Original Research

Copula Approach for Reliability and Performance Estimation of Manufacturing System

Ibrahim, Yusuf [1*]. Abdullahi, Sanus[2]

Received: 05 Jan 2022/ Accepted: 03 Jan 2023/ Published online: 03 Jan 2023

*Corresponding Author, iyusuf.cs@buk.edu.ng

1- Department of Mathematical Sciences Bayero University, Kano.
2- Bayero University, Kano.

Abstract
This study focuses on an investigation into the analysis of reliability measures used to determine the strength of a serial manufacturing system comprising of three subsystems A, B, and C in the form of units, conveyors, and processors. Subsystem A has three parallel active units, whereas subsystems B and C each have two units. The system is analyzed using the linear differential difference equation, supplementary variable technique and Gumbel-Hougaard family of copula to obtain expressions of reliability measures for determining system strength such as availability, reliability, mean time to failure (MTTF), and profit function. Numerical examples are provided to illustrate the obtained results and analyze the effects of various system parameters. The current study may assist manufacturing, industries, reliability engineers and their repairers in alleviating some of the challenges faced by repairers in certain manufacturing and industrial systems operating in harsh environments or under unfavorable weather conditions.

Keywords- reliability; availability; mean time to failure; profit; manufacturing system

INTRODUCTION

The copula approach is a technique for calculating joint distributions using marginal distributions, in which the variables are non-normal. Copulas can also be used to analyze pairs of random variables in a nonparametric manner. The copula of a multivariate distribution describes not only the correlations of the random variables, but also the dependence structure. It has the advantage in that it does not assume marginal to be normal or independent and marginal can be any number. Sklar was the first to introduce Copula (1973). Since then, copula analysis has adopted new dimensions and analyses. Several researchers have previously presented copula methods in the field of system reliability and performance analysis by examining system performance under different conditions. Nelsen (2006), for example, used a copula to relate a multivariate distribution to a one-dimensional marginal distribution function. Abubakar and Singh (2019) analyzed the performance of industrial system using copula linguistics. Gulati et al. (2016) focused on the performance of a complex system in series configuration with various failure and repair scenarios. Gahlot et al. (2018) presented a system performance assessment for serial configuration. Tyagi et al. (2021) presented a copula analysis for parallel system with fault coverage. Chopra and Ram (2021) presented reliability measures of two dissimilar units in parallel using Gumbel-Hougaard copula. Maihulla et al. (2021) discussed the copula approach for performance and reliability evaluation of series parallel.

Sha (2021) presented copula reliability analysis for hybrid systems. The conditional copula and its application to time series analysis were introduced by Patton (2009). The application of copula in financial management was the topic of Rodriguez (2007). Trivedi and Zimmer (2007) capture the application of copula in multivariate distributions. Chopra and Ram (2019) used the Gumbel-Hougaard family copula to examine the reliability of a dissimilar parallel system with two units. Ram and Singh (2008) investigated the availability and cost analysis of a complex parallel system subject to two types of failures and preemptive resume repair using the Gumbel-Hougaard family copula. Ram and Singh (2010) used the Gumbel-Hougaard family copula to analyze the MTTF, cost, and availability of a system undergoing preventive maintenance.

Many manufacturing companies cannot operate without dependable systems. As a result, working engineering systems are expected to run at full efficiency for the longest possible time, that is stable operation. Many manufacturing companies, such as those in the chemical, sugar, sofa or curtain, cardboard paper, and fertilizer industries, have realized the value of stable operation.

The performance of a production system essentially depends on the performance of the floor of the workshop (Farahani, 2019). To achieve maximum output and profit, it is critical that all processes remain in continuous service for an extended period of time. Though failure is unavoidable due a variety of factors that results in significant production losses. However, these failed systems can be repaired or replaced and returned to service in the shortest amount of time possible. To achieve this, modeling and study of repairable complex structures are the only ways. When systems are mathematically modeled and evaluated in real-world environments, their efficiency can be quantified in terms of mean time to failure, availability, cost benefit and reliability.

