

An availability-based design optimization by using a fuzzy goal programming approach

Zahra Sobhani¹ · Mahmoud Shahrokhi^{2*} · Alain Bernard³

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* Corresponding Author, Shahrokhi292@yahoo.com

1- Department of Industrial Engineering, University of Kurdistan, Sanandaj, Iran, Zahra.sobhani7@gmail.com

2- Department of Industrial Engineering, University of Kurdistan, Sanandaj, Iran, Shahrokhi292@yahoo.com

3- Digital Sciences Laboratory, LS2N UMR CNRS 6004, Ecole Centrale de Nantes, France, Alain.bernard@ec-nantes.fr

Abstract

Selecting systems configuration is a critical step in the safe design of systems. Optimizing systems configuration means maximizing their availability and minimizing their overall cost. In this regard, this paper aims to present a novel binary nonlinear fuzzy goal programming (FGP) model to choose components suppliers of multistate parallel series systems based on availability and manufacturing costs. Quantity-based discounts, components purchase cost, and penalties for delaying system construction were also considered. In addition, a fuzzy target programming model was applied to minimize deviations from the goal values of expenses. A system reliability block diagram illustrates the system's status. The Markov chain model describes a sequence of possible events in which the probability of each event depends on the reliability of system components. The other effect of ordering several pieces from the same supplier is considered (reducing the unit price of elements and increasing their delivery lead time). Model results indicate the practical application of this method to optimize system components reliability, taking into account life cycle parameters, including system construction cost and operational reliability.

Keywords - Goal programming; reliability (RBD); Concurrent engineering; Markov model; Fuzzy theory

INTRODUCTION

Unexpected shutdowns of strategic industrial systems cause enormous costs and risks. One of the best ways to prevent these unexpected failures is to optimize systems configuration at the design phase. In this paper, optimizing multistate parallel series systems configurations is investigated. A new binary nonlinear fuzzy goal programming (FGP) approach is presented to choose components suppliers of the feedwater system (FWS) of heat recovery steam generator (HRSG) boilers in combined-cycle power plants. FWS is a serial/parallel multistate system. By dedicating more budget to purchasing systems components, higher quality components can be ordered from suppliers who offer lower failure rates and higher repair rates. It increases the system's availability during operation. On the other hand, the orders should be planned based on the suppliers' delivery lead time which affects the system construction delay penalties. This research proposes a new design for a reliability approach using a goal programming model and considering the effects of components similarity on the components purchase price and project completion penalty. The model objective functions optimize the components purchase cost and the system construction delay penalty for a typical industrial multistate system. The proposed mathematical model considers the quantity-based discounts and delivery time variation for components unit prices. So, ordering similar elements to a supplier reduces the order cost and profit quantity-based discounts. However, it increases the components' delivery lead time because the supplier needs more time to

prepare large orders. FWS may work in full-capacity (FC), half capacity (HC), or shutdown (SD), due to the system components failure. The Markov model calculates the proportional time the system spends in different states. This FGP model minimizes product costs, including the purchase cost and the system construction delay penalties, and maximizes the system's availability at FC and HC. The following sections present the most critical research on the reliability allocation problem, the proposed mathematical model, model results, and discussion. In conclusion, it reviews the research results and offers several recommendations for future research.

LITERATURE REVIEW

Important industries such as power plants are exposed to rapid technological developments. Based on the reliability-centered maintenance (RCM) approach [1] presented a systematic framework for maintenance strategy and improves reliability and availability of manufacturing process machinery. It integrated conventional RCM procedure with the fuzzy computational process to improve failure mode effect criticality analysis (FMECA). Reference [2] introduced an optimization model of particle swarm optimization (FAPSO) and firefly algorithm with genetic algorithm (FAGA) are to improve the design performance of the water distribution networks. Mirzaei et al. used a neuro-fuzzy system to predict repair time and used Monte Carlo simulation to calculate indicators for equipment preventive maintenance, availability, repair rate and availability [3]. Using human experience, fuzzy logic, and Monte Carlo simulations [4] estimated repair times and time-dependent availability for gas turbine power plant components. Results show the fuel and lubrication systems have lower availability, with the highest system availability at 95% in the 16th year and the lowest at 87% in the 19th year. Reference [5] determined the availability of continuous mining systems at open pits using fuzzy logic and fuzzy inference systems. It synthesized partial indicators of availability, reliability, and maintenance convenience, offering a quick assessment tool for planning and maintenance, enhancing stable production and cost reduction. By examining how reliability, logistics management, and Prognostics and Health Management (PHM) approaches influence availability [6] focused on operational availability, emphasizing its importance in critical systems.. The methodology, adaptable to other availability types, highlights the significance of maintaining high availability to avoid operational disruptions and financial losses. To determine the availability of continuous mining systems using a neuro-fuzzy inference system, by combining fuzzy logic and artificial neural networks, [7] evaluated partial indicators based on historical data. The model accurately predicts availability quarterly, considering specific system data and external influences. Reference [8] evaluated the reliability of repairable systems by developing a fuzzy steady-state availability model. Using Zadeh's extension principle, it formulates mathematical programs to find α -cuts and derives a closed-form expression for the membership function, illustrated with numerical examples. scheduling approach. Such method can complement the conventional cost-based industry practices."

