Original Research

Aircraft routing problem considering various maintenance operation factors: A literature review

Masoumeh Mirjafari[1]. Alireza Rashidi Komijan[2*]. Ahmad Shoja[3]

Received: 05 June 2023/ Accepted: 21 Aug 2023/ Published online: 21 Aug 2023

*Corresponding Author, rashidi@azad.ac.ir

1- Department of Industrial Engineering, Roudehen Branch, Islamic Azad University, Roudhen, Iran
2 - Department of Industrial Engineering, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran
3- Department of Mathematics Engineering, Roudehen Branch, Islamic Azad University, Roudhen, Iran

Abstract
The companies in the aviation industry require exact scheduling and operation due to their complex and costly activities. The aircraft routing problem (ARP) which meets all of the requirements related to maintenance operations and achieves the minimum costs is among the significant issues for an airline. Solving the ARP includes creating all of the routes and defining aircraft maintenance inspections. The present study aims to review and categorize the recent research on ARP and maintenance operation. To this aim, four significant categories including type of model, maintenance and repair factors, disruption and robustness, as well as objective function and solution approach were defined. Based on the literature review, the integrated study of the airline schedule steps provides better results than the multi-stage review. In addition, defining the combined framework of different maintenance factors generates a more accurate schedule to control the maintenance requirements. Further, applying multiple hybrids meta-heuristic approach leads to significant results.

Key words- Airline scheduling; Aircraft routing problem; Maintenance operations

INTRODUCTION

The aviation industry is among the largest in the world to counter the large-scale problems, complex operations, high costs, competitive field, and various unpredictable problems and disruptions (Cadarso et al., 2017; Ben Ahmad et al., 2017). Accordingly, all of the airlines seek to schedule their activities tactically and operationally. The tactical scheduling is planned before the operating date, while the operational one is conducted during the operational step (Tekiner et al., 2009; Wang and Jacquillat, 2020; Wen et al., 2020b). As Fig. 1 shows, robust scheduling, disruption management, and recovery activities are considered more than traditional scheduling (Wen et al., 2021).

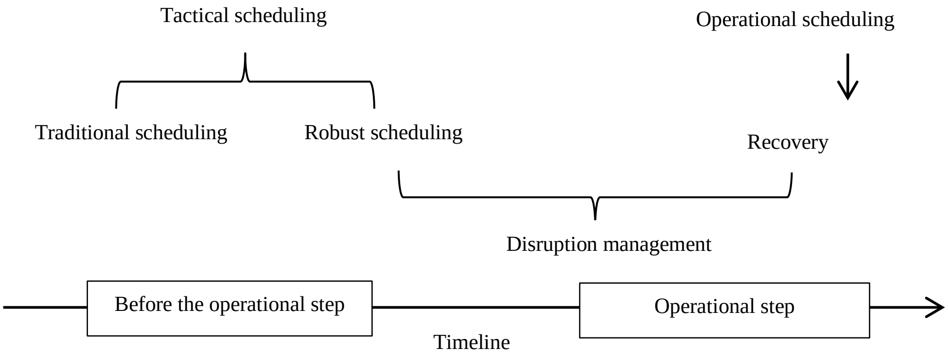

FIGURE 1
CLASSIFICATION OF THE AIRLINE SCHEDULING

Airline scheduling which has attracted a lot of attention focuses on four problems including flight scheduling, fleet assignment, maintenance routing, and crew assignment (Bazargan, 2010; Bae, 2010) which are considered in a multi-stage or integrated manner. Problems are often eliminated in a multi-stage and sequential method so that the output of one stage is regarded as the input of the next, resulting in reducing computational time and complexity. A large number of researchers have integrated the above-mentioned problems considering that the solutions may not be sufficiently optimal or unsolvable (Fig. 2) (Muter et al., 2013; Shao et al., 2017; Ben Ahmed et al., 2018).

