Original Research

Multiple Dependent States Repetitive Sampling Control Chart for Monitoring Rayleigh Distributed Data

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Abstract

An attribute control chart for Rayleigh distribution based on the multiple dependent state repetitive sampling (MDSRS) is developed in this paper. The performance of the proposed chart is evaluated in terms of average run length for the design of the proposed chart. Furthermore, the control chart constants for instance test time multiplier; inner and outer control limits coefficients are determined by considering the process in-control average run length (ARL) in support of different sample sizes. The efficiency of the proposed chart is compared with some competing control charts using another sampling method such as single sampling (SS), multiple dependent states (MDS) and repetitive sampling (RS). The application of the proposed chart is illustrated using simulated data, which showed the superiority of the proposed chart as compared to the competing charts. Based on the ARL performance of the proposed chart and the application example, it is recommended that the process engineers use the proposed chart when monitoring the number of failed products that follow the Rayleigh distribution.

Keywords- Attribute control chart; single sampling; Rayleigh distribution; control chart; multiple dependent state, average run length; repetitive sampling; chart coefficients.

INTRODUCTION

Control charts are popular statistical process control apparatus used extensively in manufacturing and non-manufacturing for monitoring of processes to quickly detect shifts. In industrial manufacturing, it helps to develop product corresponding to the specified specifications by monitoring the quality of product from deviation. Basically control charts could be classified as attribute and variable control chart. The variable control charts are used in a process when the quality characteristic is quantifiable, on the other hand attribute control charts are developed for non-measurable data which could be classified as conforming or non- conforming. The rationale behind the design of both charts is to quickly detect shifts in the process. A shift in the process is detected by the control charts and helps the process engineer to quickly rectify the abnormal process situation. The control chart is typically made up of the upper and lower control limits. The process is declared as in-control (IC) if the plotting statistic falls within the upper control limit (UCL) and the lower control limit (LCL), otherwise, it is declared as out-of-control (OOC). Many control charts have been developed to monitor process based on the normal distribution, however, in real-life, some process may follow some non-normal distribution. Hence, monitoring the process using control charts under the assumption of normality may mislead the process engineers if the process variable follows some non-normal distribution.

Productivity and performance are proof of total efficiency of production process and also a subject of maximization (Namdari et al., 2017). Many researchers have developed control charts for non-normal distributions in the statistical process control (SPC) literature (see Amin et al., 1995; McCracken and Chakraborti, 2013; Ahmad et al, 2014; Wu and Wang, 2007; De Araujo Rodrigues et al, 2011; Al-Oraini and Rahim, 2003). More recently, designing of attribute control charts for monitoring process characteristic for some non-normal distributions has attracted the attention of researchers based on truncated life test. Adeoti and Ogundipe (2021) proposed an attribute chart for generalized exponential distribution, Aslam and Jun (2015) and Aslam et al. (2016) designed a control chart based on Weibull distribution and Pareto distribution of second kind under truncated life test. Joekes and Barbosa (2013) proposed an improved control chart for monitoring non-conforming proportion in high quality processes. Aslam et al. (2020) proposed an attribute chart based on multiple dependent state repetitive sampling (MDSRS) for monitoring the lifetime of the product for some non-normal distributions. Farahani et al. (2020) presented a new mixed integer nonlinear programming model to investigate overall equipment effectiveness (OEE) with integrated optimization of preventive maintenance and quality control chart. Adeoti and Rao (2021) developed a control chart for the Rayleigh and inverse Rayleigh distribution. Balamurali and Jeyadurga (2019) proposed economic design of an attribute control chart for monitoring mean life based on multiple deferred state sampling. Jeyadurga et al. (2018) proposed an attribute chart for process monitoring based on repetitive group sampling under truncated life tests. Farahani et al. (2019) have presented an integrated model for optimizing statistical process control policies (sampling interval, sample size and control limit) and preventive maintenance (the preventive maintenance interval). Control chart for non-normal distribution by repetitive sampling and multiple dependent state (MDS) under truncated test was proposed as efficient sampling scheme to improve the run length characteristics of the single sampling control chart by Aslam et al. (2017).

The multiple dependent chart state repetitive sampling was introduced by Aldosari et al (2017) by combining repetitive sampling and MDS scheme. The proposed chart is found more efficient than repetitive sampling and MDS in decreasing the ARL values. Quality of labor or inspector productivity has the potential to contribute to the development of inspection performance and efficiency (Namdari et al., 2017). Aldosari et al. (2019) proposed the MDSRS for multivariate Poisson distribution. Aslam et al. (2020) designed MDSRS chart for the Birnbaum-Saunders distribution. Adeoti and Rao (2022) proposed control chart for Rayleigh distribution using repetitive sampling under truncated life test. The Rayleigh distribution is an important statistical skewed distribution useful in reliability engineering for modelling lifetime of objects. It has been studied by many researchers to model respiratory signals (Li and Li, 2015) and to fit the data of signal voltage (Mutlu, 2014). It is used as an appropriate model when the normality assumption is not satisfied. By exploring the SPC literature, no work on MDSRS for Rayleigh distribution exist, therefore, this paper presents the MDSRS chart to monitor the Rayleigh quality characteristic in reliability engineering. The design structure of the control chart and comparative study is presented in this paper. The rest of this paper is organized as follows: The design of the proposed control chart is given in Section 2. Performance evaluations of proposed control chart are given in Section 3. A comparative study is given in Section 4 and simulation study is presented in Section 5. Finally, some conclusions are offered in Section 6.