By developing an availability-centered reliability-based framework, [9] enhanced gas pipeline maintenance. Using Monte Carlo and discrete event simulations, it evaluated maintenance strategies, revealing that a combination of wrap and replacement actions. It optimized availability per unit cost, by aiding pipeline professionals in maintenance planning. By using particle swarm optimization (PSO) and Markov modeling [10] optimized the availability of a thermal power plant's turbo-generator subsystem. Results show that turbine lubrication and generator excitation are critical, achieving a maximum availability of 98.9394% with 30 particles, guiding maintenance strategies. By applying reliability, availability, and maintainability (RAM) methodologies [11] enhanced productivity in automotive manufacturing. Using RAM analysis and multi-attribute utility theory, it identified forklifts and loading equipment as bottlenecks and optimizes maintenance intervals, improving operational performance and sustainability. Using reliability modeling, [12] identified the impact of gas turbine rotor vibration. It analyzed failure effects on the system and environment, applying fuzzy modeling to optimize reliability and availability, ultimately improving system exploitation and monitoring. Using Markov processes and genetic algorithms, [13] enhanced a steel casting system's performance. It modeled four interdependent subsystems, develops differential equations for steady-state availability, and optimizes failure/repair rates, achieving optimal plant availability. Reference [14] evaluated and optimize the performance of a water circulation system in a coal-fired power plant using RAM analysis. Methods include reliability block diagrams, fault tree analysis, and Markov modeling. Results show the boiler feed pump is most critical, with optimized maintenance strategies enhancing system availability. Using deviation models and the probabilistic model-checker Prism, [15] introduced a novel approach to analyze availability in automated production systems. It verified availability requirements, reduces redundant models, and evaluates different positioning sensors' effects on a Pick-and-Place Unit, enhancing system productivity. By considering geo-environmental features and pipeline operation criticality, [16] developed failure prediction models and a maintenance planning framework for gas pipelines.. Using historical data, regression analyses, and Monte Carlo simulations, it predicts corrosion failures and optimizes maintenance, enhancing pipeline reliability and availability. By considering three interval availability indexes and using aggregated stochastic processes and Laplace transforms, [17] provided an analytical solution for interval availability in Markov repairable systems. The results are derived in closed form and validated against Monte Carlo simulations, applied to a fault-tolerant database system. Reference [18] improved robustness in availability predictive analysis under uncertainty and incorporated variabilities and uncertainties using interval methods and arithmetic. The approach accounts for MTTR and MTTF variabilities, enhancing the reliability of availability evaluations. By considering the sale of worn-out components [19] enhanced system reliability through a two-stage redundancy allocation approach and

optimized reliability at launch and during operation, using budget allocation and replacement strategies, significantly reducing costs. The model is validated with an example. By considering stochastic interdependence effects and a fractional-order model, [20] assessed the dynamic availability of a multi-component production system. It evaluated system states (impacted, degraded, failed) and validates the model with a numerical example, offering practical insights for maintenance planning and decision-making.

Levitin et al. presented a model to optimize the reliability of components and redundancy of different subsystems in multistate series-parallel systems [21]. Seyed Esfahani et al. used fuzzy theory to develop a model to maximize system reliability by determining the number of redundant components at each stage of multi-stage systems with parallel-series structures and using standby components subjected to cost and weight constraints [22]. Özder et al. considered the products' quality one of the supplier selection criteria [23]. They used preemptive goal programming and TOPSIS methods to develop a model for selecting the most appropriate supplier in the automotive production sector. Chin-Chia et al. integrated multistate reliability and transit time to optimize the network flow by considering its components' reliability [24]. Rui et al. developed a reliability optimization algorithm to calculate the system configuration by considering efficiency sharing to minimize its overall cost [25]. Yi et al. developed a reliability optimization model by studying the possibility of selecting component reliability from alternative levels by considering the system's overall performance as a function of each component's failure rate [26]. They also defined proper safety management procedures and assessed the effectiveness of safety measures and controls. Montoro-Cazorla et al. have studied a system subject to shocks that can cause damage or fail several units [27]. They developed a state-vector for the system units with the Markov model using intermediate functions and indicators. They compared the results of using different redundant units on the system performance.

Ge et al. developed an optimization model to allocate critical components' reliability in a serial system [28]. A penalty cost is applied when the total system downtime exceeds a predetermined level and uncertainty in component failure and repair rates. An enumeration procedure is applied to determine the optimal solution. Carpitella et al. presented a multi-objective model using mathematical programming and TOPSIS to optimize the system configuration. They developed a mathematical model for calculating the stationary availability of a k-out-of-n system, consistent with the fundamental theorem of Markov chains [29]. Kumar et al. analyzed the steady-state mechanical systems availability that followed condition-based maintenance and evaluated the optimal condition monitoring interval with the semi-Markov process model [30].

Arezki Mellal and Zio Considered the availability optimization problem for series-parallel systems and offered a multi-objective model [31]. They investigated systems with failure dependencies and solved the presented model using particle swarm optimization (PSO) and cuckoo optimization algorithms (COA). Results showed that COA achieved better outcomes. Akrouche et al. worked on an optimization method for the configuration of multistate systems considering availability [32]. They considered aleatory and epistemic uncertainties in their research and applied a Markovian approach combined with interval contraction techniques for the model. Their goal was to choose the best configuration for a system out of several given configurations regarding availability, cost, and imprecision. At the same time, failure and repair rates were proposed in intervals. Kumar and Punia presented a model to maximize system availability and manage suitable maintenance strategies [33]. They applied Markov Methods to model steady and transient state availability for a complex repairable screw manufacturing system and utilized Particle Swarm Optimization (PSO) algorithm to choose optimal parameters. They obtained failure and repair rates of the different components using maintenance records of the failures.

Kumar et al. utilized metaheuristic algorithms for Steady-state availability (SSA) of a cooling tower in a steam turbine power plant [34]. An efficient stochastic model in this research was presented to overcome flaws in these metaheuristic algorithms. They also benefited from the Markovian birth-death process for the Chapman-Kolmogorov differential-difference equations. Then, system availability is optimized for this system, including six subsystems using genetic algorithm (GA) and particle swarm optimization (PSO).

Based on our best knowledge, optimizing systems availability and configuration using components' quantity-based discount and delivery lead time has not been considered in the literature. It is a possible problem, especially in essential industries; this paper proposes a model to cover this gap. This paper presents a model to construct a specific series-parallel multistate industrial system concerning availability, purchase price, the effects of similarity in ordering elements, and system construction delay cost. A binary mathematical program simultaneously optimizes the system costs during the construction and exploitation phases. The following sections present an FGP model to plan components orders and construction programs and optimize costs during the system construction phase and its availability during its exploitation phase.

PROBLEM STATEMENT

The purpose of this article is to provide a suitable method for selecting the best suppliers for the components of a system that includes component A and three similar components B, C and D.

