a), modified digital logic (Manshadi et al. 2007; Torrez et al. 2012), Z-transformation (Fayazbakhsh and Abedian 2010; Fayazbakhsh et al. 2009), and quality function deployment (Mayyas et al. 2011; Prasad 2013; Prasad and Chakraborty 2013). Two groups of researchers have reviewed the applications of MADM methods in materials selection (Jahan and Edwards 2013a; Jahan et al. 2010b).

Almost all the aforementioned methods have a key feature that is moderate to the extreme complexity of their mathematical models. Utilization of these techniques seems to be difficult, requiring advanced mathematical knowledge (Karande and Chakraborty 2012a). Accordingly, an undemanding MADM method can be a real blessing for decision makers. The multi-objective optimization on the basis of ratio analysis (MOORA) method proposed by Brauers and Zavadskas (2006) has uncomplicated mathematics. Therefore, it can be employed effortlessly and effectually for selection of materials. The MULTIMOORA method is a comprehensive form of the MOORA technique. As discussed by Brauers and Ginevičius (2010), because the final rank is generated by the integration of three subordinate ranks in the MULTIMOORA technique, its results can be more robust than traditional MADM methods in which a single rank is obtained. The MOORA and MULTIMOORA techniques have been used in different applications like decision making in manufacturing environment (Chakraborty 2011), robot selection (Datta et al. 2013), supplier selection (Farzamnia and Babolghani 2014; Karande and Chakraborty 2012b; Mishra et al. 2015), evaluating the risk of failure modes (Liu et al. 2014a), project selection (Rached-Paoli and Baunda 2014), selection of health-care waste treatment (Liu et al. 2014b), ranking of banks (Brauers et al. 2014), and student selection (Deliktas and Ustun 2015).

In the present paper, we extended the MULTIMOORA method using entropy weight based on Shannon information theory for application in materials selection. Our study is closely related to Karande and Chakraborty (2012a). They used the MOORA technique in the materials selection process of four practical cases. The novelties of our paper comparing the study of Karande and Chakraborty (2012a) are as follows: First, they did not calculate the final ranking of the MULTIMOORA method and only reported the three subordinate ranks. The third subordinate rank of the MULTIMOORA method, i.e., the full multiplicative form rank, was incorrectly called the MULTIMOORA ranking in

their study. In this paper, we employed the dominance theory to integrate the three subordinate ranks into the final ranking, named the MULTIMOORA ranking. This aggregate final ranking is more robust than each of the subordinate ranks as stated by Brauers and Ginevičius (2010). Second, Karande and Chakraborty (2012a) did not utilize any relative significance for attributes. However, we used two forms of attributes weighting, i.e., subjective and the general Shannon entropy weights, to generate two solution modes named the weighted and entropy-weighted MULTIMOORA rankings, respectively. Third, Karande and Chakraborty (2012a) employed Voogd ratio (Voogd 1983) for normalization, whereas we utilized the original MULTIMOORA normalization equation that is the most robust option among various ratios as shown by Brauers and Zavadskas (2006). A few studies on assigning weights for the MOORA and MULTIMOORA techniques exist. Brauers and Zavadskas (2006) mentioned that giving importance to each attribute is possible, but they did not discuss on the specifications of these significance factors. Özçelik et al. (2014) assigned weight for the reference point approach of the MOORA method. In their study, the fuzzy analytic hierarchy process was utilized for the determination of significance coefficients of attributes. El-Santawy (2014) used a new form of entropy weight to develop the MOORA method. Derivation of their significance factors differs from Shannon entropy weight. In addition, they did not develop the MULTIMOORA method with their suggested weights. To the best of the authors' knowledge, no study has been conducted on combination of Shannon entropy weight with MULTIMOORA technique. In our proposed approach, the general form of entropy weight was utilized that includes subjective and objective parts. The subjective significance coefficient is obtained directly from decision makers opinions. The objective part is calculated based on the entropy concept through analyzing the data regardless of decision makers' comments. The general form of entropy weight improves the initial values of decision matrix and reliability of the ranking of alternatives obtained by the MULTIMOORA approach. We evaluated two practical examples in the field of materials selection. The results were compared with other studies that have considered these two problems. Eventually, concluding remarks were cited to make a summary of our work and to present an overview of the developed MULTIMOORA method and its application in materials selection.

The MULTIMOORA method

The MOORA method proposed by Brauers and Zavadskas (2006) is formed from two parts: the ratio system and the reference point approach. Brauers and Zavadskas (2010)

developed the concept by utilizing the full multiplicative form. The updated method, called MULTIMOORA, is composed of MOORA parts and the full multiplicative form. The MULTIMOORA method begins with a decision matrix $\mathbf{X}$ in which x_{ij} presents the performance index of ith alternative respecting jth attribute, $i = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n$:

$$\mathbf{X} = [x_{ij}]_{m \times n}. \tag{1}$$

To make the performance indices dimensionless and comparable, the decision matrix is normalized. This normalization ratio is a comparison between each response of an alternative to an attribute, as a numerator, and a denominator that is a representative for all alternative performances with respect to that attribute. In the MULTIMOORA method, the dominator is selected as the square root of the sum of squares of performance indices per attribute as shown in the following:

$$x_{ij}^* = \frac{x_{ij}}{\left[\sum_{i=1}^{m} x_{ij}^2\right]^{1/2}}, \tag{2}$$

in which x_{ij}^* denotes the normalized performance index of ith alternative respecting jth attribute. Brauers and Zavadskas (2006) proved that this ratio is the most robust selection among different normalization equations for the MULTIMOORA method.