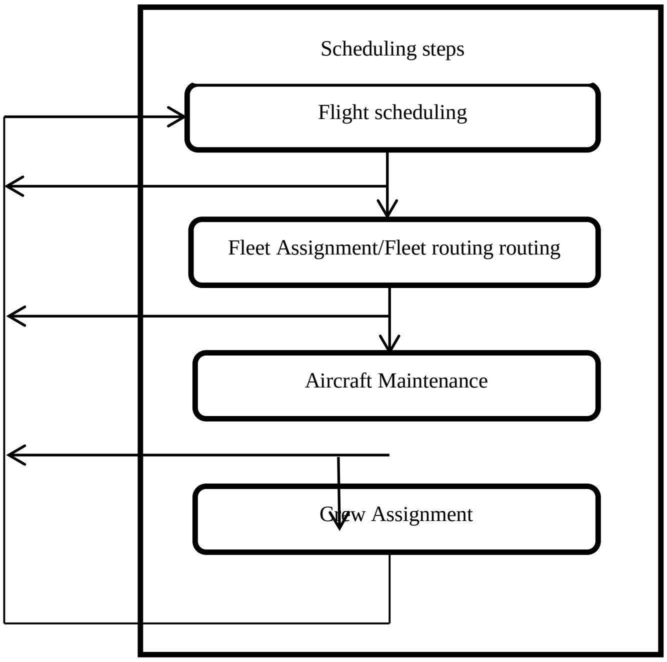

FIGURE 2
AIRLINE SCHEDULING

Flight scheduling, as the first step in all of the airlines, aims to create a timetable to display origins, destinations, numbers, and times. The aforementioned stage assesses various factors including information related to previous periods, forecasts, competitive market situation and demand, resources (aircrafts and manpower), regulations, and airline decision (Barnhart, 2008; Bazargan, 2010; Eltoukhy et al., 2017). Fleet assignment focuses on determining the type of fleet for each flight and identifying the network of fleet flow. In other words, the above-mentioned stage evaluates the type of fleet, not a specific aircraft (Sherali et al., 2006; Dozic and Kalic, 2015). Aircrafts are allocated to flight legs at aircraft routing stage to meet all of the maintenance requirements (Papadakos, 2009; Basdere and Bilge, 2014; Al-Thani et al., 2016). The aircraft maintenance is among the most critical problems. Finally, the crew assignment determines the sequences of flight legs and assigns their crew member. A sequence defines a pairing starts and ends at the same base. A large number of factors including airline decision and labor union rules are considered at the aforementioned stage (Kasirzadeh et al., 2015). The crew assignment, as a complex and difficult problem, is divided into crew pairing and rostering. This study discusses the aircraft routing and maintenance operations. In order to determine the appropriate time for performing aircraft maintenance operations, different factors have been considered during the recent years. The reference values of these factors are included in the aircraft manufacturer's manuals and maintenance planning documents. Furthermore, every part of the aircraft is maintained according to its usage function. Whichever of these reference values is reached first, the aircraft must enter into maintenance. Aircraft maintenance checks are normally driven by the number of days as well as, the number of hours an aircraft has flown, and also the total number of takeoffs cycle. The most significant of which includes performing such operations at the end of the day and during a three-day period. In other words, the aircrafts should be maintained during three days and necessarily at the end of the first, second, or third working day. The efficiency of the above-mentioned factor reduces because the flight hours of an aircraft in a day can be more or less. Therefore, the factor of threshold flight hours (maximum or cumulative flight time) was considered, as well. The number of takeoffs which leads to maintenance and repair of aircrafts before reaching the pre-determined limit is regarded as another critical factor. It is unrealistic and unreasonable to accept only one factor to control the aircraft maintenance. As can be seen recently, much attention is paid to define the framework of various factors for the maintenance schedule to control all the unforeseen conditions and instabilities. The aforementioned issues can be examined by investigating the studies more efficiently and finding the noticeable gaps. Therefore, the articles related to maintenance operations were categorized based on four categories. Here, 43 studies published in 24 journals during 2008-2022 are analyzed considering four categories including type of mode, maintenance and repair factor, disruption and robustness, as well as objective function and solution approach (Table 1). The aircraft routing and maintenance operation literature are studied as follows.

TABLE 1
THE NUMBER OF PUBLISHED PAPERS IN JOURNALS

Name of Journal	Number of papers
Transportation Research Part C: Emerging Technologies	2
Journal of Cleaner Production	1
Journal of Quality in Maintenance Engineering	1
European Journal of Operational Research	2
Computers & Operations Research	7
Management	1
Transportation Research Procedia	1
Transport. Sci	2
Omega	4
Computers & Industrial Engineering	3
Eur. J. Oper. Res.	1
Journal of Air Transport Management	5
Research gate	1
Industrial. Management & Data Systems	1
Mathematical programming	1
Transportation Research Part B	2
Int. J. Sustainable Aviation	1
WPOM-Working Papers on Operations Management	1
Journal of Aeronautics and Space Technologies	1
Expert Systems with Applications	1
Oper. Res	1
Transportation Research Part E	2
Ann. Tourism Res.	1

LITERATURE REVIEW

The aviation industry is involved in a growing trend of research in aircraft routing planning and maintenance operations. Recent studies on aircraft routing are assessed here. The existing literature is categorized from several perspectives for a more detail and through review of the studies. The above-mentioned category includes type of model, maintenance and repair factors, disruption and robustness, as well as objective function and solution approach, which are evaluated as follows.

- *Type of model*

As indicated, some studies have considered the steps related to airline scheduling including flight scheduling, fleet assignment, aircraft routing, and crew assignment in a multi-stage or integrated manner. The following researchers addressed and integrated the ARP with other problems related to airline planning. Ben Ahmed et al. (2022) assessed fleet assignment, aircraft routing, and crew assignment to formulate the large-scale integer programming. In addition, Mirjafari et al. (2022) developed an integrated model to assign crew and aircraft to flights considering the risk of COVID-19 infection, and designed another model for aircraft routing and crew scheduling regarding the aircraft replacement, fairly night flights assignment, and long-life aircraft (Mirjafari et al., 2020). Further, Parmentier and Meunier (2020) described the large-scale problems for the aircraft and crew assignment.

Furthermore, Ben Ahmed et al. (2018) proposed an integrated model for aircraft routing and crew pairing based on the disruption, delay, cancelation, and maintenance requirements. In another study, Cacchian and Salazar-González (2019) examined the flight scheduling considering its re-timing. Salazar-González (2014) investigated aircraft routing, crew scheduling, and fleet assignment at the Canary Islands Airlines (Spain). In addition, Diaz Ramirez et al. (2014) studied the aircraft routing and crew scheduling. To this aim, the aforementioned problems were formulated and eliminated in sequential steps. Then, an integrated model was provided to eliminate the above-mentioned problems simultaneously. Further, Dunbar and et al. (2014 and 2012) reviewed the aircraft routing and crew assignment. Furthermore, Wade et al. (2010) discussed aircraft routing and crew scheduling simultaneously, resulting in achieving the minimum cost and robust schedule.

TABLE 2
TYPE OF MODEL

		Type of model: Integrated		
Authors / Year	Flight scheduling	Fleet assignment	Aircraft routing	Crew scheduling
Ben Ahmad et al. (2022)		*	*	*
Mirjafari et al. (2022)			*	*
Parmentier and Meunier (2020)			*	*
Mirjafari et al. (2020)			*	*
Cacchian and Salazar-Gonzalez (2019)	*	*	*	*
Ben Ahmed et al. (2018)			*	*
Diaz-Ramirez et al. (2014)			*	*
Salazar-Gonzalez (2014)		*	*	*
Dunbar and et al. (2014)			*	*
Dunbar and et al. (2012)			*	*
Weide et al. (2010)			*	*

- *Maintenance and repair factors*

Working days have been used the most among the several factors for the necessity of maintenance operations (Shaukat et al., 2020; Parmentier and Meunier, 2020; Cacchian and Salazar-González, 2019; Bugaj et al., 2019; Safaei and Jardine, 2018; Hu et al., 2015; Salazar-González, 2014; Dunber et al., 2014; Weide et al., 2010). Various factors including the number of working days, aircraft flight hours, number of takeoffs, and their combined framework have been offered for implementing maintenance operations due to the different flight hours of the aircrafts and necessity of maintenance operation during the working day, not necessarily at the end of the day. Table 3 indicates the studies which utilized other maintenance factors, along with the working days. Thus, the aircraft should undergo maintenance operations when its flight hours meet the threshold (Barreto et al., 2021; Mirjafari et al., 2020; Ben Ahmed et al., 2018; Eltoukhy et al., 2017 and 2018).