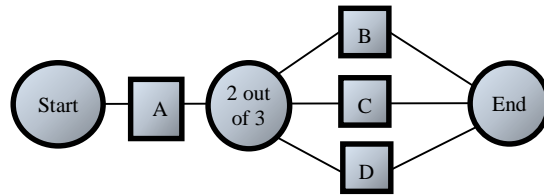


FIGURE 1
STUDIED SERIES-PARALLEL SYSTEM

Operating the system at full capacity requires at least two operational components from components A, B and C, each of which alone provides 50% of the capacity. Figure (4) shows a set of all possible states, based on the Markov model, along with their relationships. Oval, trapezoidal and rectangular shapes represent half load (FC), full load (HC) and full stop (SD) modes, respectively. This system is available when operating in FC and HC modes. In the full capacity mode, component A plus at least two of the three components A, B, and C must work. Half-capacity conditions occur when component A and only one of the other three components are working, and the system shuts down when A or all three components fail. Each of the suppliers has different components quality (i.e. failure and repair rate), prices and delivery time. Also, ordering similar items from the same supplier, their unit price will be discounted, but instead, because the supplier needs more time too prepare larger orders, it increases the delivery time. The operation of components A, B, and C are identical, and by ordering them separately from supplier j , delivery lead time and price will be L_{Bj} and C_{Bj} , respectively. By ordering two components to supplier j , the unit price and delivery lead time of these two components will be L_{Bj} and C_{Bj} , respectively. By ordering two components to supplier j , the unit price and delivery lead time of these two components will be C'_{Bj} and L'_{Bj} , respectively. Finally, by ordering all these three components together to supplier j , their unit price and delivery lead time will be C''_{Bj} and L''_{Bj} , respectively.

Reduced capacity and outages create hourly costs during operation. Components that are more reliable and repairable are more expensive and have longer delivery times. Delays in assembling components will result in a fixed daily penalty. Therefore, components should be ordered in such a way that their assembly is completed with the least possible delay. In addition, minimum system reliability and the budget available for purchase the components are limited. This model minimizes the total cost including components purchase, delay penalty and system operation cost. Therefore, the proposed model optimizes the following objectives, simultaneously:

1. Minimizing the total cost of building the system, including the purchase price of components and the penalty for delay in building the system
2. Maximize the expected ratio of the system in FC and HC states during its use

The deviation from the optimal value of the above goals should be minimized and these deviations have a weighting factor, according to their importance. Sometimes the probability of occurrence of DS and HC does not have a linear effect on the satisfaction of decision makers. The use of H and O utility functions shows the non-linear effect of SD and HC probabilities on the fulfillment of decision makers' expectations.

MODEL ASSUMPTIONS

The model assumptions include:

1. The system construction project includes purchasing and assembling components. It has a deadline and a daily penalty for the delays.
2. The alternative components supplies offer different qualities (repair and failure rates), prices, and lead times.
3. The functions of components A, B, and C are identical and independent of their quality. Each provides 50% of the required (nominal) system capacity.
4. The similarity of components A, B, and C causes discounts on their price and increases their delivery lead time.
5. There are three system states FC, HC, or SD, according to the condition of its components.
6. The components repair time is an exponential random variable with known parameters.
7. The components purchase price includes their order, delivery, and assembly costs.
8. Assembling processes of components A, B, and C should begin after the completion of component A. Their parallel implementation does not change their construction process times.
9. The time and cost of construction processes are deterministic and known, independent of the components' quality.
10. The fuzzy weighting factors present the importance of the goals.

a. Indices

$i = A, B, C, D$: The components

$j = 1, 2, \dots, J$: The component suppliers

$k = 1, 2, \dots, K$: The assembly process

$l = 1, 2, \dots, L$: The system states

$m = 1, 2, 3$: The goals

b. Parameters

C_{Aj} = The purchasing price of component A from supplier j

C_{Bj} = The purchase price of each component A, B, and C from supplier j, when purchased individually

C'_{Bj} = The offered unit purchase price by supplier j for two similar pumping lines

C''_{Bj} = The offered unit purchase price by supplier j for three similar pumping lines

R_{ij} = The reliability of component i offered by supplier j

A = The minimum requested system availability

T = The system construction project deadline

B = The available budget for purchasing components

n = The number of components

C^D = The daily delay penalty for the system construction project

L_{Aj} = The delivery lead time of component A by the supplier j

L_{Bj} = The delivery lead time of one component A, B, and C from the supplier j

L'_{ij} = Delivery lead times of two similar components A, B, and C from the supplier j

L''_{ij} = Delivery lead time of three similar components A, B, and C from the supplier j

F_{Ak} = time of assembly process k for components A

F_{Bk} = time of assembly process k for each component A, B, and C

K_A = Set of required assembly processes for component A

K_B = set of assembly processes done on the components A, B, and C

μ_{ij} = The rate of repair component i from supplier j

λ_{ij} = The rate of failure component i from supplier j

b_m = The value of goal m

g_m = The normalizing factor for goal m

w_m = Weighting factor of the undesirable deviation from goal m

c. Decision variable

A_1 = The components purchase cost

A_2 = The system construction delay penalty

y_{ij} = Binary variable with values 1, if component i is ordered from supplier j, and zero otherwise.

R_i = The reliability of component i

A_e = The estimated entire system availability

P_0 = The proportion of system shutdown time during its use

P_{50} = The proportion of HC during the system use

S_l = The proportion of time that the system spends in state l

μ_i = The rate of repair of component i

λ_i = The rate of failure component i

T^c = The completion time of the system construction project

T_c^i = The completion time of component i assembly

T^B = The completion time of delivery of all pumping lines

d_m^+, d_m^- = Positive and negative deviations from goal m

Q = The total utility function

H = Utility function of the system at SD states

O = Utility function of the system at HC states

$x_1, x_2, x_3, x_4, x_5, u_1, u_2, u_3, u_4, z_1, z_2, z_3, z_4,$
 z_5, v_1, v_2, v_3, v_4

= Auxiliary variables for the linearization of the utility functions

Equations (1) to (37) show the objective functions and constraints of the mathematical model.

$$\text{Min } Z = w_1(g_1d_1^+) + w_2(g_2d_2^-) \tag{1}$$

Subjected to:

$$A_1 = \sum_{j=1}^3 y_{Bj}y_{Cj}y_{Dj}(3C_{Bj}'') + \sum_{j=1}^3 (y_{Bj}y_{Cj}(1 - y_{Dj}) + y_{Bj}y_{Dj}(1 - y_{Cj}) + y_{Cj}y_{Dj}(1 - y_{Bj})) \times (2C_{Bj}') + \sum_{j=1}^3 (y_{Bj}(1 - y_{Cj})(1 - y_{Dj}) + y_{Cj}(1 - y_{Bj})(1 - y_{Dj}) + y_{Dj}(1 - y_{Bj})(1 - y_{Cj})) \times C_{Bj} + \sum_{j=1}^3 y_{Aj}C_{Aj} \tag{2}$$

$$P_0 = S_8 + S_9 + S_{10} + S_{11} + S_{12} + S_{13} + S_{14} + S_{15} \tag{3}$$