The ratio system

Equation (2) justifies the appellation of this technique as the ratio system. For this method, the normalized performance indices are added for beneficial attributes (in case of maximization) or deducted for non-beneficial attributes (in case of minimization) as follows (Brauers and Zavadskas 2006):

$$y_i^* = \sum_{j=1}^{g} x_{ij}^* - \sum_{j=g+1}^{n} x_{ij}^*, \tag{3}$$

in which g indicates the number of beneficial attributes and $(n - g)$ is the number of non-beneficial attributes. y_i^* denotes the assessment value of ith alternative regarding all attributes for the ratio system. The optimal alternative based on the ratio system has the highest assessment value (Datta et al. 2013):

$$A_{RS}^* = \left\{ A_i \middle| \max_i y_i^* \right\}. \tag{4}$$

The reference point approach

As the second part of the MOORA method, the reference point approach is also based on the ratio system, i.e., Eq. (2). A maximal objective reference point is utilized in the method. The

ith co-ordinate of the maximal objective reference point vector is defined as follows (Brauers and Zavadskas 2006):

$$r_j = \begin{cases} \max_i x_{ij}^* & \text{in case of maximization} \\ \max_i x_{ij}^* & \text{in case of minization} \end{cases}. \tag{5}$$

Deviation of a performance index from the reference point r_j can be obtained as $(r_i - x_{ij}^*)$. Afterwards, maximum value of the deviation for each alternative respecting all attributes can be calculated as:

$$z_i^* = \max_j \left| r_j - x_{ij}^* \right|. \tag{6}$$

To reach the optimal alternative based on the reference point approach, the minimum value of Eq. (6) among all alternatives should be found. The optimal alternative of the reference point approach can be calculated as (Datta et al. 2013):

$$A_{RP}^* = \left\{ A_i \middle| \min_i z_i^* \right\}. \tag{7}$$

The full multiplicative form

Brauers and Zavadskas (2010) developed the full multiplicative form as the third part of the MULTIMOORA method. The formula of the method can be determined as follows:

$$U_i' = \frac{\prod_{j=1}^{g} x_{ij}}{\prod_{j=g+1}^{n} x_{ij}}, \tag{8}$$

in which g is defined similarly as aforementioned for the ratio system. The numerator of Eq. (8) indicates the product of performance indices of ith alternative relating to beneficial attributes. The denominator of Eq. (8) represents the product of performance indices of ith alternative relating to non-beneficial attributes.

Using the normalized decision matrix, an equivalent form of U_i' can be established as:

$$U_i^* = \frac{\prod_{j=1}^{g} x_{ij}^*}{\prod_{j=g+1}^{n} x_{ij}^*}. \tag{9}$$

The assessment values of U_i^* differ from U_i'; however, the ranking calculated by both equations is analogous. Accordingly, to preserve a harmony between all parts of the MULTIMOORA method, we use Eq. (9) as the full multiplicative form representation.

Similar to the ratio system, an optimal alternative can be distinguished by searching for maximum among all assessment values of U_i^* as:

$$A_{MF}^* = \left\{ A_i \middle| \max_i U_i^* \right\}. \tag{10}$$

The final ranking of the MULTIMOORA method based on the dominance theory

The dominance theory was employed as a tool for consolidation of subordinate rankings of the MULTIMOORA method (Brauers et al. 2011; Brauers and Zavadskas 2011, 2012). After the calculation of the subordinate ranks as above, they can be integrated into a final ranking, named the MULTIMOORA rank, based on the dominance theory. For a detailed explanation of the dominance theory, readers can refer to the study of Brauers and Zavadskas (2012).

Shannon entropy weight

Entropy concept has been widely employed in social and physical sciences. Economics, spectral analysis, and language modeling are a few typical practical applications of entropy. A mathematical theory of communication was proposed by Shannon (1948). Entropy evaluates the expected information content of a certain message. Entropy concept in information theory can be considered as a criterion for the degree of uncertainty represented by a discrete probability distribution.

Entropy idea can be effectively employed in the process of decision making, because it measures existent contrasts between sets of data and clarifies the average intrinsic information transferred to decision maker.

To determine objective weight through Shannon entropy, the following procedure should be adopted (Hwang and Yoon 1981):

Step 1 Normalization of the arrays of decision matrix (performance indices) to obtain the project outcomes p_{ij}:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{11}$$

Step 2 Computation of the entropy measure of project outcomes using the following equation:

$$E_j = -k \sum_{i=1}^{m} p_{ij} \ln p_{ij}, \tag{12}$$

in which $k = 1/\ln(m)$.

Step 3 Defining the objective weight based on the entropy concept:

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n} (1 - E_j)} \tag{13}$$

Step 4 Calculating the general form of the entropy weight, if the decision maker assigns subjective weight s_j. By considering s_j, Eq. (13) transforms into the following:

$$w_j^* = \frac{s_j w_j}{\sum_{j=1}^{n} s_j w_j}, \tag{14}$$

in which subjective and objective weights (s_j and w_j) are combined to produce the general form of Shannon entropy weight w_j^*.