The importance of reliability and performance evaluation in various industrial, manufacturing and production settings have caught the attention of many researchers in the light of the aforementioned reality. To mention few, Wang et al. (1993) considered the reliability of a versatile manufacturing method based on fuzzy data. Using novel Markov models, Du et al. (2018) estimated the final product's quality state based on the state transfer likelihood of key parameters in the manufacturing process. Mogil et al. (2013) used the reliability analysis methodology in the shoe manufacturing industry. The shoe upper manufacturing unit system is taken into account in order to determine the system’s time-dependent and long-term availability. Chiang (2006) investigated the efficiency of a serial manufacturing system with a buffer and a testing unit, demonstrating the Markov reliability model’s and Bernoulli model’s application areas. Meerkov (2009) modeled serial manufacturing as a Bernoulli production line and concluded that as machine reliability improves, system efficiency appears to increase monotonically. He et al. (2015) presented a reliability modeling and optimization strategy for multi-station manufacturing systems based on the RQR chain. Kumar et al. (2010) used fuzzy reliability in their mathematical simulation and study of a stainless-steel utensil manufacturing unit. Li et al. (2010) proposed a grey model-based prediction model for manufacturing system reliability. Muthiah and Huang (2006) conducted a literature review on measuring and improving the efficiency of manufacturing systems. He et al. (2018) presented a cost analysis for predictive maintenance using a cyber manufacturing system. He et al. (2015) developed model for reliability and optimization plan for manufacturing system based on RQR chain. He et al. (2017) created a mission reliability model based on a quality state task network for a multi-station manufacturing system. Kumar and Kumar (2010) used a fuzzy reliability approach to develop mathematical models for the analysis of a stainless-steel utensil manufacturing unit. Lin and Jun (2008) discussed the estimation of manufacturing system reliability based on operational data. Quality and reliability maintenance for multistage manufacturing systems subject to condition monitoring were discussed by Lu and Zhou (2019). Li and Ni (2008) discussed manufacturing system reliability estimation based on operational data. Li et al. (2010) created a model for predicting reliability in manufacturing systems. Lin and Jun (2008) discussed the estimation of manufacturing system reliability based on operational data. Quality and reliability maintenance for multistage manufacturing systems subject to condition monitoring were discussed by Lu and Zhou (2019). Li and Ni (2008) discussed manufacturing system reliability estimation based on operational data. Li et al. (2010) created a model for predicting reliability in manufacturing systems. Savsar (2000) examined the reliability of a flexible manufacturing cell. Zhang et al. (2017) discussed the modeling and performance analysis of a multistage serial manufacturing system that takes rework and product polymorphism into account. Zhou and Lu (2018) created preventive maintenance scheduling for serial multi-station manufacturing systems with station reliability and product quality interactions. Youssef et al. (2008) discussed the performance analysis of modular machine-based manufacturing systems using the universal generating function. Youssef et al. (2006) presented an assessment of the availability of multi-state manufacturing systems using a universal generating function. Sun et al. (2008) investigated the reliability of a serial-parallel hybrid multi-operational manufacturing system in terms of dimensional quality, tool degradation, and system configuration. For the performance analysis, Ye et al. (2020a) developed models for competing failure for automated manufacturing systems with imperfect quality inspection. Ye et al. (2019) investigated the reliability of a series manufacturing system with imperfect inspection, taking into account the interaction of quality and degradation. Ye et al. (2020b) investigated the dependability of a manufacturing machine with degradation and low-quality feedstocks. Chen et al. (2019) discussed mission reliability evaluation for multistate manufacturing systems based on operational quality data. Chang et al. (2017) investigated the dependability of a multi-state manufacturing network with shared buffer stations. Based on operational data, Han et al. (2019) investigated the mission reliability driven manufacturing system. Garg et al. (2010) investigated crank-case manufacturing availability in the two-wheeler automobile industry. Duan et al. (2019) created models for assessing the reliability of repairable non-series manufacturing systems with finite buffers. Chen and Jin (2005) were concerned with the quality reliability modeling of complex manufacturing processes.