$$P_{50} = S_5 + S_6 + S_7 \tag{4}$$

$$H(P_0) = \begin{cases} 125 - 125P_0 & 0.4 < P_0 \leq 1 \\ 103 - 70P_0 & 0.1 < P_0 \leq 0.4 \\ 101 - 50P_0 & 0.02 < P_0 \leq 0.1 \\ 100 & P_0 \leq 0.02 \end{cases} \tag{5}$$

$$O(P_{50}) = \begin{cases} 120 - 95P_{50} & 0.4 < P_{50} \leq 1 \\ 108 - 65P_{50} & 0.2 < P_{50} \leq 0.4 \\ 105 - 50P_{50} & 0.1 < P_{50} \leq 0.2 \\ 100 & P_{50} \leq 0.1 \end{cases} \tag{6}$$

$$Q = (3H + O)/4 \tag{7}$$

$$Q + d_2^- = b_2 \tag{8}$$

$$T^A = \sum_{j=1}^3 y_{Aj}L_{Aj} + \sum_{k \in k_A} F_{Ak} \tag{9}$$

$$T^B = \sum_{j=1}^3 y_{Bj}y_{Cj}y_{Dj}L_{Bj}'' + \left(\sum_{j=1}^3 (y_{Bj}y_{Cj}(1 - y_{Dj}) + y_{Bj}y_{Dj}(1 - y_{Cj}) + y_{Cj}y_{Dj}(1 - y_{Bj})) \right) \times \text{Max}(L_{Bj}, L'_{Bj}) + \left(\sum_{j=1}^3 (y_{Bj}(1 - y_{Cj})(1 - y_{Dj}) + y_{Cj}(1 - y_{Bj})(1 - y_{Dj}) + y_{Dj}(1 - y_{Bj})(1 - y_{Cj})) \right) \times \text{Max}(L_{Bj}, L'_{Bj}) \tag{10}$$

$$T^C = \max(T^B, T^A) + \sum_{k \in k_B} F_{Bk} \tag{11}$$

$$A_2 = \max(0, T^C - T)C^D \tag{12}$$

$$A_1 + A_2 - d_1^+ = b_1 \tag{13}$$

$$A_1 \leq B \tag{14}$$

$$\mu_i = \sum_{j=1}^3 y_{ij}\mu_{ij} \quad \forall i \tag{15}$$

$$\lambda_i = \sum_{j=1}^3 y_{ij}\lambda_{ij} \quad \forall i \tag{16}$$

$$\sum_{j=1}^3 y_{ij} = 1 \quad \forall i \tag{17}$$

$$y_{ij} \in \{0,1\} \quad \forall i,j \tag{18}$$

$$\sum_{l=1}^{15} S_l = 1 \tag{19}$$

$$A_e = 1 - P_0 \tag{20}$$

$$A_e \geq A \tag{21}$$

$$P_{100} = S_1 + S_2 + S_3 + S_4 \tag{22}$$

Equation (1) shows the objective function, which minimizes the deviation of two goals, including the total costs and the total utility function, from desired predefined values. g_1 and g_2 are required to normalize these two goals because of the difference

between their modality. In addition w_1 and w_2 are the goals weighing factors, indicating the goals' priorities and importance. In equation (2), the binary variables calculate the cost of purchasing all components according to the size of ordered similar elements and the supplier. The first statement indicates the cost of buying three identical components (B, C, and D), which is three times the unit price cost by:

$$\sum_{j=1}^3 y_{Bj} y_{Cj} y_{Dj} (3C_{Bj}'')$$

If two of the components are similar, the cost will be calculated by adding two identical components to the price of one other element, as follows:

$$\sum_{j=1}^3 (y_{Bj} y_{Cj} (1 - y_{Dj}) + y_{Bj} y_{Dj} (1 - y_{Cj}) + y_{Cj} y_{Dj} (1 - y_{Bj})) \times (2C_{Bj}')$$

If the components B, C, and D are dissimilar, the total cost is calculated by adding the cost of each of them, according to the suppliers' offered price, as follows:

$$\sum_{j=1}^3 (y_{Bj} (1 - y_{Cj}) (1 - y_{Dj}) + y_{Cj} (1 - y_{Bj}) (1 - y_{Dj}) + y_{Dj} (1 - y_{Bj}) (1 - y_{Cj})) \times C_{Bj}$$

In addition, the price of component A depends on its suppliers and is calculated by:

$$\sum_{j=1}^3 y_{Aj} C_{Aj} \tag{23}$$

Equations (3) and (4) calculate the expected proportion of the system's time at shutdown and half capacity states, respectively, according to the Markov process illustrated in Figure (4). Equations (5) and (6) define the predicted utility functions relating to the proportion of time that the system spends in shut down and half capacity states, respectively. These functions are defined as piecewise linear functions to facilitate modeling. The values of the utility functions will be increased by reducing the proportion of the time that the system is shutting down and is working at half capacity states, as shown in the following Figures:

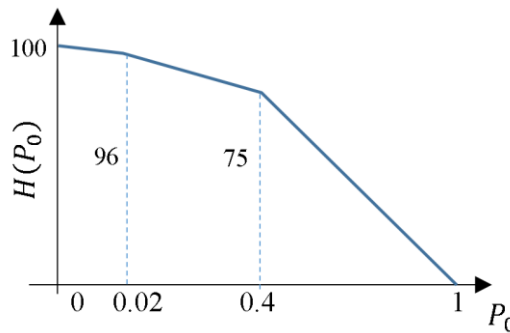


FIGURE 2
THE UTILITY FUNCTION FOR THE SHUTDOWN PROBABILITY

It indicates that the decision-makers are very sensitive to the SD occurrence and only low SD probabilities are acceptable. Similarly, the utility function of the probability of half-load work is as follows:

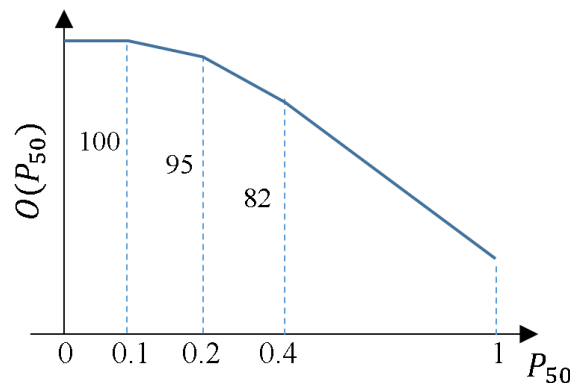


FIGURE 3
THE UTILITY FUNCTION FOR THE HC PROBABILITY

Figure (3) shows that the decision-makers may accept an extended range of the HD probability. Equation (7) calculates the value of the expected total utility function of the system states as a weighted average of the computed values in (5) and (6). Equation (8) calculates the deviation from the second goal, reflected in the objective function. Equation (9) calculates the completion time of assembly of component A by adding its delivery lead time and the sum of its assembly processes, according to its supplier. Equation (10) calculates components B, C, and D's delivery lead time according to their suppliers and order sizes. Supposing that the delivery lead time of larger orders is always longer than small orders, this equation can be written in a more simplified form by removing the max operators; however, it is written in a general form. This equation uses the same logic previously used in equation (2). Constraint (11) calculates the completion time of the system construction project by adding the possible earliest time of starting assembly processes of components B, C, and D, with the time of their assembly processes. The max operator is used because beginning these processes requires delivery of all components and completion of assembly of component A.

The construction delay penalty value is calculated using equation (12), as the multiplication of the daily delay penalty by the difference between the project end time and its deadline. Equation (13) calculates the total construction cost by adding the component purchasing and construction delay penalty. Since the assemblies' processes' prices are constant and independent of the decision variables of the model, they are ignored. Constraint (14) limits the maximum components' purchasing cost to the predefined level. Equations (15) and (16) determine the failure and repair rates of the elements, respectively, by considering their suppliers. Constraint (17) indicates that each component should be ordered from one and only one supplier. Constraint (18) determines the domain of binary variables. Condition (19) ensures that the sum of all proportions of times provided by the latest equations equals one (because the system should always be in one of these states). Constraint (20) calculates the expected availability of the entire system as the proportion of times that it is not shutting down, and (21) ensures achieving the requested minimum level of reliability required of the whole system. Constraint (22) calculates the expected proportion of times the system works at the full-capacity states.

It is estimated to facilitate the analyses of the results. Constraints related to equilibrium equations of the continuous Markova processes are written in the appendix, according to Figure (4), and ensure that the sum of the input streams to each state, in the long term, is equal to the sum of the output streams. These constraints also determine the proportion of time the system spends in each of the particular states. To facilitate solving the model, the utility functions (i.e., $H(P_0)$ and $O(P_{50})$), shown in (5) and (6), are transformed into simple linear constraints. These constraints are given in the appendix section.

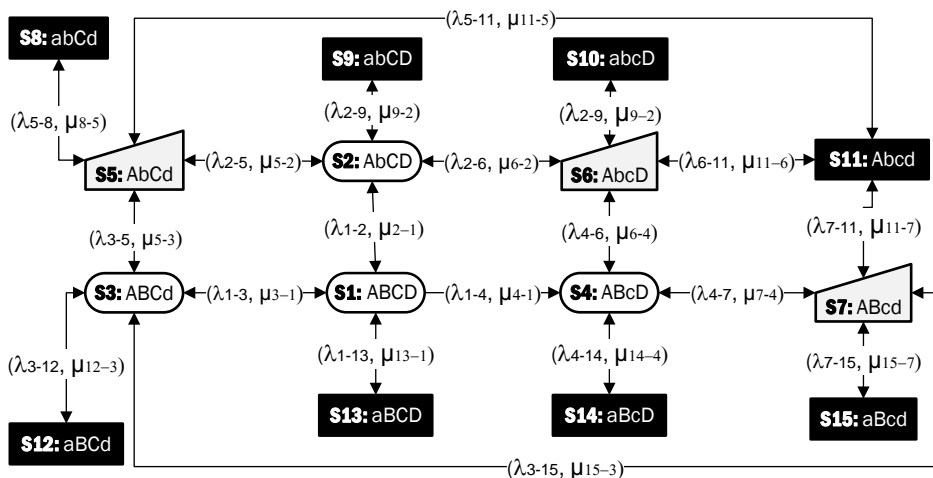


FIGURE 4
DIAGRAM OF RELATIONSHIPS BETWEEN DIFFERENT SYSTEM STATES

NUMERICAL RESULTS

Tables (1) to (3) show the model parameter values. Table (1) illustrates the system and the components' data.

TABLE I
THE PARAMETERS VALUES

| Parameter | Value | Parameter | Value |
|----------------|-------|-------------|-------|
| B | 1100 | μ_{i1} | 0.05 |
| T | 75 | μ_{i2} | 0.07 |
| C^D | 300 | μ_{i3} | 0.1 |
| λ_{i1} | 0.05 | w_1 | 1 |
| λ_{i2} | 0.03 | w_2 | 5 |
| λ_{i3} | 0.01 | b_1 | 6100 |
| R_{i1} | 0.9 | b_2 | 400 |
| R_{i2} | 0.95 | A | 0.8 |
| R_{i3} | 0.99 | g_2 | 0.01 |
| g_1 | 0.001 | $i=1,2,3,4$ | |

Table (2) concerns the suppliers' data.

TABLE 2
THE SUPPLIERS' DATA

| Supplier (j) | | | Reliability |
|--------------|------|-----|-------------|
| 3 | 2 | 1 | |
| 0.99 | 0.95 | 0.9 | |
| 240 | 220 | 200 | C_{Aj} |
| 380 | 340 | 300 | C_{Bj} |
| 320 | 280 | 250 | C'_{Bj} |
| 280 | 240 | 200 | C''_{Bj} |
| 17 | 7 | 5 | L_{Aj} |
| 31 | 19 | 6 | L_{Bj} |
| 37 | 24 | 8 | L'_{Bj} |
| 42 | 30 | 12 | L''_{Bj} |

Table (3) presents the time of the components assembly processes.

TABLE 3
TIME OF THE COMPONENTS ASSEMBLY PROCESSES

| Assembly process (k) | 1 | 2 | 3 | 4 | 5 |
|----------------------|---|----|----|---|---|
| F_{Ak} | 3 | 5 | 7 | 4 | 2 |
| F_{Bk} | 6 | 13 | 16 | 5 | - |

Table (4) presents the results of the model.