The extended MULTIMOORA method based on Shannon entropy weight

In the initial paper on the MOORA method, Brauers and Zavadskas (2006) allocated a section for the importance given to an attribute. They mentioned that a significance coefficient can be considered to affix more importance to a specific attribute. Their weighted form of the MOORA method confines to general representation of the main formulas and no details concerning characteristics of significance coefficient have been cited. This concept was later updated to encompass all subsections of MULTIMOORA method (Brauers and Zavadskas 2010, 2011).

Significance coefficient can be subjective weight gained directly from the decision makers similar to the routine procedure of the majority of MADM methods. The coefficient can also be regarded as an objective factor like Shannon entropy weight. The inclusive significance coefficient is the combination of subjective and objective factors like the general Shannon entropy weight.

In the present paper, we designate two forms of weight as significance coefficient of attributes. If significance coefficient only consists of subjective weight s_j earned from the decision makers, the resultant approach is named the weighted MULTIMOORA method. Application of the general Shannon entropy weight that is a combined subjective and objective significance coefficient leads to the so-called entropy-weighted MULTIMOORA technique. Based on Shannon entropy weight and the original MULTIMOORA approach, the following methodology is attained.

The extended ratio system

Significance coefficient or importance weight of attributes can be added to the ratio system. As mentioned above, the two forms of weighting were considered in this paper. By considering Eq. (3), the extended ratio system can have two parts, as follows:

$$y_i^w = \sum_{j=1}^{g} s_j x_{ij}^* - \sum_{j=g+1}^{n} s_j x_{ij}^*, \tag{15}$$

$$y_i^{ew} = \sum_{j=1}^{g} w_j^* x_{ij}^* - \sum_{j=g+1}^{n} w_j^* x_{ij}^*. \tag{16}$$

y_i^w and y_i^{ew} represent the assessment values of ith alternative regarding all attributes for the weighted and entropy-weighted ratio systems, respectively. The resultant optimal alternatives based on these techniques can be identified as follows:

$$A_{WRS}^* = \left\{ A_i \middle| \max_i y_i^w \right\}. \tag{17}$$

$$A_{EWRS}^* = \left\{ A_i \middle| \max_i y_i^{ew} \right\}. \tag{18}$$

The extended reference point approach

The reference point approach can also be developed using subjective and the general Shannon entropy weights:

$$z_i^w = \max_j \left| s_j r_j - s_j x_{ij}^* \right|, \tag{19}$$

$$z_i^{ew} = \max_j \left| w_j^* r_j - w_j^* x_{ij}^* \right|. \tag{20}$$

Then, alternatives can be listed in ascending order based on the assessment values of Eqs. (19) and (20) to find the optimal alternatives of the weighted and entropy-weighted reference point approaches, respectively, as:

$$A_{WRP}^* = \left\{ A_i \middle| \min_i z_i^w \right\}. \tag{21}$$

$$A_{EWRP}^* = \left\{ A_i \middle| \min_i z_i^{ew} \right\}. \tag{22}$$

The extended full multiplicative form

Brauers and Zavadskas (2012) showed that considering weights as coefficients is meaningless for the full multiplicative form. Instead, the weights should be employed as exponents. The weighted and entropy-weighted full multiplicative forms can be formulated, respectively, as:

$$U_i^w = \frac{\prod_{j=1}^{g} \left(x_{ij}^* \right)^{s_j}}{\prod_{j=g+1}^{n} \left(x_{ij}^* \right)^{s_j}}, \tag{23}$$

$$U_i^{ew} = \frac{\prod_{j=1}^{g} \left(x_{ij}^* \right)^{w_j^*}}{\prod_{j=g+1}^{n} \left(x_{ij}^* \right)^{w_j^*}}, \tag{24}$$

The optimal alternatives based on the two techniques have the greatest assessment value:

$$A_{WMF}^* = \left\{ A_i \middle| \max_i U_i^w \right\}, \tag{25}$$

$$A_{EWMF}^* = \left\{ A_i \middle| \max_i U_i^{ew} \right\}. \tag{26}$$

The final ranking of the extended MULTIMOORA method based on the dominance theory

By utilizing the dominance theory, we integrated the subordinate rankings into a final ranking.

Application of the extended MULTIMOORA method in materials selection

Karande and Chakraborty (2012a) utilized the MOORA technique to choose materials for different applications. However, they altered the original normalization ratio of the method, i.e., Eq. (2), into another form. They used Voogd ratio (Voogd 1983) as normalization formula that is $x_{ij}^* = x_{ij} / \sum_{i=1}^{m} x_{ij}$.

Brauers and Zavadskas (2006) established that among different choices for the denominator of the normalization ratio, the square root of the sum of each alternative performance index, $\left[\sum_{i=1}^{m} x_{ij}^2 \right]^{1/2}$, is the most robust option. Therefore, the results of the study of Karande and Chakraborty (2012a) may not be as robust as the original MULTIMOORA method. Thus, we do not verify our results with their outcomes.

In the following subsections, we calculated the weighted and entropy-weighted MULTIMOORA rankings for two material selection problems cited in the study of Karande and Chakraborty (2012a). Besides, we compared our results with the related studies on the field.