Most production scheduling models assume that machines are always available; however, in real production environments, the machine breaks down and may not be available at some time (Farahani, 2020). Researchers mentioned above have made significant contributions to improving the efficiency and performance of various manufacturing systems. However, much is needed in the measures of dependability used to address the effectiveness, strength, efficacy, and performance of manufacturing systems. There are very few studies which consider the influence of the role of units, conveyor and processor on effectiveness, strength, efficacy, and performance of series–parallel hybrid manufacturing systems. In this paper, the reliability analysis of repairable series–parallel manufacturing system consisting of three units as subsystem 1, two conveyors as subsystem 2 and two processors as subsystem 3 is established, and some reliability indicators such as cost function, availability, system mean time to failure (MTTF) and reliability are given. To strengthen the analysis, Gumbel Hougaard family copula distribution is employed in reliability modelling and analysis of the manufacturing system in this study.

The reliability characteristics of a serial manufacturing system were discussed in this paper. In comparison to previous studies, the effect of Copula repairs has been captured. The following is the paper's structure. The notations, assumptions, and description of the system used in the investigation are presented in section 2. In section 3, the initiation of the models and solutions are presented. Section 4 concentrates on the study's analytical portion, in which certain specific situations are discussed. Section 5 presents the results of our numerical simulations and discussion, and Section 6 concludes the paper.

NOTATIONS, ASSUMPTIONS, AND DESCRIPTION OF THE SYSTEM

- Notations

t: Time variable on a time scale.

s: variable for all expressions of Laplace transform variable

$\beta_1 / \beta_2 / \beta_3$: rate of failure of unit/ conveyer/processor.

$\phi(x)/\phi(y)/\phi(z)$: rate of repair of unit/ conveyer/processor.

$\mu_0(x)/\mu_0(y)/\mu_0(z)$: rate of repair when the system is not functional due to complete failure of unit/ conveyer/processor.

$p_i(t)$: Probability of being the system in Si state at instants for i =0 to 10

$\overline{P}(s)$: Laplace transformation of state transition probability $p(t)$

$P_i(x, t)$: The probability that a system is in state S_i for i=1……., the system under repair and elapse repair time is (x, t) with repair variable x and time variable t

$P_i(y, t)$: The probability that a system is in state S_i for i=1……., the system under repair and elapse repair time is (y, t) with repair variable y and time variable t

$P_i(z, t)$: The probability that a system is in state S_i for i=1……., the system under repair and elapse repair time is (y, t) with repair variable z and time variable t

$E_p(t)$: Expected profit during the time interval [0, t)

K_1, K_2: Revenue and service cost per unit time, respectively.

$\mu_0(x)$: The expression of joint probability (failed state S_i to good state S_0) according to Gumbel-Hougaard family copula definition

$$\mu_0(x) = c_\theta(u_1, u_2(x)) = exp\left(x^\theta + \{log\,\phi\,(x)^\theta\}^{\frac{1}{\theta}}\right)\ 1 \le \theta \le \infty. \text{ Where } \mu_1 = \phi(x), \text{ and } u_2 = e^x$$

- Assumptions

a. Firstly, all subsystems are assumed to be operational.
b. Any unit failure leads to adequate system output.
c. If a system unit fails, it can be repaired when it is still operational, or it can fail completely.
d. All failures are expected to follow an exponential distribution.
e. General distribution is used to bring back partially failed states, while Gumbel-Hougaard family Copula distribution is used to distribute completely failed states.
f. The load is ready for the system's successful output as soon as the failed device is reset.

- System Descriptions

The system consists of three subsystems arranged in series namely three units, two conveyors and two processors as depicted in Figure 1 below. Subsystem 1: Consist of three units in active parallel whose failure cause complete failure of the entire system. Failure of one unit, the system to work in reduced capacity. Failure of three units at a time will cause complete failure of the system.
Subsystem 2: Consists of two in active parallel conveyers. Failure of one unit, the system to work in reduced capacity. Failure of two conveyors at a time will cause complete failure of the system. Subsystem C: Consisting of two processors in active parallel. Failure of one unit, the system to work in reduced capacity. Failure of the system occurs when all the two processors have failed. The processors received items to be process from the units and channel it to any of the conveyor for the process in either processor.