TABLE 4
THE EXAMPLE RESULTS

| Decision variable | Value | Decision variable | Value |
|-------------------|---------|-------------------|--------|
| R_1 | 0.99 | A_1 | 1080 |
| R_2 | 0.95 | A_2 | 5100 |
| R_3 | 0.9 | H | 92.268 |
| R_4 | 0.9 | O | 88.664 |
| Z | 252.666 | P_0 | 0.153 |
| A_e | 0.847 | P_{50} | 0.297 |
| T^c | 92 | P_{100} | 0.549 |
| d_1^+ | 80 | d_2^- | 34.533 |

According to this table, component A was ordered with the highest reliability. Component A has no redundancy, so this component is critical for the reliability and availability of the whole system and has an important role. One of the pumping lines (component B) was chosen with middle reliability (0.95), while the other two were selected with the lowest reliability (both with 0.9). As these components have the same use in the system, they are redundant. As a result, they can work instead of each other in case of failure. That is why these components have been selected with lower reliability than component A. Although two of the components have the lowest reliability, the overall reliability of the whole system is about 0.85. The system is not working just at 15% of its lifetime. Also, it works with FC for about 55% of its lifecycle. At about 30% of its lifetime, the system works with half capacity because the reliability of components B, C, and D are lower than 0.99. Besides, the deviation from the first goal is larger than the second one, indicating the second goal's higher importance. The delay penalty has a more significant impact on selecting components and has a more decisive role. Given that delivery lead time increases by increasing components' reliability, the model keeps components B, C, and D at lower reliability levels to reach a shorter completion time. This way, the cost of delays will decrease.

DISCUSSION

The available components purchasing budget increase shows that purchasing price increases and the delay penalty decreases. The delay penalty (the second objective) has a more critical role than purchase costs (the first objective). As shown in Figure (6), increasing the available budget reduces both the proportion of SD and HC time because more budget allows decision-makers to choose components with higher reliability. Thus, the reliability and availability of the system increase, which means minor failure and capacity reduction. This improvement continues until the budget reaches 1100 because further availability leads to higher components purchasing costs and project delay penalties, which increases system costs. So it is optimal to dedicate at most 1100 units to buy elements.

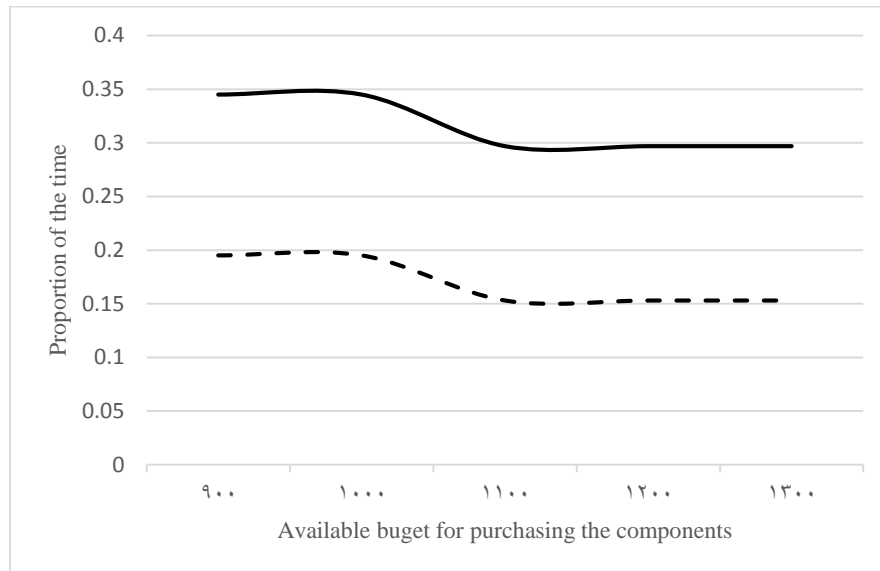


FIGURE 6
PROPORTION OF THE TIME OF THE SYSTEM SHUTDOWN (DASHED LINE) AND WORKING AT HALF CAPACITY (SOLID LINE), IN TERMS OF AVAILABLE MAXIMUM COMPONENTS PURCHASE BUDGET

Figure (7) shows that by increasing the first goal deviation's weighting factor, the purchase cost does not change, while the delay penalty remains constant after a significant reduction. Minimum requested availability and high nonconformity from the second goal impede other components' quality reduction. Ordering similar high-reliability elements let profit the discounts, and the purchase cost remains constant at 1080 units. By considering $W1 = 0$, the delay penalty will be too much due to the components' similarity and high components delivery lead times. Also, it turns out that increasing $W1$ more than 2 makes no difference.

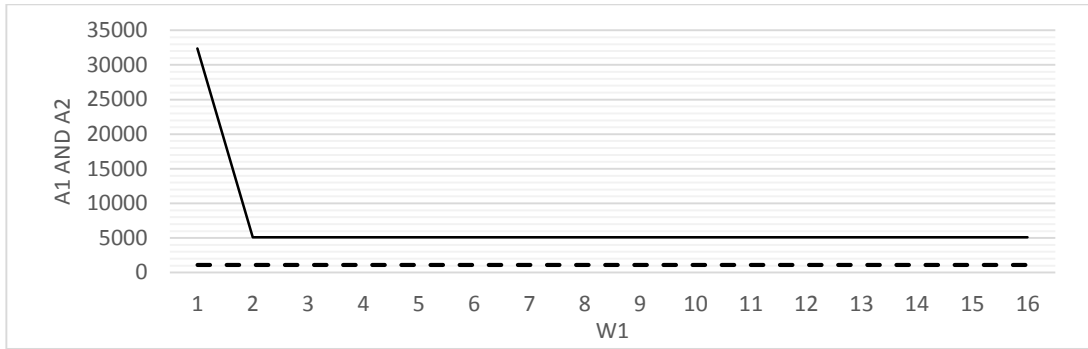


FIGURE 7

VARIATION OF PURCHASE COSTS (DASHED LINE) AND DELAY PENALTY (SOLID LINE) IN TERMS OF CHANGING THE WEIGHTING FACTOR OF THE FIRST GOAL DEVIATION

Figure (8) shows that when $W1=0$, the probabilities of SD and HC are low. By increasing $W1$, first, these probabilities increase (till $W1=2$) and then remain constant at a higher level. As shown in Fig. 5, $W1=0$ results in the highest possible quality for components. As a result, the system has the highest reliability, which means the lowest failures, shutdowns, and capacity reductions. By increasing $W1$ more than 2, although this factor tends to lower components' reliability, the minimum requested availability constraint does not allow components to be selected with low qualities.