Example 1: Material selection for flywheel

The problem addresses materials selection for a flywheel. Other studies have solved this practical case using various methods (Chatterjee et al. 2009; Jahan et al. 2010a; Jee and Kang 2000). The main requirements in the design of a flywheel are to save the maximum amount of kinetic energy as well as to prevent fatigue and fracture. Stored kinetic energy per unit mass of a thin flywheel is as follows (Lewis 1990):

$$\frac{u}{m} = \frac{s k_s}{(1-v)\rho}, \tag{27}$$

in which u, m, s, v, and ρ are kinetic energy, mass, failure strength, Poisson ratio, and density, respectively. k_s is a factor related to the extent of material anisotropy. Fatigue

strength σ_f can be considered as failure strength s for a flywheel. By ignoring effects of the values of v and k_s, a general relation obtains from Eq. (27). That is, if σ_f/ρ increases, u/m will be greater. Thus, the first attribute is specific strength σ_f/ρ. Waterman and Ashby (1991) showed that the criterion for minimization of the disc weight is s/ρ. Therefore, σ_f/ρ can concurrently be a measure for fatigue strength, kinetic energy maximization, and weight minimization. The fracture strength can be represented by fracture toughness K_{IC}. Thus, to minimize the probability of brittle fracture, K_{IC}/ρ is taken as the second attribute. The third important index can be price per unit mass. Fragmentability is an essential feature of a given flywheel that ensures safety. Hence, fragmentability is regarded as the last attribute. Only price per unit mass attribute is non-beneficial and the rest of the attributes are beneficial. Ten candidate materials for the engineering materials selection problem and their properties are gathered in Table 1. Table 2 shows the decision matrix for the problem. The performance indices can be normalized using Eq. (2) as displayed in Table 3.

E_j and w_j were calculated using Eqs. (12) and (13), respectively, as shown in Table 4. Four different sets of subjective weight s_j exist for the flywheel problem in the study of Jee and Kang (2000). In the present paper, we considered case one for the subjective weight. By applying the subjective and objective weighting factors, the general Shannon entropy weight w_j^* was obtained according to Eq. (14) as listed in Table 4.

The assessment values of the weighted and entropy-weighted MULTIMOORA methods and their resultant rankings are shown in Tables 5 and 6, respectively. Assessment values presented in Tables 5 and 6 are related to the three parts of the weighted and entropy-weighted MULTIMOORA approaches that can be obtained using

Eqs. (15), (19), and (23) besides Eqs. (16), (20), and (24), respectively. In Tables 5 and 6, the rankings for the first and third parts were calculated based on descending order. In contrast, the assessment values for the second part of the proposed method that is the reference point approach were arranged in ascending order. The last columns were allocated to the final ranks determined based on the dominance theory (Brauers and Zavadskas 2012). The optimal material can be found using the related A^* equations. From the assessment values of Tables 5 and 6, $A_{WRS}^* = A_{EWRS}^* = A_{WRP}^* = A_{EWRP}^* =$ Kevlar 49–epoxy FRP and $A_{WMF}^* = A_{EWMF}^* =$ S glass–epoxy FRP. Final ranking has more importance because it is the integrated form of subordinate ranks. Kevlar 49–epoxy FRP, Carbon–epoxy FRP, Kevlar 29–epoxy FRP, and S glass–epoxy FRP obtain the first to third positions, respectively, in the final rankings of the both weighted and entropy-weighted MULTIMOORA methods.

Table 7 shows the final ranks of the proposed model and other approaches for the flywheel materials selection problem. The optimal material in all the methods is similar that is Kevlar 49–epoxy FRP. However, similarity or contrast may exist between our materials ranks and the others.

To show an association between the materials ranks of our methods and other approaches listed in Table 7, we utilized Spearman rank correlation coefficient. Figure 1 illustrates Spearman coefficients for Example 1. By considering the coefficients related to the weighted MULTIMOORA, because of considering entropy concept for weight calculation, the TOPSIS method (Jee and Kang 2000) has the lowest value 0.76. Because other techniques exploited subjective weights, they show more concordance with the proposed weighted MULTIMOORA results. The ELECTRE approach (Chatterjee et al. 2009) outranks with

Table 1 Candidate materials and their properties for Example 1 (Jee and Kang 2000)

Materials	Properties				
	Fatigue strength (Mpa)	Fracture toughness (Mpa·m$^{1/2}$)	Density (g/cm^3)	Price/mass (10^3 US$/t)	Fragmentability
300 M	800	68.9	8	4.2	3 (poor)
2024-T3	140	38	2.82	2.1	3 (poor)
7050-T73651	220	35.4	2.82	2.1	3 (poor)
Ti–6Al–4V	515	123	5	10.5	3 (poor)
E glass–epoxy FRP	140	20	2	2.735	9 (excellent)
S glass–epoxy FRP	330	50	2	4.095	9 (excellent)
Carbon–epoxy FRP	700	35	2	35.47	7 (fairly good)
Kevlar 29–epoxy FRP	340	40	1	11	7 (fairly good)
Kevlar 49–epoxy FRP	900	50	1	25	7 (fairly good)
Boron–epoxy FRP	1000	46	2	315	5 (good)