Some studies have defined a framework for the number of takeoffs and flight hours for maintenance operations. Therefore, the aircraft undergoes maintenance operation when the number of its take-offs and flight hours reach the pre-defined value and allowed threshold, respectively (Ben Ahmad et al., 2022; Ruan et al., 2021; Sanchez et al., 2020; Liang et al., 2018). In another study, Jamili (2017) and Altani et al. (2016) defined the number of working days and aircraft takeoffs as a maintenance framework. Mirjafari et al. (2022) analyzed the number of days, take-offs, and flight hours for scheduling aircraft maintenance operations. Maintenance operation is performed whenever any of the aforementioned factors reaches the pre-defined limit.

TABLE 3
MAINTENANCE AND REPAIR FACTORS

	Maintenance and repair factors		
Authors / Year	Number of day	Number of takeoffs	Flight hour
Ben Ahmad et al. (2022)		*	*
Mirjafari et al. (2022)	*	*	*
Barreto et al. (2021)			*
Ruan et al. (2021)		*	*
Sanchez et al. (2020)		*	*
Shaukat et al. (2020)	*		
Parmentier and Meunier (2020)	*		
Mirjafari et al. (2020)			*
Cacchian and Salazar-González (2019)	*		
Bugaj et al. (2019)	*		
Safaei and Jardine (2018)	*		
Ben Ahmed et al. (2018)			*
Liang et al. (2018)		*	*
Eltoukhy et al. (2018)			*
Eltoukhy et al. (2017)			*
Jamili (2017)	*	*	
Altani et al. (2016)	*	*	
Hu et al. (2015)	*		
Salazar-González (2014)	*		
Dunber et al. (2014)	*		
Weide et al. (2010)	*		

- *Disruption and robustness*

An airline faces unpredictable occurrences such as global events, delays, cancellations, diseases, weather conditions, air accidents, and the like in its internal and external environment. Disruptions and costs arise for all of the airlines when such events occur. The method of controlling alterations, reducing costs, and benefitting from a robust schedule plays a critical role. Thus, retiming, robustness, as well as rescheduling and recovering operations have attracted a lot of attention. Wade et al. (2010) applied an iterative approach to solve the integrated model and assessed using the airline dataset, resulting in adopting less costly and more robust approach. In addition, Dunbar et al. (2014 and 2012) evaluated the aircraft routing, crew assignment, and reassignment due to unexpected events. Further, Hu et al. (2015) discussed the aircraft reassignment and recovery, as well as the problems created for passengers due to the above-mentioned disruption.

Furthermore, Jamili (2017) examined the method of assigning flight schedule to fleets and aircraft assignment considering issues such as disruptions in flights and fleet legs, as well as alternation in the aircraft by the crew during their flight to destination. In another study, Liang et al. (2018) investigated the cancellation, delay, and recovery schedule. Cacchian and Salazar-González (2019) highlighted the flight retiming and increasing the robustness of the solution to consider delays. Finally, Ben Ahmed et al. (2022) studied the method of minimizing delays and cancelling the flights.

- *Objective function and solution approach*

This section reviews the literature considering the objective function and solution approach. Belien et al. (2010) analyzed the maintenance routing considering the crew maintenance scheduling and decisions, as well as minimum cost, addressed the issue of inspections emphasizing that aircraft should be subjected to short inspection operations between their arrival and departure times, formulated the mixed integer linear programming (MILP), and used the heuristic approach to solve the proposed model based on the real-world data. In addition, Orhan et al. (2011) proposed an ILP to maximize the remaining time for maintenance and utilized a simultaneous mathematical modeling to the daily flight route and maintenance schedule. The model is solved applying GAMS software. The adopted approach was applied to the airlines, which obtained acceptable results. Further, Dunbar and et al. (2014) assessed the method of minimizing the unforeseen delays and its costs, as well as finding the relatively optimal solution. Furthermore, Dunber et al. (2012) evaluated the approaches to solve their integrated model, which included a column CG algorithm and an iterative approach to estimate delays. The results indicated a significant reduction in delays in the case study.

In another study, Basder and Bilidge (2014) studied the aircraft maintenance by providing a rapid and feasible maintenance routes for each aircraft to maximize the total remaining time of the fleet. To this aim, the exact and heuristic method was used, indicating that the first one does not eliminate the large-scale problem and the second one provides the possible solution. Diaz Ramirez et al. (2014) collected real datasets including the flight schedule for three Latin American airlines (522 flights). Considerable improvement was observed in large-scale problems although the integrated model generated a slight improvement over solving the models separately. In addition, Salazar-González (2014) formulated the ILP model and solved its model for each day during 7 am-11 pm utilizing multi-objective solution software, leading to a satisfactory result. Further, Hu et al. (2015) examined the method of assigning aircraft to minimize the cost of aircraft reassignment and passengers. Furthermore, Al-Thani et al. (2016) defined the aircraft routing as one of the most complex issues in the airline scheduling. To this aim, all of the aircrafts were assigned to flights after proposing the mathematical model to minimize the flight time, as well as considering the maintenance inspection. The model was solved applying the heuristic algorithm based on the real data (354) related to flights and eight aircrafts. Based on the results, the algorithm provided relatively optimal solutions. In another study, Jamili (2017) formulated a mathematical model and solved with the simulated annealing (SA) and particle swarm optimization (PSO). To this aim, random examples were generated. Based on the results, an integrated algorithm in large size scales gave a more efficient and robust solution.