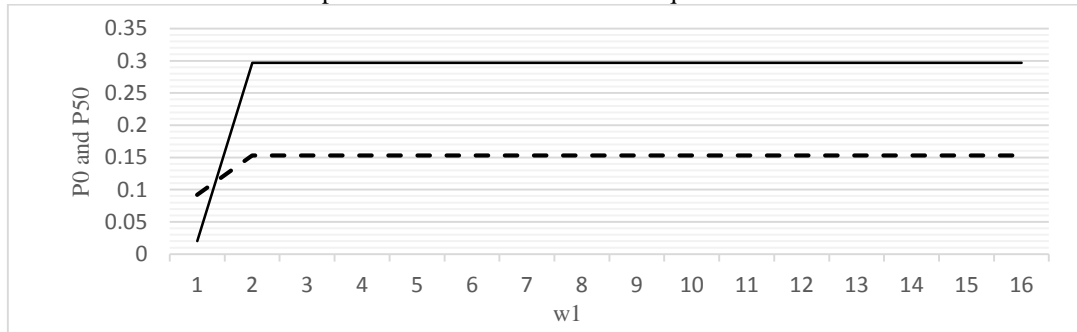


FIGURE 8

VARIATION OF PROPORTION OF THE TIME THAT THE SYSTEM SPENDS AT SHUTDOWN (DASHED LINE) AND WORKING AT HALF CAPACITY (SOLID LINE) STATES, IN TERMS OF WEIGHTING FACTOR OF DEVIATION FROM THE FIRST GOAL

Figure (9) shows the effects of variation of $W2$ on purchase cost and delay penalty. It can be seen that changing $W2$ does not affect components purchase cost and project delay penalty. The requested minimum availability constraint prevents the model from selecting components with low reliabilities despite their shorter delivery lead time. Besides, $W1$ does not allow purchase costs to grow much due to buying high-quality elements.

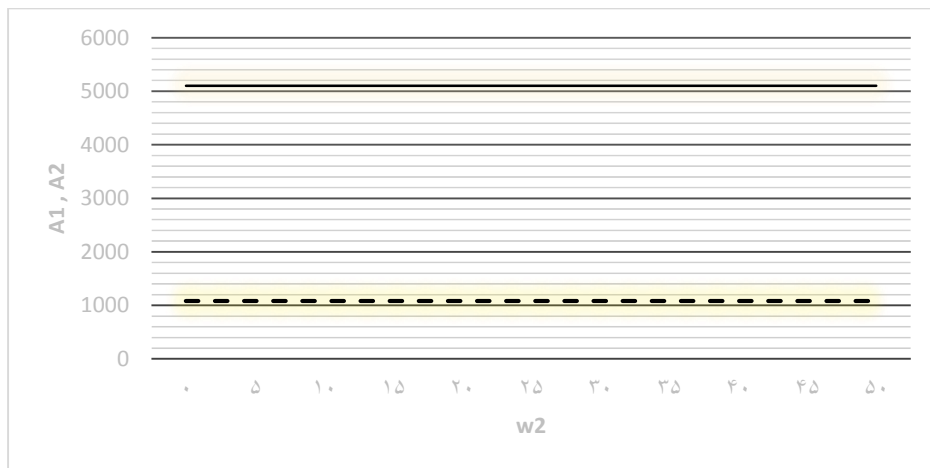


FIGURE 9

VARIATION OF THE COMPONENTS PURCHASE COSTS AND PROJECT DELAY PENALTY IN TERMS OF WEIGHTING FACTOR OF DEVIATION FROM THE SECOND GOAL

Figure (10) illustrates the impact of W_2 values on system shutdown and half capacity. As it can be observed, SD and HC probabilities do not change when W_2 goes up. It also occurs due to the requested minimum availability constraint and the first goal of the model.

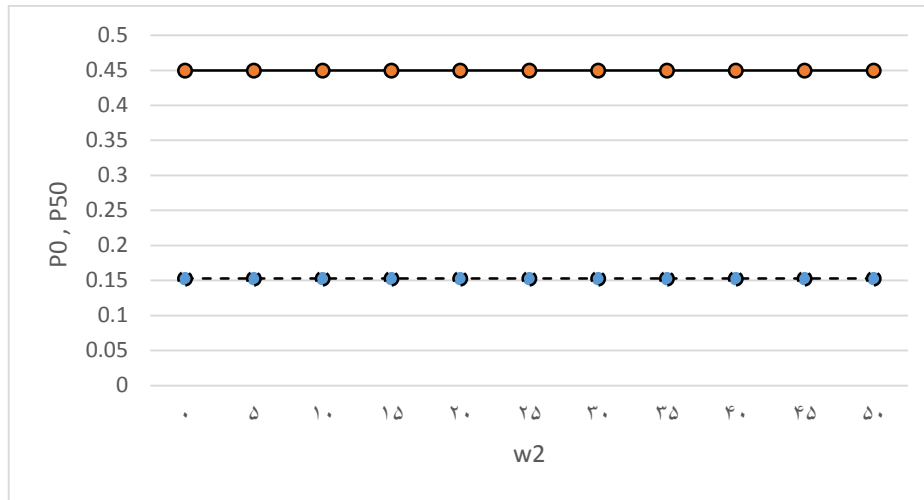


FIGURE 10
PROPORTION OF THE TIME OF SYSTEM SHUTDOWN AND WORKING AT HALF CAPACITY, IN TERMS OF DEVIATION FROM THE SECOND GOAL

Table (5) shows the numerical results by considering $W_2 = 0$, and releasing the minimum availability constraint. Because of removing this constraint, elements have been selected with lower reliabilities. For example, element A has minimum reliability while this component has an essential role in total system reliability. Elements B and C have the lowermost reliability, too. As a result, the system works with half capacity at 14% and total capacity at just 35%. However, the purchasing cost and project delay penalty remain constant despite changing the components' quality.

Results of the model can be assessed through simulation. Failure rates and repair rates can be simulated for each way of choosing all four elements, and then all the simulation results can be compared to find the best category. This optimal solution can be compared with table (4) results.