Table 2 Decision matrix for Example 1 (Jee and Kang 2000)

| Materials | Beneficial and non-beneficial attributes | | | |
	MAX Specific strength (kN·m/kg)	MAX Specific toughness (kPa·m$^{1/2}$/kg/m^3)	MIN Price/mass (10^3 US$/t)	MAX Fragmentability
300 M	100	8.613	4.2	3
2024-T3	49.645	13.475	2.1	3
7050-T73651	78.014	12.553	2.1	3
Ti–6Al–4V	108.879	26.004	10.5	3
E glass–epoxy FRP	70	10	2.735	9
S glass–epoxy FRP	165	25	4.095	9
Carbon–epoxy FRP	440.252	22.013	35.47	7
Kevlar 29–epoxy FRP	242.857	28.571	11	7
Kevlar 49–epoxy FRP	616.438	34.247	25	7
Boron–epoxy FRP	500	23	315	5

Table 3 Normalized decision matrix for Example 1

| Materials | Beneficial and non-beneficial attributes | | | |
	MAX Specific strength	MAX Specific toughness	MIN Price/mass	MAX Fragmentability
300 M	0.103	0.124	0.013	0.156
2024-T3	0.051	0.194	0.007	0.156
7050-T73651	0.080	0.181	0.007	0.156
Ti–6Al–4V	0.112	0.375	0.033	0.156
E glass–epoxy FRP	0.072	0.144	0.009	0.468
S glass–epoxy FRP	0.170	0.360	0.013	0.468
Carbon–epoxy FRP	0.453	0.317	0.111	0.364
Kevlar 29–epoxy FRP	0.250	0.412	0.035	0.364
Kevlar 49–epoxy FRP	0.634	0.493	0.079	0.364
Boron–epoxy FRP	0.514	0.331	0.989	0.260

Table 4 Entropy measure and weighting factors for Example 1 (Jee and Kang 2000)

| Entropy and weights | Beneficial and non-beneficial attributes | | | |
	MAX Specific strength	MAX Specific toughness	MIN Price/mass	MAX Fragmentability
s_j	0.4	0.3	0.2	0.1
E_j	0.861	0.963	0.415	0.960
w_j	0.174	0.047	0.730	0.050
w_j^*	0.296	0.060	0.623	0.021

the value 0.95. The ranking of the ELECTRE approach (Chatterjee et al. 2009) is almost similar to our outcome that can be observed in Table 7, as well. In the entropy-weighted MULTIMOORA method category, the highest 0.89 is for the TOPSIS method (Jee and Kang 2000). The reason is that among the four studies, only Jee and Kang (2000) considered entropy weight in the formulation of their method. In this category, the linear assignment method (Jahan et al. 2010a), by 0.53, has the lowest agreement with the results of the present paper.

Example 2: Material selection for cryogenic storage tank

We considered materials selection problem of a cryogenic pressure vessel for storing liquid nitrogen as the second example. The material of a cryogenic storage tank should be adequately strong and stiff. Moreover, weldability and processability of the vessel must be high. The other important properties for a pressure vessel or storage tank are density, specific heat, thermal expansion coefficient,

Table 5 Assessment values and rankings of the weighted MULTIMOORA method for Example 1

Materials	Assessment values			Rankings			
	y_i^w	z_i^w	U_i^w	y_i^w Rank	z_i^w Rank	U_i^w Rank	Final Rank
300 M	0.091	0.212	0.425	10	7	9	9
2024-T3	0.093	0.233	0.422	9	10	10	10
7050-T73651	0.101	0.222	0.495	8	8	6	7
Ti–6Al–4V	0.166	0.209	0.510	5	6	5	5
E glass–epoxy FRP	0.117	0.225	0.469	7	9	8	8
S glass–epoxy FRP	0.220	0.186	0.802	4	4	2	4
Carbon–epoxy FRP	0.290	0.072	0.724	2	2	4	2
Kevlar 29–epoxy FRP	0.253	0.154	0.779	3	3	3	3
Kevlar 49–epoxy FRP	0.422	0.014	1.014	1	1	1	1
Boron–epoxy FRP	0.133	0.197	0.482	6	5	7	6

Table 6 Assessment values and rankings of the entropy-weighted MULTIMOORA method for Example 1

| Materials	Assessment values			Rankings			
	y_i^{ew}	z_i^{ew}	U_i^{ew}	y_i^{ew} Rank	z_i^{ew} Rank	U_i^{ew} Rank	Final Rank
300 M	0.033	0.157	6.404	8	6	5	8
2024-T3	0.026	0.173	8.230	9	9	3	9
7050-T73651	0.034	0.164	9.370	7	7	1	6
Ti–6Al–4V	0.038	0.155	3.965	5	5	8	5
E glass–epoxy FRP	0.035	0.167	7.774	6	8	4	7
S glass–epoxy FRP	0.074	0.138	8.233	4	4	2	4
Carbon–epoxy FRP	0.092	0.065	2.834	2	2	9	2
Kevlar 29–epoxy FRP	0.085	0.114	5.003	3	3	6	3
Kevlar 49–epoxy FRP	0.176	0.045	3.997	1	1	7	1
Boron–epoxy FRP	−0.438	0.612	0.752	10	10	10	10

Table 7 Comparison between the materials ranks of the proposed model and other methods for Example 1