Eltoukhy et al. (2017) offered an ILP for aircraft maintenance routing using two critical maintenance constraints including the cumulative flight time limit and capacity of the workforce. To this aim, a new approach was presented and validated utilizing 12 real data from the Egypt airline. The results indicated that the approach provided the optimal outcomes in less time. In addition, Eltoukhy et al. (2018) expanded their model and described a solution approach based on the heuristic ones, resulting in solving the aircraft maintenance routing in a shorter time. Each aircraft exhibits different maintenance requirements at different times, resulting in complicating the related decisions. Therefore, the frequent type of maintenance operation is commonly considered. Further, Safaei and Jardine (2018) focused on the aircraft routing and maintenance operation considering the full range of maintenance requirements in order to reduce maintenance misalignment. To this aim, the iterative approach and real data were applied to investigate the proposed model. Furthermore, Liang et al. (2018) reviewed the airport capacity and flexibility maintenance, delay and cancellation, as well as flight swapping to minimize the recovery cost and satisfy the maintenance requirements. To this aim, the column generation (CG) approach was used to solve the model. Based on the results, the cost of recovery reduced and large-scale problems were eliminated in a short time.

In another study, Ben Ahmed et al. (2018) focused on scheduling robust routes to reduce costs and meet all of the constraints. To this aim, the model was solved by a general-purpose solver and real data from major airlines were utilized. The results indicated the cost-effective and robust solutions. Bugaj et al. (2019) addressed several factors including size and age of fleet, type of aircraft and maintenance operations, ownership or leasing status of aircraft, and effect of environment on the security and profitability. The results indicated that the maintenance reserve is considered as an effective factor in re-timing and keeping the profitability. Bugaj et al. (2019) focused on managing the reserved operations maintenance and applied the real-world data to analyze the proposed problem. Flight re-timing includes correcting the pre-defined departure times to reduce costs and provide better services.

In addition, Cacchian and Salazar-González (2019) discussed flight scheduling and re-timing, and used four heuristic algorithms and real dataset to solve their mixed integer model. Based on the results, the solution reduced costs and created a sustainable schedule. Further, Mirjafari et al. (2020) proposed the integrated method and solved the model utilizing Lagrangian relaxation and PSO algorithm, as well as GAMS software. The numerical examples in small, medium, and large sizes were generated. Based on results, Lagrangian relaxation approach provided better solutions with a small gap to optimum solution. Furthermore, Sanchez et al. (2020) considered a multi-objective integer programming for the aircraft maintenance routing to minimize the number of faults in maintenance regulations and control the number of aircraft which cannot fly. To this aim, a framework including flight hours, number of takeoffs, and workforce capacity was defined to control the maintenance operation and an iterative method was presented to eliminate the problem applying real datasets (16,000 flights, 529 aircraft, and 8 maintenance workshops). The results indicated that the solution approach can create almost optimal maintenance programs in minutes.

In another study, Shaukat et al. (2020) analyzed the aircraft maintenance routes and proposed the mixed integer problem by assigning duties to maintenance opportunities in order to reduce the deviation from the pre-determined schedule, delay, and cost, as well as increasing the flexibility. To this aim, the heuristic and exact methods, as well as real datasets were used to solve the model. Based on the results, the problems were eliminated in ten minutes with improved maintenance operations. Parmentier and Meunier (2020) assessed the method of assigning aircraft and crew by the model and used the CG algorithm and real-world data considering the source limitations. The results indicated that the solution approach can solve the large cases. In addition, Deng et al. (2020) focused on optimizing the schedule of operations inspections and proposed that the model minimizes unused flight hours which directly reduce the number of aircraft inspections. The dynamic programming (DP) approach was applied based on the real dataset. The results presented a reduction in maintenance costs and an increase in revenue for more aircraft due to the available aircraft.

Further, Ruan et al. (2021) evaluated the aircraft maintenance routing. First, an integer programming framework was defined considering the maximum flight hours, number of takeoffs, and work-force capacity to control the maintenance operations. Then, a new learning-based algorithm was developed to eliminate the problem effectively. The algorithm generated quality solutions for medium and large-scale maintenance routing.
Furthermore, Bulbul and Kasimbeyli (2021) examined the aircraft maintenance as a traveling salesman problem considering the limitations of maintenance operations, available resources, and fleet size with a large-scale network. To this aim, a hybrid approach including Lagrangain relaxation and ACO algorithm was used to solve the model and acquire the quality results. In another study, Barreto et al. (2021) investigated the aircraft assignment based on the preventive and corrective maintenance requirements. To this aim, the mixed integer problem was formulated and preventive maintenance and failure prognostics were considered to minimize the total connection costs between ordinary and cancelled flights. Flight hours of aircraft were determined to control the maintenance operation. The results indicated an improvement in the overall aircraft routing scheduling.

Mirjafari et al. (2022) focused on airports as high-risk places in COVID-19 pandemic to minimize the difference between crew sit times. Flight hours and number of days and take-offs were defined for maintenance requirements. Several numerical problems were solved utilizing GAMS and PSO which enhanced CPU time and gap by 98.279 and 1.902% on average, respectively. In addition, Ben Ahmed et al. (2022) formulated the large-scale integer programming to maximize the total profit (total revenue - fleet assignment costs). Model was solved applying the meta-heuristic algorithm. Based on results, the approach presented high-quality solutions.

Heuristic, meta-heuristic, and hybrid heuristic algorithms were used to obtain a relatively optimal solution due to the complexity and unsolvable nature of the aforementioned scheduling problems. Table 4 presents a summary for the recent studies on aircraft routing problem and maintenance operation considering the objective function and solution approach.