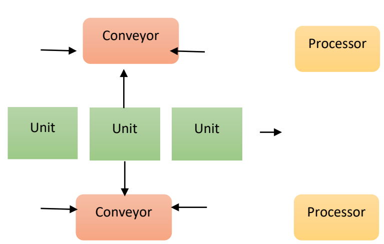

Figure 1: Reliability block diagram

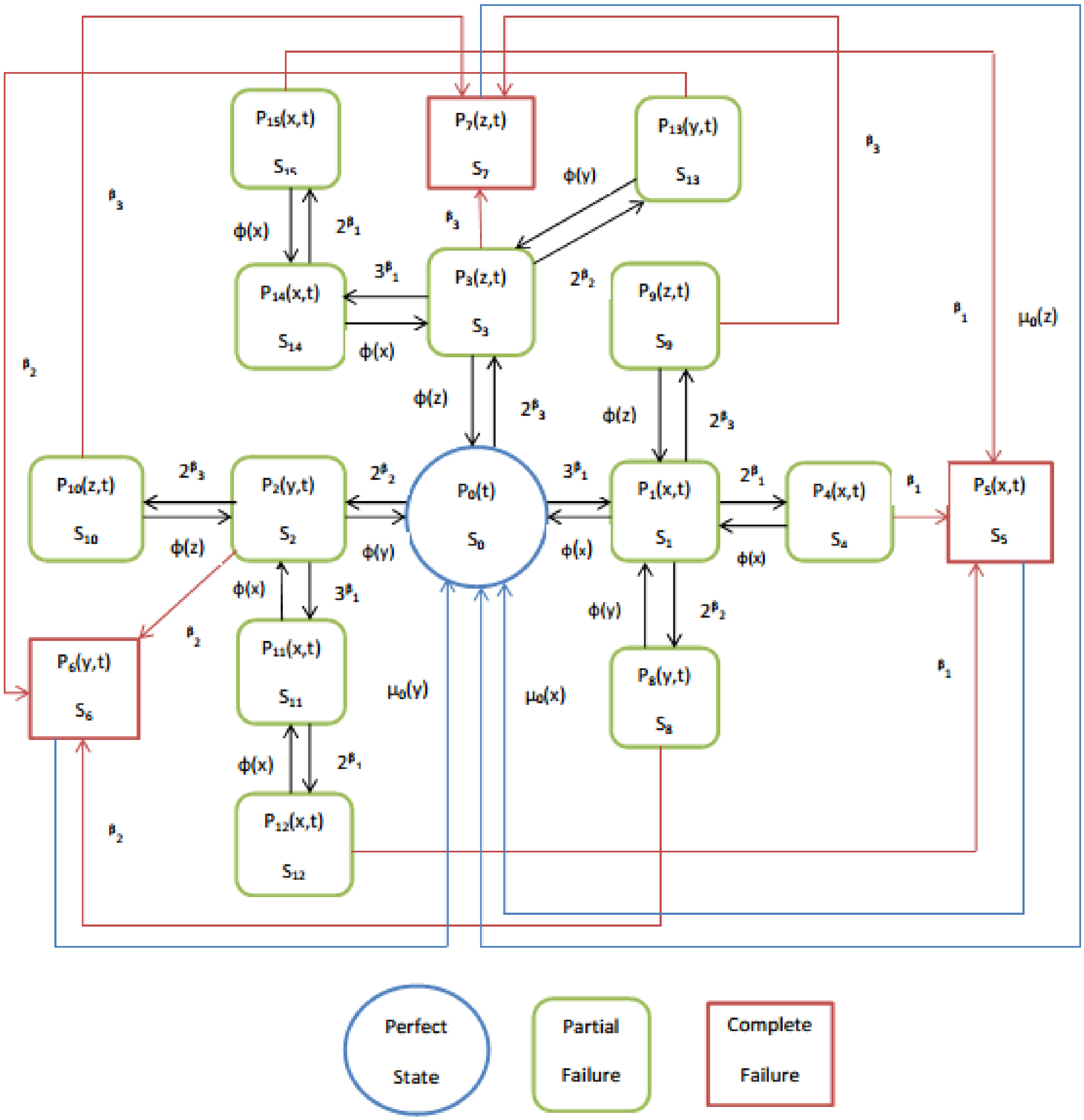

Figure 2: Transition diagram of the system

INITIATION OF MATHEMATICAL MODEL

By the probability of considerations and continuity of arguments as in Nelson (2006), Gulati et al. (2016), Singh and Ayagi (2017), Gahlot et al. (2018), Lado et al. (2018), Lado and Singh (2019), and Singh and Poonia (2019), the system of partial differential difference equations generated from Figure 2 is shown below.

$$\left(\frac{\partial}{\partial t}+3\beta_1+2\beta_2+2\beta_3\right)p_0(t)=\int_0^\infty \phi(x)\,p_1(x,t)dx+\int_0^\infty \phi(y)\,p_2(y,t)dy+\int_0^\infty \phi(z)\,p_3(z,t)dz+$$

$$\int_0^\infty \mu_0(x)\,p_5(x,t)dx+\int_0^\infty \mu_0(y)\,p_6(y,t)dx+\int_0^\infty \mu_0(z)\,p_7(z,t)dz \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\beta_1+2\beta_2+2\beta_3+\phi(x)\right)p_1(x,t)=0 \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+3\beta_1+\beta_2+2\beta_3+\phi(y)\right)p_2(y,t)=0 \tag{3}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+3\beta_1+2\beta_2+\beta_3+\phi(z)\right)p_3(z,t)=0 \tag{4}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_1+\phi(x)\right)p_4(x,t)=0 \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)p_5(x,t)=0 \tag{6}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)p_6(y,t)=0 \tag{7}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\beta_2+\mu_0(z)\right)p_7(z,t)=0 \tag{8}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta_2+\phi(y)\right)p_8(y,t)=0 \tag{9}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\beta_3+\phi(z)\right)p_9(z,t)=0 \tag{10}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\beta_3+\phi(z)\right)p_{10}(z,t)=0 \tag{11}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\beta_1+\phi(x)\right)p_{11}(x,t)=0 \tag{12}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_1+\phi(x)\right)p_{12}(x,t)=0 \tag{13}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta_2+\phi(y)\right)p_{13}(y,t)=0 \tag{14}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\beta_1+\phi(x)\right)p_{14}(x,t)=0 \tag{15}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_1+\phi(x)\right)p_{15}(x,t)=0 \tag{16}$$

Boundary condition

$$p_1(0,t)=3\beta_1 p_0(t) \tag{17}$$

$$p_2(0,t)=2\beta_2 p_0(t) \tag{18}$$

$$p_3(0,t)=2\beta_3 p_0(t) \tag{19}$$

$$p_4(0,t)=2\beta_1 p_1(0,t) \tag{20}$$

$$p_5(0,t)=\beta_1\left(p_4(0,t)+p_{12}(0,t)+p_{15}(0,t)\right) \tag{21}$$

$$p_6(0,t)=\beta_2\left(p_2(0,t)+p_8(0,t)+p_{13}(0,t)\right) \tag{22}$$

$$p_7(0,t)=\beta_2\left(p_3(0,t)+p_9(0,t)+p_{10}(0,t)\right) \tag{23}$$

$$p_8(0,t)=2\beta_2 p_1(0,t) \tag{24}$$

$$p_9(0,t)=2\beta_3 p_1(0,t) \tag{25}$$

$$p_{10}(0,t)=2\beta_3 p_2(0,t) \tag{26}$$

$$p_{11}(0,t)=3\beta_1 p_2(0,t) \tag{27}$$

$$p_{12}(0,t) = 2\beta_1 p_{11}(0,t) \tag{28}$$

$$p_{13}(0,t) = 2\beta_2 p_3(0,t) \tag{29}$$

$$p_{14}(0,t) = 3\beta_1 p_3(0,t) \tag{30}$$

$$p_{15}(0,t) = 2\beta_1 p_{14}(0,t) \tag{31}$$

SOLUTION OF THE MATHEMATICAL MODEL

Taking Laplace transformation of the equations (1) - (31) with help of boundary conditions