I. Design of Proposed Control Chart

In this section, the design of the proposed attribute control chart for the Rayleigh distribution using the multiple dependent state repetitive sampling (MDSRS) is provided. Suppose that the lifetime of the product, denoted T follows the Rayleigh distribution with scale parameter θ. The probability density function (PDF) and cumulative distribution function (CDF) of the Rayleigh distribution is given by

$$f(t,\theta) = \frac{t}{\theta^2} e^{-(t^2/2\theta^2)} \qquad t > 0, \theta > 0 \qquad (1)$$

and

$$F(t,\theta) = 1 - e^{-(t^2/2\theta^2)} \qquad t > 0, \theta > 0 \qquad (2)$$

The mean life of a product that follows the Rayleigh distribution is given as

$$E(T) = \mu = \theta\sqrt{\frac{\pi}{2}} \qquad (3)$$

Let $t_0 = a\mu_0$ be the truncated time when the process is in-control, where a is the truncated constant and μ_0 is the specified mean life. The target mean for the in-control process will be

$$\mu_0 = \theta_0\sqrt{\frac{\pi}{2}}. \qquad (4)$$

The probability of a failed product denoted p_0 by time t_0 where $t_0 = a\mu_0$ for IC process is expressed as

$$p_0 = P(T < t|\theta_0) = 1 - e^{-(t_0^2/2\theta^2)} = 1 - e^{-(a^2\pi/4)}. \qquad (5)$$

The proposed attribute control chart for the Rayleigh distribution using MDSRS under time truncated life test is given in the following steps as follows:

Step 1: Select a random sample of size *n* at each subgroup. Conduct a time truncated life test on the selected samples at pre-assigned time t_0 and record the number of failed product denoted D by time t_0

Step 2: The process is declared as in-control if $LCL_2 \leq D \leq UCL_2$ for control limit coefficients L_1 and L_2 ($L_1 > L_2$). Declare the process as out-of-control if $D > UCL_1$ or $D < LCL_1$. Otherwise go to step 3

Step 3: The process is considered as in-control if *m* preceding subgroup have been in-control state (i.e. $UCL_2 \leq D \leq UCL_1$ or $LCL_1 \leq D \leq LCL_2$). Otherwise repeat step 1.

The control limits of the proposed control chart for Rayleigh distribution using the MDSRS scheme is given as

$$UCL_1 = np_0 + L_1\sqrt{np_0(1 - p_0)} \qquad (6a)$$

$$LCL_1 = \max\left[0, np_0 - L_1\sqrt{np_0(1 - p_0)}\right] \qquad (6b)$$

$$UCL_2 = np_0 + L_2\sqrt{np_0(1 - p_0)} \qquad (6c)$$

$$LCL_2 = \max\left[0, np_0 - L_2\sqrt{np_0(1 - p_0)}\right]. \qquad (6d)$$

The proposed control chart is an *np* chart with parameters n and p_0, where p_0 is the probability of a failed product before time t_0 for an in-control process. Note that UCL_1, UCL_2, LCL_1 and LCL_2 are the outer and inner control limits that must be computed using data when the process is in-control and m is any value ($m \geq 1$) that can be specified by the process engineers, L_1 and L_2 are the control limit coefficients that are determined to achieve a pre-assigned in-control average run length (IC ARL) value.

According to Aldosari et al. (2019), "the MDSRS have advantages over the MDS and repetitive sampling (RS) as it allows process repetition if decision about the condition of the process cannot be achieve with *m* preceding subgroups". Thus, it is envisaged that the proposed control chart for Rayleigh distribution using MDSRS will perform better than single sampling (SS), MDS or RS chart. Note that the proposed control chart becomes the single sampling attribute control chart for Rayleigh distribution when $L_1 = L_2$ and $UCL_1 = UCL_2$. The proposed chart becomes the Rayleigh attribute chart using MDS when probability of repetition is zero and becomes the attribute control chart for Rayleigh distribution based on RS scheme when $m = 0$.

When the probability of a failed product before t_0 for an in-control process is unknown to the process engineers, then the control limits of the number of failed products based on the average number of failed product (denoted D) can be used for process monitoring is given as

$$UCL_1 = \overline{D} + L_1\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)} \tag{7a}$$

$$LCL_1 = max\left[0, \overline{D} - L_1\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)}\right]. \tag{7b}$$

$$UCL_2 = \overline{D} + L_2\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)} \tag{7c}$$

$$LCL_2 = max\left[0, \overline{D} - L_2\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)}\right]. \tag{7d}$$

where $\overline{D}$ denotes the average number of failed product obtained from the samples taken when the process is IC. Next, the performance of the proposed control chart is investigated in terms of ARL using the control limits in Equation (6). Firstly, the probability of declaring the process to be in-control at probability p_0 on the basis of single subgroup and MDS is given as

$$p_{in,0}^{(0)} = P(LCL_2 \le D \le UCL_2) + \{P(LCL_1 \le D \le LCL_2) + P(UCL_2 \le D \le UCL_1)\}\big(P(LCL_2 \le D \le UCL_2)\big)^m.$$

or

$$p_{in,0}^{(0)} = \sum_{d=LCL_2+1}^{UCL_2}\binom{n}{d}p_0^d(1-p_0)^{n-d} + \left\{\begin{matrix}\sum_{d=LCL_1+1}^{LCL_2}\binom{n}{d}p_0^d(1-p_0)^{n-d} + \\ \sum_{d=UCL_2+1}^{UCL_1}\binom{n}{d}p_0^d(1-p_0)^{n-d}\end{matrix}\right\}. \left(\sum_{d=LCL_2+1}^{UCL_2}\binom{n}{d}p_0^d(1-p_0)^{n-d}\right)^m \tag{8}$$