TABLE 4
OBJECTIVE FUNCTION AND SOLUTION APPROACH

Authors / Year	Objective function	Solution approach
Ben Ahmad et al. (2022)	Maximize the total profit	Decomposition approach Proximity search
Mirjafari et al. (2022)	Minimize the Sit time	Particle swarm optimization
Barreto et al. (2021)	Minimize the total connection costs	branch, price and cut algorithms
Bulbul and Kasimbeyli (2021)	maximize the total through value of the route	Lagrangian relaxation Ant colony optimization
Ruan et al. (2021)	Maximize the expectation of the cumulative rewards	Learning-based algorithm
Deng et al (2020)	Minimize the wasted interval between checks	Dynamic programming
Parmentier and Meunier (2020)	Reduce the total computing time	Column generation
Shaukat et al. (2020)	Minimize the deviation from this schedule	Integrated exact solution Sequential approach
Sanchez et al. (2020)	Minimize maintenance violence	Iterative algorithm
Mirjafari et al. (2020)	Minimize the cost	Lagrangian relaxation Particle swarm optimization
Cacchian and Gonzalez (2019)	Minimize the cost	Column generation
Bugaj et al. (2019)	Reduce the costs Provide better service	New algorithm
Ben Ahmed et al. (2018)	Reduce the costs	General-purpose solver
Liang et al. (2018)	minimize the recovery cost	Column generation
Safaei and Jardine (2018)	Minimize the maintenance misalignments	Branch-and bound, Local search
Eltoukhy et al. (2018)	Improve the profit	Compressed annealing
Eltoukhy et al. (2017)	Improve the profit Solution in much less time	Ant colony optimization Simulated annealing Genetic algorithm
Jamili (2017)	Provide the robust solution	Simulated Annealing Particle Swarm Optimization
Altani et al. (2016)	Improve the solvability	Neighborhood search
Hu et al. (2015)	Minimize the cost	Exact algorithm
Basder and Bilidge (2014)	Maximize the Remaining time	Branch-and-bound
Diaz-Ramirez et al. (2014)	Improve the profit Maximize the profit	Column generation
Salazar-Gonzalez (2014)	Minimize the worst-case features among assignment	General-purpose solver
Dunbar and et al. (2014)	Minimize delay propagation Improve the profit	Iterative algorithm
Dunbar and et al. (2012)	Minimize the cost of propagated delay	Iterative algorithm
Orhan et al. (2011)	Maximize the Remaining time	Exact algorithm
Belien et al. (2010)	Minimize the cost	Branch-and-bound
Weide et al. (2010)	Minimize the cost Provide the robust solution	Iterative solution

- *Network description*

The flight network includes a series of arcs and nodes. Each event (takeoffs) creates the nodes and the line connecting the nodes indicates a city and its events. All of the directional lines in the flight network are regarded as arcs, which are divided into two categories including flight and ground ones.

• A flight arc is located between two cities and the time for the end node is after that for the beginning node.

• A ground arc connects two consecutive nodes in a city.

Sequences of flight network arcs on which a type of aircraft can exist are considered as a flight cycle. A loop in a network is regarded as a flight cycle and the first node in the first arc is considered as the same as the end node of its last arc. A pattern contains a series of loops covering the entire flight network and allocates the existing fleet aircraft to the pattern loops. Finally, all of the flights can be operationalized. Fig. 3 illustrates the aircraft routing network G = (V, A) considering maintenance requirements. As observed, V and A include set in all of the nodes, while arcs in G and N represent all of the legs. In addition, set E includes all of the legal joints among the legs. Further, s and t indicate starting and ending nodes (source and sink). Refer to the Diaz Ramirez et al. (2014) for more details.

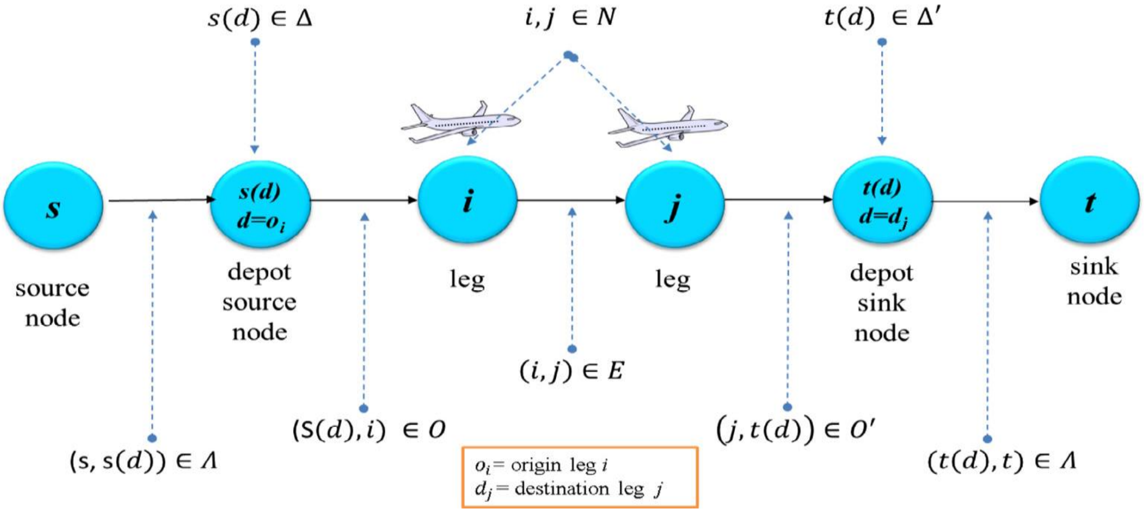

FIGURE 3
CONNECTION NETWORK

Aircraft routing model

Generally, the aircraft routing model seeks to minimize the total cost of selected routes. Model details including sets, indices, and variables are described as follows. F ($i \in F$) and R ($j \in R$) indicate the set of flights and feasible routings, respectively. In addition, C_j is regarded as the cost of route. Further, j and N represent the total number of aircraft in the fleet. Furthermore, a_{ij} is considered as a binary parameter indicating whether flight i is covered by route j. x_{ij} (decision variable) equals 1 and otherwise 0 when route j is selected.