Materials	Methods					
	Weighted MULTIMOORA	Entropy-weighted MULTIMOORA	TOPSIS (Jee and Kang 2000)	ELECTRE (Chatterjee et al. 2009)	VIKOR (Chatterjee et al. 2009)	Linear assignment (Jahan et al. 2010a)
300 M	9	8	5	10	9	7
2024-T3	10	9	9	9	10	10
7050-T73651	7	6	7	8	8	8
Ti–6Al–4V	5	5	6	6	6	6
E glass–epoxy FRP	8	7	8	7	7	9
S glass–epoxy FRP	4	4	3	3	5	5
Carbon–epoxy FRP	2	2	4	2	2	3
Kevlar 29–epoxy FRP	3	3	2	4	4	4
Kevlar 49–epoxy FRP	1	1	1	1	1	1
Boron–epoxy FRP	6	10	10	5	3	2

thermal conductivity, and sufficient toughness at the operating temperature (Manshadi et al. 2007). The decision matrix of Example 2 consists of seven engineering materials and their properties as displayed in Table 8. The beneficial attributes are toughness index, yield strength, and elastic modulus, whereas density, thermal expansion
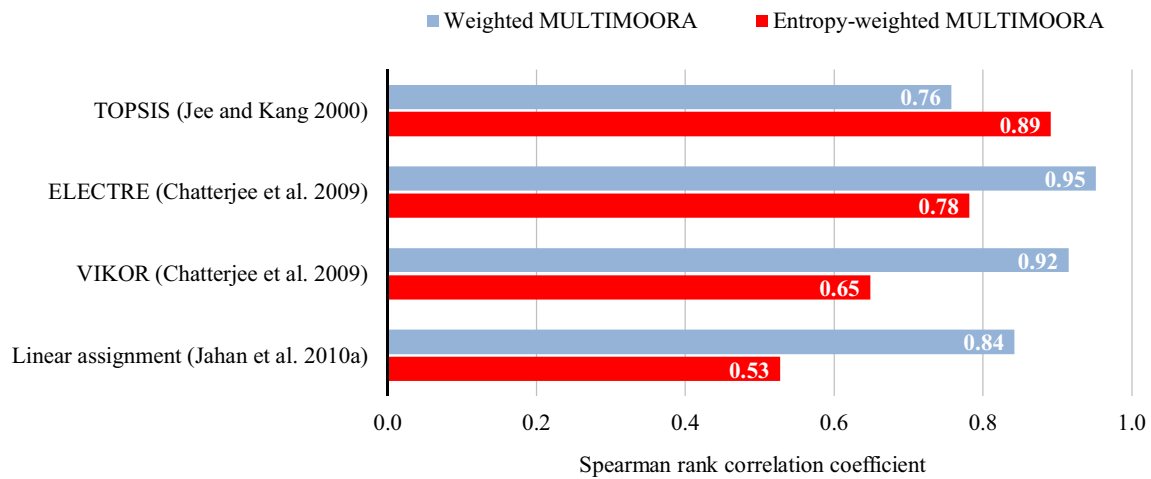
Fig. 1 Correlation between the materials ranks of the proposed model and other methods for Example 1

Table 8 Decision matrix for Example 2 (Manshadi et al. 2007)

Materials	Beneficial and non-beneficial attributes						
	MAX Toughness index (MPa)	MAX Yield strength (MPa)	MAX Elastic modulus (GPa)	MIN Density (g/cm^3)	MIN Thermal expansion (10^{-6}/°C)	MIN Thermal conductivity (cal/s/cm/°C)	MIN Specific heat (cal/g/°C)
Al 2024-T6	75.5	420	74.2	2.80	21.4	0.370	0.16
Al 5052-O	95	91	70	2.68	22.1	0.330	0.16
SS 301-FH	770	1365	189	7.90	16.9	0.040	0.08
SS 310-3AH	187	1120	210	7.90	14.4	0.030	0.08
Ti–6Al–4V	179	875	112	4.43	9.4	0.016	0.09
Inconel 718	239	1190	217	8.51	11.5	0.310	0.07
70Cu–30Zn	273	200	112	8.53	19.9	0.290	0.06

coefficient, thermal conductivity, and specific heat are the non-beneficial attributes. The arrays of the decision matrix were normalized as revealed in Table 9.

Table 10 indicates the values of entropy measure and weights for Example 2. Subjective weights are allocated based on the study of Manshadi et al. (2007). The last row belongs to the general entropy weight w_j^*.

Tables 11 and 12 exhibit the assessment values related to two scenarios of the proposed model and their resultant rankings for Example 2. The final ranks obtained for the materials selection problem of the nitrogen storage tank are presented in the end columns of Tables 11 and 12. Comparison of the subordinate and final ranks reveals that the optimal material A^* is identical ($A_{WMF}^* = A_{EWMF}^* =$ SS 301-FH). Tables 11 and 12 show a nearly identical final ranking except for SS 310-3AH and Inconel 718 that have different standings in the weighted and entropy-weighted MULTIMOORA scenarios. Al 5052-O is the worst option for selection in both scenarios.