The probability of repeated (repetitive) sampling is given as

$$p_{rep}^{(0)} = \{P(LCL_1 \le D \le LCL_2) + P(UCL_2 \le D \le UCL_1)\}\left\{1 - \big(P(LCL_2 \le D \le UCL_2)\big)^m\right\}.$$

or

$$p_{rep}^{(0)} = \left\{\sum_{d=LCL_1+1}^{LCL_2}\binom{n}{d}p_0^d(1-p_0)^{n-d} + \sum_{d=UCL_2+1}^{UCL_1}\binom{n}{d}p_0^d(1-p_0)^{n-d}\right\} \left\{1 - \left(\sum_{d=LCL_2+1}^{UCL_2}\binom{n}{d}p_0^d(1-p_0)^{n-d}\right)^m\right\}. \tag{9}$$

Thus, the IC probability of the proposed attribute Rayleigh chart based on MDSRS when it is actually in-control is given as

$$p_{in}^{(0)} = \frac{p_{in,0}^{(0)}}{1-p_{rep}^{(0)}}. \tag{10}$$

Now, suppose that there is a shift in the scale parameter from θ_0 to $\theta_1 = c\theta_0$ where c is a constant. The probability of a failed product before t_0 when the process is shifted to a new scale parameter is given as

$$p_1 = 1 - e^{-(a^2\pi f^2/4)}. \tag{11}$$

Therefore, the probability that the process is considered to be in-control on basis of single subgroup and MDS when the process is shifted is given as

$$p_{in,1}^{(1)} = \frac{\sum_{d=LCL_2+1}^{UCL_2} \binom{n}{d} p_1^d (1-p_1)^{n-d} + \begin{Bmatrix} \sum_{d=LCL_1+1}^{LCL_2} \binom{n}{d} p_1^d (1-p_1)^{n-d} + \\ \sum_{d=UCL_2+1}^{UCL_1} \binom{n}{d} p_1^d (1-p_1)^{n-d} \end{Bmatrix}}{\left(\sum_{d=LCL_2+1}^{UCL_2} \binom{n}{d} p_1^d (1-p_1)^{n-d}\right)^m}. \qquad (12)$$

The probability of repetitive sampling for the shifted process is

$$p_{rep}^{(1)} = \left\{ \sum_{d=LCL_1+1}^{LCL_2} \binom{n}{d} p_1^d (1-p_1)^{n-d} + \sum_{d=UCL_2+1}^{UCL_1} \binom{n}{d} p_{10}^d (1-p_1)^{n-d} \right\}$$
$$\left\{1 - \left(\sum_{d=LCL_2+1}^{UCL_2} \binom{n}{d} p_1^d (1-p_1)^{n-d}\right)^m\right\}. \qquad (13)$$

The probability that the process is declared as in-control when the process is shifted to θ_1 for Rayleigh distribution based on MDSRS is given as

$$p_{in}^{(1)} = \frac{p_{in,1}^{(1)}}{1-p_{rep}^{(1)}}. \qquad (14)$$

II. Performance Evaluation of Proposed Control Chart

The performance of the proposed control chart for Rayleigh distribution using MDSRS technique is evaluated based on the average run length (ARL). The ARL is the average number of sample to first signal OOC. The in-control ARL (ARL_0) of the proposed chart is given as

$$ARL_0 = \frac{1}{1-p_{in}^{(0)}}. \qquad (15)$$

The out-of-control ARL (ARL_1) and standard deviation run length (SDRL) of the proposed chart is given as

$$ARL_1 = \frac{1}{1-p_{in}^{(1)}}. \qquad (16)$$

$$SDRL_1 = \frac{\sqrt{1-p_{in}^{(1)}}}{p_{in}^{(1)}}. \qquad (17)$$

Tables 1-4 display the ARL_1 and $SDRL_1$ values of the proposed control chart for sample sizes *n*=20 and 30 and target IC ARL (R_0)= 300 and 370 and $m = 1,2,3,4$. The step-by-step procedure to compute L_1 and L_2 and obtain the ARL in Tables 1-4 is given as follows:

i. Specify the target IC ARL (R_0), *m* and *n*.

ii. Determine the value of L_1 and L_2 and truncated constant a for which $ARL_0 \geq R_0$ where R_0 is the pre-specified ARL value.

iii. Compute the ARL_1 values using the values of parameters L_1 and L_2 and a in (ii) for different shifts sizes ranging from 1 to 2.