$$\sum_{j\in J} c_j\, x_j \tag{1}$$

$$\sum_{j\in R} a_{ij}\, x_j \qquad For\ all\ i \in F \tag{2}$$

$$\sum_{j\in R} x_j \le N \qquad For\ all\ j \in R \tag{3}$$

$$x_j \in \{0,1\} \tag{4}$$

The objective function (1) aims to minimize the cost. Equation (1) indicates each flight is covered by only one route. Equation (2) limits the number of selected routes to aircraft. Hein et al. (1995) modeled the aircraft assignment problem as a flow in the multi-commodity network which minimizes the total cost of the assignment. The model contains C and F which define the set of airports and available legs, while S (f) describes the number of aircrafts. In addition, L indicates a set of scheduled flights which are displayed as {i} or {odt} including the origin-destination-time. Further, O (f) represents the set of flight arcs. Furthermore, H is regarded as a set of consecutive flights to be operated by one aircraft and N is considered as a set of network nodes {fot}. Parameter C_{fi} determines the operating cost of aircraft including the costs of fuel and lost passenger. The binary variable x_{fodt} or x_{fi} equals 1; otherwise, it is 0 when an aircraft flies between two cities. The number of aircrafts on the ground in the city is defined by variable y_{fott^+}. Assigning aircraft to flight was proposed as a mathematical model which is described as follows. Refer to Hane and et al. (1995) for more details.

$$\sum_{i\in L}\sum_{f\in F} C_{fi}x_{fi} \quad (5)$$

$$\sum_{f} x_{fi} = 1 \qquad \forall i \in L \quad (6)$$

$$\sum_{d} x_{fodt} + y_{fott^{+}} + y_{fot^{-}t} + \sum_{d} x_{fdot} = 0 \qquad \forall\{\text{fot}\} \in \text{N} \quad (7)$$

$$x_{fi} + x_{fj} = 0 \qquad \forall (i,j) \in H, f \in F \quad (8)$$

$$\sum_{i\in o(f)} x_{fi} + \sum_{t\in T} y_{fot_n t_1} \le S(f) \qquad \forall f \in F \quad (9)$$

$$y_{fott^{+}} \ge 0 \qquad \forall\{\text{fot}\} \in \text{N} \quad (10)$$

$$x_{fi} \in \{0,1\} \qquad \forall i \in L, f \in F \quad (11)$$

Díaz-Ramírez et al. (2014) formulated the AMRP as a longest path problem considering the maintenance requirements to maximize the total profit (p_{ij}) of each arc (i, j). N indicates the nodes which present the flight legs (i∈ N) including the identification number (id_i), origin (o_i), destination (d_i), as well as departure and arrival time (d_{ti},a_{ti}). In addition, E= {(i, j) |i, j∈ N} is regarded as the set of arcs which present legal connections. Further, Δ= $\{s(d), \forall d\}$ and Ά= {t (d),$\forall d$} represent set of source and sink, respectively. y_{ij} equals 1 and otherwise 0 when arc (i, j) is considered as part of the route. The objective function (12) seeks to maximize the profit. Equation (13) ensures the balance of flow-in and flow-out to a node. Constraints (14) and (15) include the solution to one route $s(d) - t(d)$. Equation (16) refers to the maintenance constraints. Finally, Equation (17) defines the type of decision variable.

$$\sum_{(i,j)\in A} p_{ij}\, y_{ij} \quad (12)$$

$$\sum_{j;(i,j)\in A} y_{ij} - \sum_{j;(j,i)\in A} y_{ij} = 0 \qquad \forall i \in V\ \{s,t\} \quad (13)$$

$$\sum_{(s,s(d))\in A} y_{s,s(D)} = 1 \qquad \forall s(d) \in \Delta \quad (14)$$

$$\sum_{(t(d),t)\in A} y_{t(d),t} = 1 \qquad \forall t(d) \in Ά \quad (15)$$

$$\sum_{v} \theta_{u,v} \sum_{(i,j)\epsilon B(v)} y_{ij} \ge 1 \qquad \forall u \quad (16)$$

$$y_{ij}, \forall (i,j) \in A \quad (17)$$

According to different conditions and factors in the aviation industry, several development models are proposed to solve the aircraft routing problem based on the general models.

CONCLUSION AND FUTURE RESEARCH REMARKS

As indicated, airline organizations face various demands, numerous laws, unforeseen events, and complex operations, which incur huge costs when they lack proper schedules to perform their operations. Thus, airline scheduling is regarded as the basis for all of the airline activities. Aircraft routing problem is among the critical steps of scheduling, which is discussed here. An extensive amount of literature was presented on the above-mentioned problem. The present study seeks to provide and categorize wide literatures on aircraft routing and maintenance operations based on four pre-defined categories including type of model, maintenance and repair factors, disruption and robustness, as well as objective function and solution approach.

To this aim, different studies are reviewed based on the type of models which are defined as multi-stage or integrated, and relatively better results are obtained in the integrated research (Table 2). Then, the studies are discussed considering the definition provided for the structure of maintenance and repairs operation so that three factors including working days, flight hours, and number of takeoffs are utilized the most. Based on the recent studies, the framework combines the factors of working days, flight hours, and number of aircraft takeoffs to generate a better and more accurate schedule to control the aircraft routing maintenance operations (Table 3).
The studies focused on the robustness of aircraft routing schedule, as well as recovery and re-timing, and disruptions or unforeseen events were categorized considering the experience of the infection during the last three years, leading to a huge alteration in the performance of all of the industries.

As represented in Table 4, the studies are analyzed based on the objective function and solution approach to find which objective function and algorithms have attracted most or least attention. As observed, applying a hybrid structure for meta-heuristic algorithms has attracted a lot of attention. The airlines should focus on an optimal planning to achieve maximum benefits and minimum cost, as well as finding and managing a robust schedule against all of the unpredicted events due to its complexity, high costs, and numerous unforeseen challenges in this field. Certainly, airline organizations are faced with high limitations in using their own resources. Based on recent research, hybrid methods to solve airline schedule problems along with defining a combined structure for maintenance and repairs make airline plans more practical and applicable. Stability, robustness and also the fastest adaptation of an airline's schedule to unpredicted challenges, needs more attention and future research. Future studies should focus on the aforementioned issue more than before.