The final ranks of the weighted and entropy-weighted MULTIMOORA methods for Example 2 were compared with those of the related studies in Table 13. The best material is SS 301-FH in all approaches. The weighted MULTIMOORA ranking is exactly similar to the fuzzy logic (Khabbaz et al. 2009) and the Z-transformation (Fayazbakhsh et al. 2009) rank lists. The GTMA (Rao 2006) ranking shows a direct correspondence with the entropy-weighted MULTIMOORA method.

Figure 2 demonstrates Spearman rank correlation coefficients for Example 2. In the weighted MULTIMOORA method category, the fuzzy logic (Khabbaz et al. 2009) and the Z-transformation (Fayazbakhsh et al. 2009), by Spearman coefficient 1, are exactly correspondent with our proposed approach. The AHP-TOPSIS (Rao and Davim 2008) and the WPM (Manshadi et al. 2007) have the lowest Spearman coefficient value that is 0.89. In the entropy-weighted MULTIMOORA method category, the best value, 1, is for the GTMA (Rao 2006). In this group, the

Table 9 Normalized decision matrix for Example 2

Materials	Beneficial and non-beneficial attributes						
	MAX Toughness index	MAX Yield strength	MAX Elastic modulus	MIN Density	MIN Thermal expansion	MIN Thermal conductivity	MIN Specific heat
Al 2024-T6	0.084	0.179	0.184	0.160	0.472	0.565	0.564
Al 5052-O	0.106	0.039	0.174	0.154	0.487	0.504	0.564
SS 301-FH	0.858	0.581	0.469	0.453	0.373	0.061	0.282
SS 310-3AH	0.208	0.477	0.521	0.453	0.318	0.046	0.282
Ti–6Al–4V	0.199	0.372	0.278	0.254	0.207	0.024	0.317
Inconel 718	0.266	0.506	0.538	0.488	0.254	0.473	0.247
70Cu–30Zn	0.304	0.085	0.278	0.489	0.439	0.443	0.211

Table 10 Entropy measure and weighting factors for Example 2

Entropy and weights	Beneficial and non-beneficial attributes						
	MAX Toughness index	MAX Yield strength	MAX Elastic modulus	MIN Density	MIN Thermal expansion	MIN Thermal conductivity	MIN Specific heat
s_j	0.28	0.14	0.05	0.24	0.19	0.05	0.05
E_j	0.855	0.879	0.954	0.953	0.979	0.819	0.964
w_j	0.404	0.338	0.127	0.132	0.058	0.505	0.101
w_j^*	0.571	0.238	0.032	0.159	0.055	0.127	0.026

Table 11 Assessment values and rankings of the weighted MULTIMOORA method for Example 2

Materials	Assessment values			Rankings			
	y_i^w	z_i^w	U_i^w	y_i^w Rank	z_i^w Rank	U_i^w Rank	Final Rank
Al 2024-T6	−0.127	0.217	0.684	6	7	6	6
Al 5052-O	−0.139	0.210	0.593	7	6	7	7
SS 301-FH	0.148	0.072	1.528	1	1	1	1
SS 310-3AH	−0.034	0.182	1.051	4	4	3	4
Ti–6Al–4V	0.004	0.184	1.243	2	5	2	2
Inconel 718	−0.029	0.166	1.045	3	3	4	3
70Cu–30Zn	−0.122	0.155	0.744	5	2	5	5

Table 12 Assessment values and rankings of the entropy-weighted MULTIMOORA method for Example 2

Materials	Assessment values			Rankings			
	y_i^{ew}	z_i^{ew}	U_i^{ew}	y_i^{ew} Rank	z_i^{ew} Rank	U_i^{ew} Rank	Final Rank
Al 2024-T6	−0.042	0.441	0.233	6	7	6	6
Al 5052-O	−0.055	0.429	0.188	7	6	7	7
SS 301-FH	0.535	0.048	1.388	1	1	1	1
SS 310-3AH	0.146	0.370	0.620	3	4	3	3
Ti–6Al–4V	0.148	0.376	0.677	2	5	2	2
Inconel 718	0.132	0.337	0.540	4	3	4	4
70Cu–30Zn	0.039	0.316	0.366	5	2	5	5