From Tables 1-4, the following are the summary of the control chart performance.

i. The ARL values increases as the values of *m* increases for fixed sample sizes *n* and shift size *f*.

ii. For small shifts in process parameter, the ARL values decreases as *n* increases.

iii. For fixed values of f and m, the proposed chart is more sensitive for large sample sizes as the ARL value decreases for large sample size.

TABLE 1
ARL AND ASN VALUES OF THE PROPOSED CONTROL AT R_0=300, N=20

f	L_1=3.007	L_2=1.621	L_1=3.006	L_2=1.213	L_1=2.957	L_2=1.613	L_1=3.281	L_2=1.067
	a=0.773 m=1		a=0.892 m=2		a=0.899 m=3		a=0.688 m=4	
	$LCL_1 = 0$	$UCL_1 = 13$	$LCL_1 = 2$	$UCL_1 = 15$	$LCL_1 = 2$	$UCL_1 = 15$	$LCL_1 = 0$	$UCL_1 = 12$
	$LCL_2 = 3$	$UCL_2 = 10$	$LCL_2 = 6$	$UCL_2 = 11$	$LCL_2 = 5$	$UCL_2 = 12$	$LCL_2 = 4$	$UCL_2 = 8$
	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$
1.00	300.39	299.89	300.15	299.65	300.45	299.95	300.03	299.53
1.02	217.64	217.14	223.91	231.41	227.83	227.33	244.84	244.34
1.04	158.71	158.21	160.71	170.21	167.10	166.60	190.94	190.43
1.06	116.79	116.29	119.96	122.46	121.11	120.61	145.13	144.63
1.08	86.80	86.30	87.10	87.40	87.68	87.18	108.93	108.43
1.10	61.16	64.66	62.81	62.31	63.71	63.21	81.35	80.85
1.12	42.39	48.88	44.99	44.48	46.54	46.04	60.72	60.22
1.14	31.77	37.27	32.35	31.84	34.21	33.71	45.41	44.91
1.16	23.14	28.63	24.37	22.87	25.31	24.80	34.08	33.58
1.18	24.65	22.15	16.98	16.47	18.85	18.34	25.70	25.19
1.20	11.75	17.24	12.43	11.92	14.14	13.63	19.48	18.98
1.25	4.95	9.44	5.95	5.43	7.19	6.67	10.07	9.56
1.30	2.85	5.32	3.16	2.61	3.96	3.43	5.54	5.01
1.35	1.63	3.09	1.95	1.36	2.45	1.88	3.31	2.77
1.40	1.22	1.86	1.42	0.77	1.72	1.11	2.20	1.62
1.50	1.04	0.75	1.09	0.30	1.18	0.46	1.33	0.66
1.60	1.01	0.35	1.02	0.13	1.05	0.22	1.09	0.32
1.70	1.00	0.17	1.00	0.06	1.01	0.11	1.03	0.16
1.80	1.00	0.08	1.00	0.02	1.00	0.05	1.01	0.08
1.90	1.00	0.04	1.00	0.01	1.00	0.02	1.00	0.04
2.00	1.00	0.02	1.00	0.01	1.00	0.01	1.00	0.02

TABLE 2
ARL AND ASN VALUES OF THE PROPOSED CONTROL AT R_0=300, N=30

f	$L_1 = 2.96$	$L_2 = 1.485$	$L_1 = 3.105$	$L_2 = 2.351$	$L_1 = 3.061$	$L_2 = 1.963$	$L_1 = 3.191$	$L_1 = 2.503$
	a=0.849 m=1		a=0.61 m=2		a=0.722 m=3		a=0.568 m=4	
	$LCL_1 = 4$	$UCL_1 = 20$	$LCL_1 = 0$	$UCL_1 = 14$	$LCL_1 = 2$	$UCL_1 = 17$	$LCL_1 = 0$	$UCL_1 = 13$
	$LCL_2 = 8$	$UCL_2 = 16$	$LCL_2 = 2$	$UCL_2 = 13$	$LCL_2 = 5$	$UCL_2 = 15$	$LCL_2 = 1$	$UCL_2 = 12$
	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$
1.00	300.49	299.99	300.48	299.98	300.38	299.88	300.18	299.68
1.02	213.39	212.89	216.44	215.94	224.31	223.81	230.04	229.54
1.04	146.58	146.08	156.45	155.95	161.63	161.12	172.96	172.46
1.06	100.21	99.71	114.12	113.62	115.38	114.88	129.41	128.91
1.08	68.99	68.49	84.22	83.71	86.70	82.20	97.13	96.63
1.10	48.02	47.52	62.93	62.43	69.87	59.37	73.45	72.94
1.12	33.83	33.32	47.63	47.13	48.86	43.35	56.06	55.56
1.14	24.11	23.60	30.51	36.01	32.53	32.02	43.24	42.73
1.16	17.37	16.87	22.33	27.83	24.42	23.91	33.69	33.19
1.18	12.66	12.15	17.24	21.74	18.54	18.03	26.53	26.03
1.20	9.33	8.82	12.67	17.16	14.23	13.72	21.10	20.59
1.25	4.61	4.08	6.40	9.89	7.70	7.18	12.41	11.90
1.30	2.56	2.00	3.51	5.99	4.47	3.94	7.71	7.20
1.35	1.66	1.04	2.32	3.79	2.83	2.28	5.06	4.53
1.40	1.27	0.59	2.03	2.48	2.08	1.39	3.51	2.96
1.50	1.04	0.21	1.18	1.18	1.29	0.61	2.00	1.42
1.60	1.01	0.08	1.10	0.62	1.19	0.30	1.41	0.76
1.70	1.00	0.03	1.11	0.35	1.12	0.15	1.17	0.44
1.80	1.00	0.01	1.04	0.20	1.05	0.07	1.06	0.26
1.90	1.00	0.01	1.01	0.11	1.00	0.03	1.02	0.15
2.00	1.00	0.01	1.00	0.06	1.00	0.01	1.01	0.08