$$(s+3\beta_1+2\beta_2+2\beta_3)\,\overline{p}_0(\mathrm{s}) = 1+\int_0^\infty \phi(x)\,\overline{p}_1(x,\mathrm{s})\,dx+\int_0^\infty \phi(y)\,\overline{p}_2(y,s)\,dy+\int_0^\infty \phi(z)\,\overline{p}_3(z,s)\,dz$$

$$\int_0^\infty \mu_0(x)\,\overline{p}_5(x,s)\,dx + \int_0^\infty \mu_0(y)\,\overline{p}_6(y,s)\,dy \int_0^\infty \mu_0(z)\,\overline{p}_7(z,s)\,dz \tag{32}$$

$$\left(s+\frac{\partial}{\partial x}+2\beta_1+2\beta_2+2\beta_3+\phi(x)\right)\overline{p}_1(x,\mathrm{s}) = 0 \tag{33}$$

$$\left(s+\frac{\partial}{\partial y}+3\beta_1+\beta_2+2\beta_3+\phi(y)\right)\overline{p}_2(y,\mathrm{s}) = 0 \tag{34}$$

$$\left(s+\frac{\partial}{\partial z}+3\beta_1+2\beta_2+\beta_3+\phi(z)\right)\overline{p}_3(z,\mathrm{s}) = 0 \tag{35}$$

$$\left(s+\frac{\partial}{\partial x}+\beta_1+\phi(x)\right)\overline{p}_4(x,t) = 0 \tag{36}$$

$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\overline{p}_5(x,s) = 0 \tag{37}$$

$$\left(s+\frac{\partial}{\partial y}+\mu_0(y)\right)\overline{p}_6(y,s) = 0 \tag{38}$$

$$\left(s+\frac{\partial}{\partial z}+\mu_0(z)\right)\overline{p}_7(z,s) = 0 \tag{39}$$

$$\left(s+\frac{\partial}{\partial y}+\beta_2+\phi(y)\right)\overline{p}_8(y,s)=0 \quad (40)$$

$$\left(s+\frac{\partial}{\partial z}+\beta_3+\phi(z)\right)\overline{p}_9(z,s)=0 \quad (41)$$

$$\left(s+\frac{\partial}{\partial z}+\beta_3+\phi(z)\right)\overline{p}_{10}(z,s)=0 \quad (42)$$

$$\left(s+\frac{\partial}{\partial x}+2\beta_1+\phi(x)\right)\overline{p}_{11}(x,s)=0 \quad (43)$$

$$\left(s+\frac{\partial}{\partial x}+\beta_1+\phi(x)\right)\overline{p}_{12}(x,s)=0 \quad (44)$$

$$\left(s+\frac{\partial}{\partial y}+\beta_2+\phi(y)\right)\overline{p}_{13}(y,s)=0 \quad (45)$$

$$\left(s+\frac{\partial}{\partial x}+2\beta_1+\phi(x)\right)\overline{p}_{14}(x,s)=0 \quad (46)$$

$$\left(s+\frac{\partial}{\partial x}+\beta_1+\phi(x)\right)\overline{p}_{15}(x,s)=0 \quad (46)$$