TABLE 5
THE EXAMPLE RESULTS FROM CONSIDERING $W_2 = 0$, AND RELEASING THE MINIMUM AVAILABILITY CONSTRAINT

| Decision variable | Value | Decision variable | Value |
|-------------------|-------|-------------------|--------|
| R_1 | 0.9 | A_1 | 1080 |
| R_2 | 0.9 | A_2 | 5100 |
| R_3 | 0.9 | H | 61.78 |
| R_4 | 0.99 | O | 98.1 |
| Z | 80 | P_0 | 0.506 |
| A_e | 0.494 | P_{50} | 0.138 |
| T^c | 92 | P_{100} | 0.356 |
| d_1^+ | 80 | d_2^- | 116.55 |

CONCLUSIONS

This paper introduces an integer nonlinear fuzzy goal programming model to optimize a typical multistate parallel-series system's design configuration. The proposed approach is essential in that, in addition to reducing the cost of building an industrial system, it also addresses the costs of operating it. It includes considering the cost of purchasing components, installing them, and the discounted effects of buying them. The impact of delays in the preparation of components on the project's completion time is also considered. The proposed approach also allows examining the effect of components reliability on the

costs of the system operation period when selecting their supplier and collects all the above in the form of an integrated model and determines its optimal solution by choosing the supplier and assigning the order. The model validation can be implemented by applying it in the real world or testing its results by numerical simulation. Results show that considering availability, delay penalties, and the effects of ordering components from the same suppliers in the designing phase improve the total costs of systems, reduce unexpected failures and risks, and better match real-world conditions. The model results also show how using fuzzy goal programming enables decision-makers to decide according to their priorities regarding the system construction and exploitation costs. Besides varying the goals weighing factors and the available components purchasing budget, it provides an appropriate trade-off between construction and exploitation costs. The problem is that this model should be changed based on the configuration of the intended system. Indeed, it is impossible to propose a general model for all system configurations.

Additionally, the numerical results show that the deviations from the objective functions goals and the requested minimum system availability constraint may significantly affect the components ordering plan. The budget ceiling also considerably impacts the system costs and availability. Future research can improve the model by considering uncertainty in some parameters. For example, failure rates, repair rates, or both can be regarded as fuzzy numbers like triangular ones. Also, the reliability of each element can be indicated using fuzzy numbers or random variables. System costs have the potential to be shown using fuzzy or random variables. Another helpful future research direction could be investigating the effect of ordering similar components on their repair rate. Because when some components are purchased from the same supplier, their spare components and repairman can be shared. So this smooth and accelerated repair rate.

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Appendix

Equilibrium equations of the continuous Markova processes:

$$(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)S_1 = \mu_2S_2 + \mu_4S_3 + \mu_3S_4 + \mu_1S_{13} \quad (1)$$

$$(\lambda_1 + \mu_2 + \lambda_3 + \lambda_4)S_2 = \lambda_2S_1 + \mu_4S_5 + \mu_3S_6 + \mu_1S_9 \quad (2)$$

$$(\lambda_1 + \mu_4 + \lambda_3 + \lambda_2)S_3 = \lambda_4S_1 + \mu_2S_5 + \mu_1S_{12} + \mu_3S_7 \quad (3)$$

$$(\lambda_1 + \mu_3 + \lambda_2 + \lambda_4)S_4 = \lambda_3S_1 + \mu_4S_7 + \mu_2S_6 + \mu_1S_{14} \quad (4)$$

$$(\lambda_1 + \mu_2 + \lambda_3 + \mu_4)S_5 = \lambda_4S_2 + \lambda_2S_3 + \mu_3S_{11} + \mu_1S_8 \quad (5)$$

$$(\lambda_1 + \mu_2 + \lambda_4 + \mu_3)S_6 = \lambda_3S_2 + \lambda_2S_4 + \mu_4S_{11} + \mu_1S_{10} \quad (6)$$

$$(\lambda_1 + \mu_3 + \lambda_2 + \mu_4)S_7 = \lambda_4S_4 + \lambda_3S_3 + \mu_2S_{11} + \mu_1S_{15} \quad (7)$$

$$\mu_1S_8 = \lambda_1S_5 \quad (8)$$

$$\mu_1S_9 = \lambda_1S_2 \quad (9)$$

$$\mu_1S_{10} = \lambda_1S_6 \quad (10)$$

$$(\mu_2 + \mu_3 + \mu_4)S_{11} = \lambda_4S_6 + \lambda_2S_7 + \lambda_3S_5 \quad (11)$$

$$\mu_1S_{12} = \lambda_1S_3 \quad (12)$$

$$\mu_1S_{13} = \lambda_1S_1 \quad (13)$$

$$\mu_1S_{14} = \lambda_1S_4 \quad (14)$$

$$\mu_1S_{15} = \lambda_1S_7 \quad (15)$$

Constraints related to linearization of utility functions:

$$P_0 = 0x_1 + 0.02x_2 + 0.1x_3 + 0.4x_4 + x_5 \quad (16)$$

$$H(P_0) = 100x_1 + 100x_2 + 96x_3 + 75x_4 + 0x_5 \quad (17)$$

$$x_1 \leq u_1 \quad (18)$$

$$x_2 \leq u_1 + u_2 \quad (19)$$

$$x_3 \leq u_2 + u_3 \quad (20)$$

$$x_4 \leq u_3 + u_4 \quad (21)$$

$$x_5 \leq u_4 \quad (22)$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \quad (23)$$

$$u_1 + u_2 + u_3 + u_4 = 1 \quad (24)$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0 \quad (25)$$

$$u_1, u_2, u_3, u_4 \in \text{Error! Bookmark not defined.} \quad (26)$$

$$P_{50} = 0z_1 + 0.1z_2 + 0.2z_3 + 0.4z_4 + z_5 \quad (27)$$

$$O(P_{50}) = 100z_1 + 100z_2 + 95z_3 + 82z_4 + 25z_5 \quad (28)$$

$$z_1 \leq v_1 \quad (29)$$

$$z_2 \leq v_1 + v_2 \quad (30)$$

$$z_3 \leq v_2 + v_3 \quad (31)$$

$$z_4 \leq v_3 + v_4 \quad (32)$$

$$z_5 \leq v_4 \quad (33)$$

$$z_1 + z_2 + z_3 + z_4 + z_5 = 1 \quad (34)$$

$$v_1 + v_2 + v_3 + v_4 = 1 \quad (35)$$

$$z_1, z_2, z_3, z_4, z_5 \geq 0 \quad (36)$$

$$v_1, v_2, v_3, v_4 \in \text{Error! Bookmark not defined.} \quad (37)$$

AUTHOR (S) INFORMATION

Zahra Sobhani, PhD student, Department of Industrial Engineering, University of Kurdistan.

Mahmoud Shahrokhi, Associate Professor, Department of Industrial Engineering, University of Kurdistan.

Alain Bernard, Professor, Digital Sciences Laboratory, LS2N UMR CNRS 6004, Ecole Centrale de Nantes.