TABLE 3
ARL AND ASN VALUES OF THE PROPOSED CONTROL AT R_0=370, N=20

f	$L_1 = 3.09$	$L_2 = 1.698$	$L_1 = 3.269$	$L_2 = 0.927$	$L_1 = 3.244$	$L_2 = 1.575$	$L_1 = 3.107$	$L_1 = 1.765$
	a=0.763 *m*=1		*a*=0.742 *m*=2		*a*=0.692 *m*=3		*a*=0.943 *m*=4	
	$LCL_1 = 0$	$UCL_1 = 13$	$LCL_1 = 0$	$UCL_1 = 13$	$LCL_1 = 0$	$UCL_1 = 12$	$LCL_1 = 3$	$UCL_1 = 16$
	$LCL_2 = 3$	$UCL_2 = 10$	$LCL_2 = 5$	$UCL_2 = 8$	$LCL_2 = 3$	$UCL_2 = 9$	$LCL_2 = 6$	$UCL_2 = 13$
	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$
1.00	370.30	369.80	370.28	369.78	370.40	369.89	370.14	369.64
1.02	269.08	268.58	273.61	273.11	296.17	295.67	335.78	335.28
1.04	196.10	195.60	198.20	197.70	228.77	228.27	271.95	271.45
1.06	141.98	143.47	142.41	141.91	173.76	173.26	205.66	205.16
1.08	106.69	106.19	109.11	101.61	131.20	130.70	150.48	149.98
1.10	79.83	79.33	80.30	72.79	99.08	98.58	108.77	108.27
1.12	60.31	59.81	62.74	52.24	75.07	74.57	78.46	77.96
1.14	45.99	45.49	48.08	37.58	57.14	56.64	58.76	56.25
1.16	35.37	34.87	37.61	27.11	43.72	43.22	44.25	40.75
1.18	27.43	26.93	29.12	19.61	33.63	33.13	34.16	29.65
1.20	21.44	20.93	24.75	14.25	26.01	25.51	27.20	21.69
1.25	11.94	11.43	12.06	6.54	14.03	13.52	14.71	10.20
1.30	3.95	6.43	4.69	3.15	5.88	7.37	6.56	5.04
1.35	4.06	3.73	4.21	1.63	4.69	4.16	5.20	2.65
1.40	1.78	2.23	1.95	0.92	2.06	2.46	2.08	1.50
1.50	1.02	0.89	1.11	0.36	1.19	1.00	1.27	0.59
1.60	1.01	0.40	1.02	0.16	1.03	0.49	1.07	0.27
1.70	1.00	0.20	1.01	0.07	1.01	0.26	1.02	0.13
1.80	1.00	0.10	1.00	0.03	1.00	0.14	1.00	0.06
1.90	1.00	0.05	1.00	0.01	1.00	0.08	1.00	0.03
2.00	1.00	0.02	1.00	0.01	1.00	0.04	1.00	0.01

TABLE 4
ARL AND ASN VALUES OF THE PROPOSED CONTROL AT R_0=370, N=30

f	$L_1 = 3.066$	$L_2 = 1.548$	$L_1 = 3.141$	$L_2 = 1.950$	$L_1 = 3.148$	$L_2 = 1.274$	$L_1 = 3.061$	$L_1 = 1.715$
	a=0.759 m=1		a=0.678 m=2		a=0.789 m=3		a=0.865 m=4	
	$LCL_1 = 2$	$UCL_1 = 18$	$LCL_1 = 1$	$UCL_1 = 16$	$LCL_1 = 3$	$UCL_1 = 19$	$LCL_1 = 5$	$UCL_1 = 21$
	$LCL_2 = 6$	$UCL_2 = 14$	$LCL_2 = 4$	$UCL_2 = 13$	$LCL_2 = 8$	$UCL_2 = 14$	$LCL_2 = 8$	$UCL_2 = 17$
	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$
1.00	370.28	369.78	370.29	369.79	370.37	369.87	370.11	369.61
1.02	255.98	255.47	266.08	265.58	274.45	273.95	333.50	333.00
1.04	175.55	175.05	188.57	188.07	190.30	189.80	256.77	256.27
1.06	121.05	120.55	133.78	133.28	148.44	127.94	180.99	180.49
1.08	84.36	83.86	95.65	95.15	98.04	85.54	123.00	122.50
1.10	59.51	59.01	69.10	68.60	77.68	57.18	82.75	82.25
1.12	42.49	41.99	50.47	49.97	51.85	38.34	55.75	55.25
1.14	30.69	30.19	35.26	36.76	36.34	25.83	37.79	37.29
1.16	22.40	21.90	24.79	27.28	25.01	17.50	25.84	25.33
1.18	16.53	16.02	16.92	20.41	17.46	11.95	17.85	17.34
1.20	11.31	11.80	11.89	15.39	12.05	8.23	12.50	11.99
1.25	6.09	5.66	6.32	7.80	6.97	3.43	7.53	5.01
1.30	2.38	2.84	2.65	4.12	2.77	1.59	2.86	2.30
1.35	1.28	1.50	1.33	2.27	1.47	0.83	1.78	1.18
1.40	1.48	0.85	1.92	1.33	1.19	0.48	1.33	0.67
1.50	1.09	0.31	1.11	0.54	1.13	0.18	1.16	0.25
1.60	1.02	0.13	1.03	0.26	1.04	0.07	1.05	0.10
1.70	1.00	0.05	1.00	0.12	1.00	0.03	1.00	0.04
1.80	1.00	0.02	1.00	0.06	1.00	0.01	1.00	0.01
1.90	1.00	0.01	1.00	0.03	1.00	0.01	1.00	0.01
2.00	1.00	0.01	1.00	0.01	1.00	0.01	1.00	0.01