REFERENCES

[1] Al-Thani, Nayla Ahmad, Ben Ahmed, Mohamed, Haouari, Mohamed, (2016). A model and optimization-based heuristic for the operational aircraft maintenance routing problem, Transportation Research Part C: Emerging Technologies, Vol. 72, No. C, pp. 29–44.

[2] Amankwah-Amoah, J. (2020) 'Stepping up and stepping out of COVID-19: new challenges for environmental sustainability policies in the global airline industry', Journal of Cleaner Production, Vol. 271, p.123000, https://doi.org/10.1016/j.jclepro.2020.123000.

[3] Bae, Ki-Hwan (2010). Integrated Airline Operations: Schedule Design, Fleet Assignment, Aircraft Routing, and Crew Scheduling, Dissertation submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Industrial and Systems Engineering.

[4] Barnhart, C. (2008). Airline scheduling: Accomplishments, opportunities and challenges. Proceedings of the International Conference on Integration of AI and OR Techniques in Constraint Programming for combinatorial Optimization Problems.

[5] Barreto, Pereira, E.A., Abrahao, F.T.M.T.M. and Olivares Loesch Vianna, W. (2021). Aircraft routing problem model for fractional fleets using fault prognostics, Journal of Quality in Maintenance Engineering, Vol. 27, No. 3, pp. 517–536. https://doi.org/10.1108/JQME-06-2020-0053.

[6] Basdere, Mehmet and Bilge, Umit (2014).Operational aircraft maintenance routing problem with remaining time consideration, European Journal of Operational Research, Vol. 235, No. 1, pp. 315-328.

[7] Bazargan, Massoud (2010). Airline Operations and scheduling second edition, Embry-Riddle Aeronautical University, USA, Ashgate publishing limite.

[8] Belien, Jeroen, Demeulemeester, Eric and Brecht (2010). Integrated staffing and scheduling for an aircraft line maintenance problem, HUB RESEARCH PAPER.

[9] Ben Ahmed, M., Ghroubi, W., Haouari, M., Sherali, H.D (2017). A hybrid optimization simulation approach for robust weekly aircraft routing and retiming, Transport. Res. C Emerg. Technol. Vol. 84, pp. 1-20, DOI:10.1016/j.trc.2017.07.010.