Spearman coefficient for the fuzzy logic (Khabbaz et al. 2009) and the Z-transformation (Fayazbakhsh et al. 2009) is 0.96. The AHP-TOPSIS (Rao and Davim 2008) and the WPM (Manshadi et al. 2007), by 0.86, have the lowest correlation with our results. From Fig. 2, it is found that the weighted MULTIMOORA rank is closer to the results of other studies than that of the entropy-weighted MULTIMOORA method. The reason is that except the GTMA (Rao 2006) and the fuzzy logic (Khabbaz et al. 2009), others have utilized nearly identical subjective weights in the derivation of the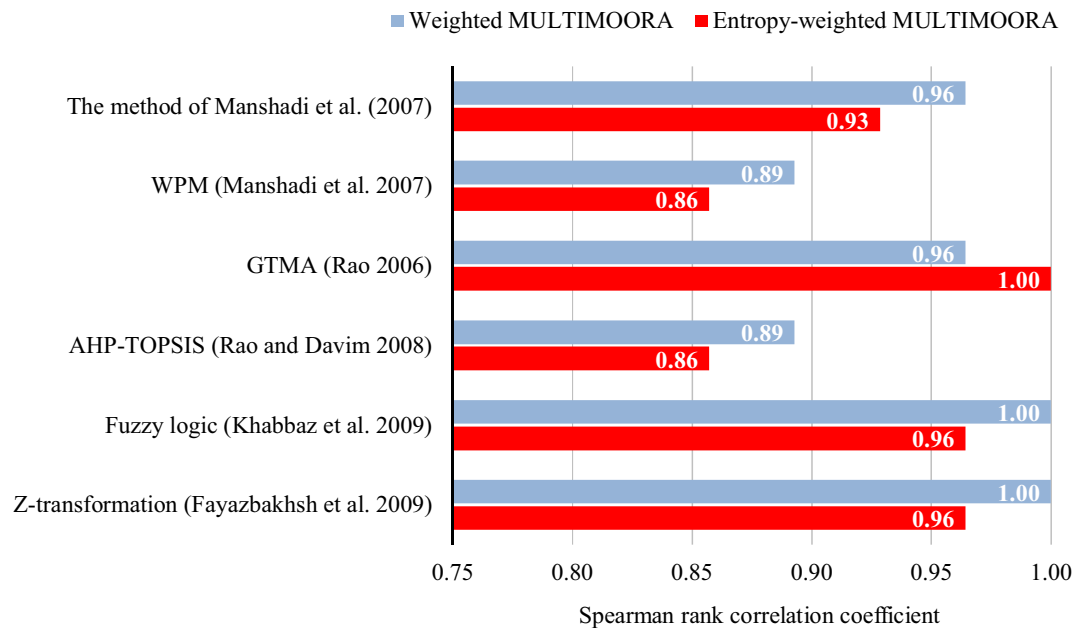ir models. A novel method of assigning subjective weights was employed in the GTMA (Rao 2006). No weighting was considered in the fuzzy logic (Khabbaz et al. 2009).

Conclusion

In the present paper, we extended MULTIMOORA method using entropy weight based on the Shannon information theory to solve materials selection problem. The extended model has two scenarios called the weighted and entropy-

Table 13 Comparison between the materials ranks of the proposed model and other methods for Example 2

Materials	Methods							
	Weighted MULTIMOORA	Entropy-weighted MULTIMOORA	The method of Manshadi et al. (2007)	WPM (Manshadi et al. 2007)	GTMA (Rao 2006)	AHP-TOPSIS (Rao and Davim 2008)	Fuzzy logic (Khabbaz et al. 2009)	Z-transformation (Fayazbakhsh et al. 2009)
Al 2024-T6	6	6	5	5	6	5	6	6
Al 5052-O	7	7	7	6	7	6	7	7
SS 301-FH	1	1	1	1	1	1	1	1
SS 310-3AH	4	3	4	4	3	4	4	4
Ti–6Al–4V	2	2	2	2	2	2	2	2
Inconel 718	3	4	3	3	4	3	3	3
70Cu–30Zn	5	5	6	7	5	7	5	5

Fig. 2 Correlation between the materials ranks of the proposed model and other methods for Example 2

weighted MULTIMOORA methods. To attach relative importance to attributes, subjective weight was considered in the first scenario whereas the combined subjective and objective weights were used in the second scenario. Subjective weight is obtained straight from decision makers' comments based on their knowledge of materials and their experiences of the engineering design process. However, objective weight is calculated using entropy idea. The two forms of weighting factor can be integrated to produce the general form of Shannon entropy weight. Each of the two scenarios has three subordinate parts. To integrate the

subordinate rankings, the dominance theory was exploited. Two practical materials selection examples were discussed to show the effect of the entropy weight on MULTI-MOORA ranking. Moreover, the final rankings of the examples were compared with those of other methods.

The comparison between our final ranks and other studies demonstrates close correspondences, especially over the best rank or the optimal material. Spearman rank correlation coefficients obtained for the two examples show that the correlation between the ranks of the weighted MULTIMOORA method and the most of the earlier studies

is higher than that of the entropy-weighted MULTI-MOORA method. This fact is due to considering subjective weights in the models of the most of the references. Because of readily comprehensible mathematical derivation, the model based on MULTIMOORA method and the entropy concept gives an efficient means for decision making in the field of materials selection. Another strong point of our model is that our final rankings that were calculated by the consolidation of three subordinate ranks are more robust than those of other studies in which a single rank has been reported. The proposed model may have practical limitations in some real-world applications. The data of decision matrix may be presented as uncertain values. In this regard, new developments of the model are required based on fuzzy, interval, green, or other uncertain numbers dependent of the type of vagueness of the data. Moreover, our suggested methodology is to be developed for the case studies in which target-based attributes exist in the decision-making process, such as biomaterials selection problems. If a large number of alternatives and attributes exist in the decision matrix for a practical case, the manual calculation may be exhausting. Thus, the algorithm of this study can be computerized for such cases.

As future research, the extended MULTIMOORA approach can be considered for application in many case studies other than materials selection problem. For instance, decision making over the selection of optimal manufacturing process and the evaluation of failure modes risks can be done using the proposed model. In the field of materials selection, only two typical practical examples were presented in this paper. Other real-world materials selection problems with a number of various alternatives and attributes can be considered. The final rankings of the proposed model for the two examples were compared with a few approaches. The comparison of the present paper results with other MADM methods or expert systems seems to be interesting. As different extensions of the MULTIMOORA method, other concepts for assigning relative importance of attributes can be utilized.

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