COMPARATIVE STUDY

In this section, the efficiency of the proposed chart over the existing attribute charts in terms of ARL_1 is compared for the single sampling, repetitive sampling and MDS sampling. The ARL_1 values of the repetitive sampling is taken from Adeoti and Rao (2022). Though, control charts for Rayleigh distribution based on SS and MDS sampling does not exist yet in the SPC literature; however, it is included for comparative study. The ARL_1 values of the proposed control chart and the competing charts are presented in Tables 5 and 6 for $ARL_0 = 300$ and 370, $n = 20{,}30$ and $m = 3$. Figure 1 shows the comparison of the proposed chart, SS, RS and MDS sampling when $ARL_0 = 370, n = 20$ and $m = 3$. From Tables 5 and 6, we observed that the proposed chart outperforms the SS, RS, MDS control charts; as the ARL values of the proposed chart decreases significantly when compared to the competing charts. For example, if *f*=1.40 the ARL values are 4.12, 5.76, 2.55 and 1.78 for RS, SS, MDS and MDSRS control chart when *n*=20. It can be seen that the control chart using MDSRS scheme performed better than the competing charts. In Figures 1 is represents the graphical representation of proposed MDSRS control chart performance over the existing MDS, RS and SS type control charts along with various shift values for ARL=370. From this figure it is clear that the proposed Rayleigh control chart based on MDSRS control chart declares more susceptible as compared to the MDS, RS and SS type control charts.

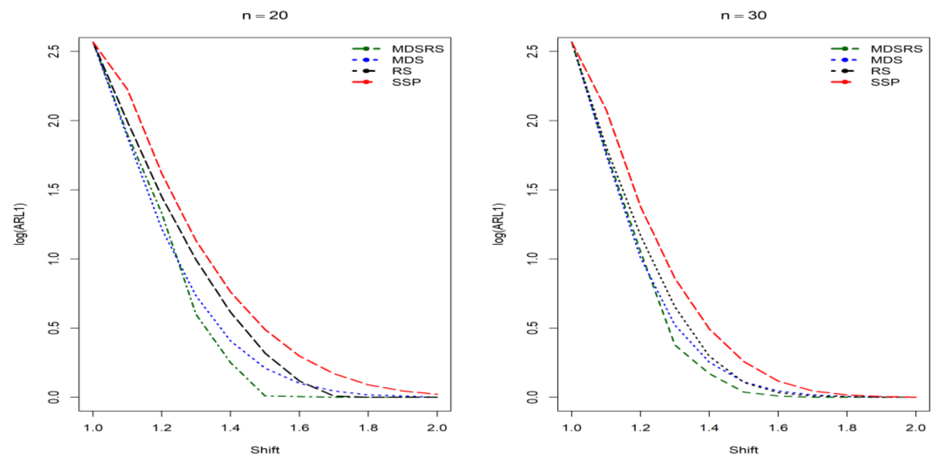

FIGURE 1
ARL CURVES OF RAYLEIGH CONTROL CHART FOR FOUR TYPES OF CONTROL CHARTS FOR ARL0=370

TABLE 5
ARL COMPARISON OF RAYLEIGH CONTROL CHART USING SS, RS, MDS AND MDSRS FOR $ARL_0 = 370$

Shift *f*	*n*=20				*n*=30			
	RS	SS	MDS	MDSRS	RS	SS	MDS	MDSRS
1.00	370.47	370.20	370.47	370.30	371.05	370.10	370.31	370.28
1.10	98.55	168.56	76.11	79.83	64.71	120.01	56.82	59.51
1.20	28.12	41.44	16.51	21.44	14.76	23.94	10.22	11.31
1.30	9.82	13.51	5.39	3.95	4.50	7.22	3.32	2.38
1.40	4.12	5.76	2.55	1.78	1.98	3.12	1.79	1.48
1.50	2.08	3.08	1.62	1.02	1.28	1.81	1.29	1.09
1.60	1.31	1.98	1.26	1.01	1.08	1.31	1.11	1.02
1.70	1.02	1.48	1.11	1.00	1.02	1.11	1.04	1.00
1.80	1.00	1.23	1.04	1.00	1.01	1.04	1.01	1.00
1.90	1.00	1.11	1.02	1.00	1.00	1.01	1.00	1.00
2.00	1.00	1.05	1.00	1.00	1.00	1.00	1.00	1.00