[10] Ben Ahmed, Mohamed, Hryhoryeva, Maryia, Hvattum, Lars Magnus, Haouari, Mohamed (2022). A meta-heuristic for the robust integrated airline fleet assignment, aircraft routing, and crew pairing problem, Computers & Operations Research, Vol. 137, No. C, p.105551.
[11] Ben Ahmed, M., Zeghal Mansour, Farah and Haouari, Mohamed (2018).Robust integrated maintenance aircraft routing and crew pairing, Management, Vol. 73, No. 1, pp.15–31.
[12] Bugaj, M., Urminsky, T., Rostas, J. and Pecho, P., (2019), Aircraft maintenance reserves – new approach to optimization, Transportation Research Procedia, Vol. 43, pp.31–40.
[13] Bulbul, K. Gulnaz, Kasimbeyli, Refail (2021), Augmented Lagrangian based hybrid subgradient method for solving aircraft maintenance routing problem, Computers & Operations Research, Vol. 132, p. 105294, https://doi.org/10.1016/j.cor.2021.105294.
[14] Cadarso, L., Vaze, V., Barnhart, C., Marín, ´A, 2017. Integrated airline scheduling: Considering competition effects and the entry of the high speed rail. Transport. Sci. Vol. 51, No. 1, pp. 132–154.
[15] Cacchiani, V. and Salazar-González, J., (2019), Heuristic approaches for flight retiming in an integrated airline scheduling problem of a regional carrier, Omega, Vol. 91, No. C, DOI: 10.1016/j.omega.2019.01.006.
[16] Diaz-Ramirez, Jenny, Huertas, Jose Ignacio and Trigos, Federico (2014). Aircraft maintenance, routing, and crew scheduling planning for airlines with a single fleet and a single maintenance and crew base, Computers & Industrial Engineering, Vol. 75, No. 1, pp.68–78.
[17] Deng, Q., Santos, B.F., Curran, R., (2020). A practical dynamic programming based methodology for aircraft maintenance check scheduling optimization. Eur. J. Oper. Res., Vol. 281, No. 2, pp. 256–273.
[18] Dozic, s. and Kalic, M. (2015). Three-stage airline fleet planning model, Journal of Air Transport Management, Vol. 43, No. C, pp. 30–39.
[19] Dunbar, Michelle, Froyland, Gary, Wu, Cheng-Lung (2014). An integrated scenario-based approach for robust aircraft routing, crew pairing and re-timing, Computers & Operations Research, Vol. 45, pp. 68–86, http://dx.doi.org/10.1016/j.cor.2013.12.003.
[20] Dunbar, Michelle, Froyland, Gary, Wu, Cheng-Lung (2012). Robust Airline Schedule Planning: Minimizing Propagated Delay in an Integrated Routing and Crewing Framework, Research gate, Vol. 46, No. 2, pp. 204–216, https://www.researchgate.net/publication/259928703.
[21] Eltoukhy, A. E., Chan, F. T., Chung, S. H., Niu, B., Wang, X. P. (2017). Heuristic approaches for operational aircraft maintenance routing problem with maximum flying hours and man-power availability considerations. Industrial Management & Data Systems, Vol. 117, No. 6, pp. 1201–1243.
[22] Eltoukhy, A. E., Chan, F. T., Chung, S. H., Niu, B. (2018). A model with a solution algorithm for the operational aircraft maintenance routing problem, Computers & Industrial Engineering, Vol. 120, pp. 346-359.
[23] Hane, C., Barnhart, C., Johnson, E., Marsten, R., Nemhauser, G. and Sigismondi, G. (1995) "The Fleet assignment problem: solving a large-scale integer problem", Mathematical programming, Vol. 70, pp. 211-232.
[24] Hu, Yuzhen, Xu, Baoguang, F. Bard, Jonathan, Chi, Hong, Gao, Mingang, (2015), Optimization of multi-fleet aircraft routing considering passenger transiting under airline disruption, Computers & Industrial Engineering, Vol. 80, No. C, pp. 132–144.
[25] Jamili, Amin (2017). A robust mathematical model and heuristic algorithms for integrated aircraft routing and scheduling, with consideration of fleet assignment problem, Journal of Air Transport Management, Vol. 58, No. C, pp. 21–30.
[26] Kim, M. and Sohn, J. (2022) 'Passenger, airline, and policy responses to the COVID-19 crisis: The case of South Korea', Journal of Air Transport Management, Vol. 98, No. C, p. 102144.
[27] Liang, Zhe, Xiao, Fan, Qian, Xiongwen, Zhou, Lei, Jin, Xianfei, Lu, Xuehua, Karichery, Sureshan (2018). A column generation-based heuristic for aircraft recovery problem with airport capacity constraints and maintenance flexibility, Transportation Research Part B, Vol. 113, No. C, pp. 70–90.
[28] Mirjafari, M., Rashidi Komijan, A. and Shoja, A. (2022) 'An integrated model of aircraft routing and crew rostering problems to develop fair schedule for the crew underCOVID-19 condition', Int. J. Sustainable Aviation, Vol. 8, No. 2, pp. 162–180.
[29] Mirjafari, Masoumeh; Rashidi Komijan, Alireza; Shoja, Ahmad (2020). An integrated model for aircraft routing and crew scheduling: Lagrangian Relaxation and meta-heuristic algorithm. WPOM-Working Papers on Operations Management, Vol. 11, No. 1, pp. 25-38. doi: https://doi.org/10.4995/wpom.v11i1.12891
[30] Muter, Ibrahim, Birbil, S. Ilker, Bulbul, Kerem, Şahin, Guvenç, Yenigun, Husnu, Tas, Duygu, Tuzun, Dilek (2013). Solving a robust airline crew pairing problem with column generation. Computers & Operations Research, Vol. 40, No. 3, pp. 815–830.
[31] Orhan, I., Kapanoglu, M., & Karakoc, t. H. (2011). Concurrent Aircraft and Maintenance Scheduling. Journal of Aeronautics and Space Tecnologies, Vol. 5, No. 1, pp. 73–79.
[32] Papadakos, N., (2009). Integrated airlines scheduling. Computers and Operations Research, Vol. 36, No. 1, pp. 176–95.
[33] Parmentier, A. and Meunier, F., (2020), Aircraft routing and crew pairing: Updated algorithms at Air France, Omega, Vol. 93, No. C, p. 102073.
[34] Ruan, J.H., Wang, Z.X., Chan, F.T.S., Patnaik, S., Tiwari, M.K., (2021). A reinforcement learning-based algorithm for the aircraft maintenance routing problem, Expert Systems with Applications, Vol. 169, p. 114399, doi: https://doi.org/10.1016/j.eswa.2020.114399.
[35] Safaei, Nima and K.S.Jardine, Andrew (2018). Aircraft routing with generalized maintenance constraints, Omega, Vol. 80, pp. 111-122, https://doi.org/10.1016/j.omega.2017.08.013.
[36] Salazar-González, Juan-José (2014). Approaches to solve the fleet-assignment, aircraft-routing, crew-pairing and crew-rostering problems of a regional carrier, Omega, Vol. 43, No. C, pp. 71–82.
[37] Sanchez, David Torres, Boyacı, Burak, Zografos, Konstantinos G., (2020). An optimization framework for airline fleet maintenance scheduling with tail assignment considerations, Transportation Research Part B: Methodological, Vol. 133, pp. 142-164.
[38] Shao, S., Sherali, H.D., Haouari, M., (2017), A novel model and decomposition approach for the integrated airline fleet assignment, aircraft routing, crew pairing problem, Transport. Sci, Vol. 51, No. 1, pp. 233-249.
[39] Shaukat, S., Katscher, M., Wu, Ch., Delgado, F., Larrain, H., (2020), Aircraft line maintenance scheduling and optimization, Journal of Air Transport Management, Vol. 89, No. C, p.101914.
[40] Sherali, H. D., Bish, E. K., and Zhu, X., (2006). Airline fleet assignment concepts, models, and algorithms. European Journal of Operational Research, Vol. 172, No. 1, pp. 1–30.
[41] Sun, X., Wandelt, S., Zheng, C. and Zhang, A. (2021) 'COVID-19 pandemic and air transportation: successfully navigating the paper hurricane', J. Air Transp. Manag., Vol. 94, No. 5, p. 102062.

[42] Tekiner, H., Birbil, S¸. ˙I. Bülbül, K., 2009. Robust crew pairing for managing extra flights. Comput. Oper. Res. Vol. 36, No. 6, pp. 2031–2048.

[43] Weide O, Ryan E, Ehrgott M. (2010). An iterative approach to robust and integrated aircraft routing and crew scheduling, Computers and Operations Research, Vol. 37, No. 5, pp. 833–844.

[44] Wang, K., Jacquillat, A., 2020. A stochastic integer programming approach to air traffic scheduling and operations. Oper. Res. Vol. 68, No. 5, pp. 1375–1402.

[45] Wen, X., Ma, H.L., Chung, S.H., Khan, W.A., 2020. Robust airline crew scheduling with flight flying time variability. Transport. Res. Part E: Logist. Transport. Rev., Vol. 144, No. C, pp.102–132.

[46] Wen, X., Sun, Xuting, Sun, Yige, Yue, Xiaohang, 2021. Airline crew scheduling: Models and algorithms. Transportation Research Part E, Vol. 149, No. C, p. 102304.

[47] Yang, Y., Zhang, H. and Chen, X. (2020) 'Coronavirus pandemic and tourism: dynamic stochastic general equilibrium modeling of infectious disease outbreak', Ann. Tourism Res., Vol. 83, No. C, p.102913.