I. Simulation Study

The efficiency of the proposed control chart is evaluated against three competing control charts such as the single sampling (SS), repetitive sampling (RS) and multiple dependent sampling (MDS) based on simulated data. The procedure describe below is employed for simulation process:

i. Suppose that the IC process parameter of the Rayleigh distribution is given as $\theta_0 = 1$. Generate the first 20 samples each of size *n*=20.

ii. Assume that the process shifted to OOC situation with *f*=1.5, i.e. $\theta_1 = f\theta_0 = 1.5(1) = 1.5$. Generate additional 10 samples each of size *n*=20 from the shifted process

iii. Compute the four control limits based on the simulated data. Plot the different statistics for SS, RS, MDS, and MDSRS against the control limits

Figures 2-5 display the control chart limits for each of the 30 samples. From Figures 2-5, we observed that the Rayleigh control chart using the SS, MDS and RS chart did not detect the shifts in the subgroup; however, using the MDSRS control chart detect the shifts at the 14th sample. Therefore, we conclude that the Rayleigh control chart using the MDSRS is more sensitive as compared to the existing charts.

TABLE 6
ARL COMPARISON OF RAYLEIGH CONTROL CHART USING SS, RS, MDS AND MDSRS FOR $ARL_0 = 300$

Shift *f*	*n*=20				*n*=30			
	RS	SS	MDS	MDSRS	RS	SS	MDS	MDSRS
1.00	300.15	300.27	301.62	300.39	300.27	300.52	300.62	300.49
1.10	58.26	77.09	124.75	61.16	69.21	57.20	56.81	48.02
1.20	13.05	23.76	28.83	11.75	17.00	15.19	9.86	9.33
1.30	3.79	9.36	8.65	2.85	5.53	5.65	3.16	2.56
1.40	1.68	4.57	3.68	1.22	2.44	2.79	1.70	1.27
1.50	1.17	2.68	2.11	1.04	1.48	1.75	1.24	1.04
1.60	1.04	1.83	1.51	1.01	1.16	1.31	1.09	1.01
1.70	1.01	1.41	1.24	1.00	1.05	1.12	1.03	1.00
1.80	1.00	1.20	1.11	1.00	1.01	1.04	1.01	1.00
1.90	1.00	1.09	1.05	1.00	1.00	1.01	1.00	1.00
2.00	1.00	1.04	1.02	1.00	1.00	1.00	1.00	1.00

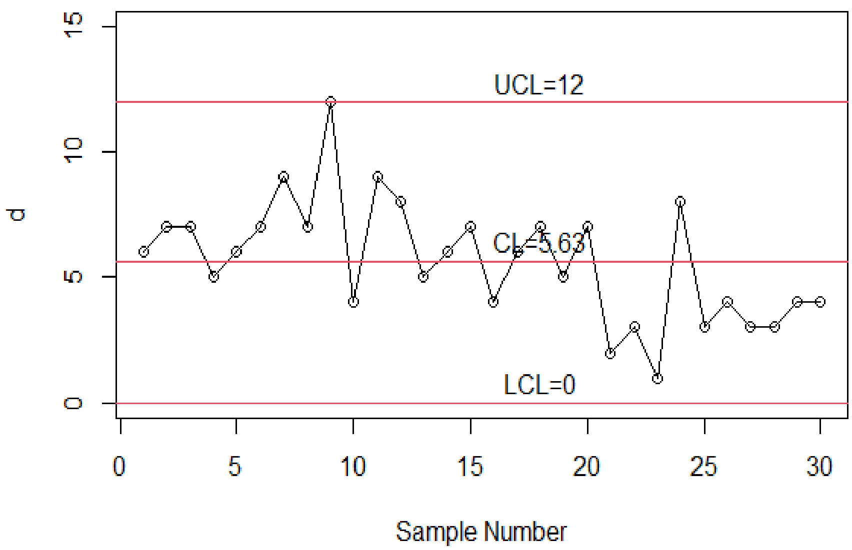

FIGURE 2
CONTROL CHART FOR THE SIMULATED EXAMPLE BASED ON THE SS METHOD

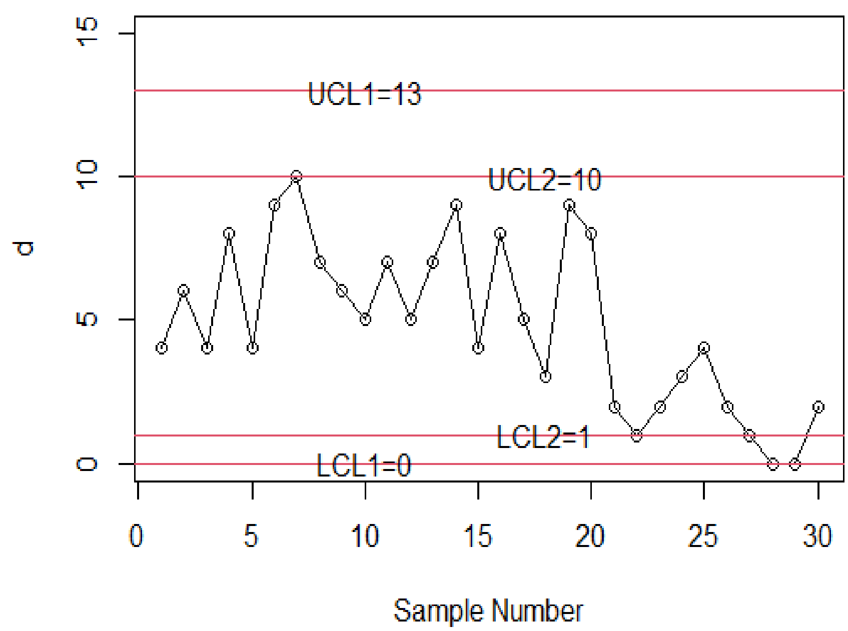

FIGURE 3
CONTROL CHART FOR THE SIMULATED EXAMPLE BASED ON THE MDS METHOD

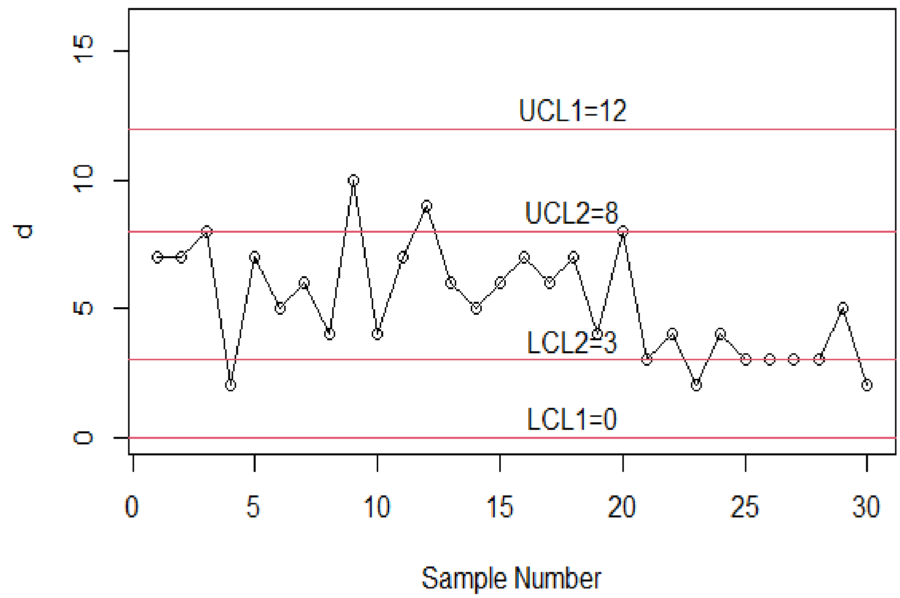

FIGURE 4
CONTROL CHART FOR THE SIMULATED EXAMPLE BASED ON THE RS METHOD

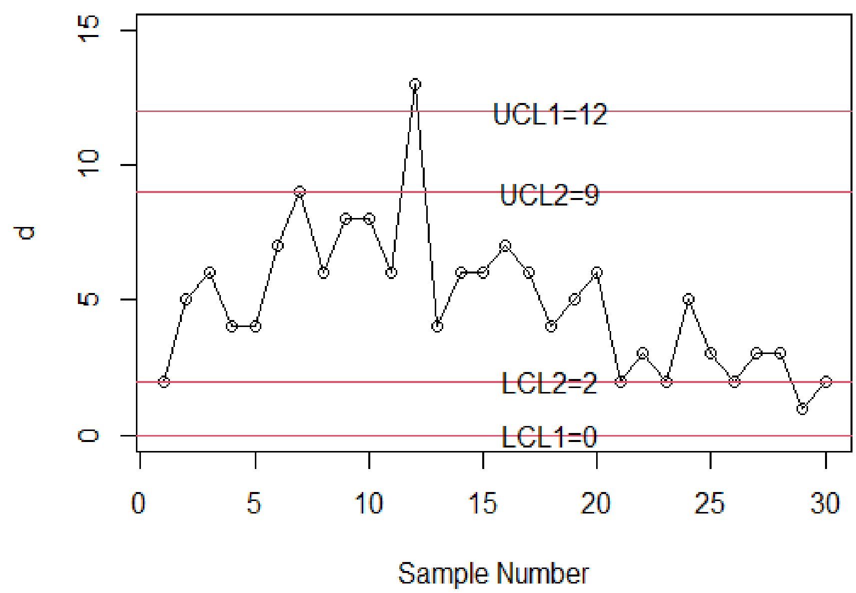

FIGURE 5
CONTROL CHART FOR THE SIMULATED EXAMPLE BASED ON THE MDSRS METHOD

CONCLUSION

In this paper, attribute control chart for Rayleigh distribution based on the MDSRS is proposed. The design of the proposed chart is given and the improvisation of the proposed chart is attributed with respect to ARLs in the paper. The improvisation of the developed chart is examined with some competing control chart using other sampling method such as SS, MDS and RS. The application of the proposed chart is demonstrated using a simulated data example which showed the superiority of the proposed chart as compared to the competing charts. Based on the ARL performance of the proposed chart and the application example, it is recommended that the process engineers to use the proposed chart when monitoring the number of failed products that follow the Rayleigh distribution.

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