BOUNDARY CONDITIONS

$$\overline{p}_1(0,s)=3\beta_1\overline{p}_0(s) \quad (47)$$

$$\overline{p}_2(0,s)=2\beta_2\overline{p}_0(s) \quad (48)$$

$$\overline{p}_3(0,s)=2\beta_3\overline{p}_0(s) \quad (49)$$

$$\overline{p}_4(0,s)=2\beta_1\overline{p}_1(0,s) \quad (50)$$

$$\overline{p}_5(0,s)=\beta_1\left(\overline{p}_4(0,s)+\overline{p}_{12}(0,s)+\overline{p}_{15}(0,s)\right) \quad (51)$$

$$\overline{p}_6(0,s)=\beta_2\left(\overline{p}_2(0,s)+\overline{p}_8(0,s)+\overline{p}_{13}(0,s)\right) \tag{52}$$

$$\overline{p}_7(0,s)=\beta_3\left(\overline{p}_3(0,s)+\overline{p}_9(0,s)+\overline{p}_{10}(0,s)\right) \tag{53}$$

$$\overline{p}_8(0,s)=2\beta_2\overline{p}_1(0,s) \tag{54}$$

$$\overline{p}_9(0,s)=2\beta_3\overline{p}_1(0,s) \tag{55}$$

$$\overline{p}_{10}(0,s)=2\beta_3\overline{p}_2(0,s) \tag{56}$$

$$\overline{p}_{11}(0,s)=3\beta_1\overline{p}_2(0,s) \tag{57}$$

$$\overline{p}_{12}(0,s)=2\beta_1\overline{p}_{11}(0,s) \tag{58}$$

$$\overline{p}_{13}(0,s)=2\beta_2\overline{p}_3(0,s) \tag{59}$$

$$\overline{p}_{14}(0,s)=3\beta_1\overline{p}_3(0,s) \tag{60}$$

$$\overline{p}_{15}(0,s)=2\beta_1\overline{p}_{14}(0,s) \tag{61}$$

Determining of equation (33) - (46) with help of Laplace transform of boundary conditions

$$\overline{p}_0(s)=\frac{1}{D(s)} \tag{62}$$

$$\overline{p}_1(s)=\frac{3\beta_1}{D(s)}\left\{\frac{1-\overline{s}_\phi(s+2\beta_1+2\beta_2+2\beta_3)}{s+2\beta_1+2\beta_2+2\beta_3}\right\} \tag{63}$$

$$\overline{p}_2(s)=\frac{2\beta_2}{D(s)}\left\{\frac{1-\overline{s}_\phi(s+3\beta_1+\beta_2+2\beta_3)}{s+3\beta_1+\beta_2+2\beta_3}\right\} \tag{64}$$

$$\overline{p}_3(s)=\frac{2\beta_3}{D(s)}\left\{\frac{1-\overline{s}_\phi(s+3\beta_1+2\beta_2+\beta_3)}{s+3\beta_1+2\beta_2+\beta_3}\right\} \tag{65}$$

$$\overline{p}_4(s)=\frac{6\beta_1^2}{D(s)}\left\{\frac{1-\overline{s}_\phi(s+\beta_1)}{s+\beta_1}\right\} \tag{66}$$

$$\bar{p}_5(s)=\frac{\left(6\beta_1^3+12\beta_1^3\beta_2+12\beta_1^3\beta_3\right)}{D(s)}\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{67}$$

$$\bar{p}_6(s)=\left(\frac{2\beta_2^2+6\beta_1\beta_2^2+4\beta_2^2\beta_3}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{68}$$

$$\bar{p}_7(s)=\left(\frac{2\beta_3^2+6\beta_1\beta_3^2+6\beta_3^2\beta_2}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{69}$$

$$\bar{p}_8(s)=\frac{6\beta_1\beta_2}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+\beta_2)}{s+\beta_2}\right\} \tag{70}$$

$$\bar{p}_9(s)=\frac{6\beta_1\beta_3}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+\beta_3)}{s+\beta_3}\right\} \tag{71}$$

$$\bar{p}_{10}(s)=\frac{6\beta_2\beta_3}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+\beta_3)}{s+\beta_3}\right\} \tag{72}$$

$$\bar{p}_{11}(s)=\frac{6\beta_1\beta_2}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+2\beta_1)}{s+2\beta_1}\right\} \tag{73}$$

$$\bar{p}_{12}(s)=\frac{12\beta_1^2\beta_2}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+\beta_1)}{s+\beta_1}\right\} \tag{74}$$

$$\bar{p}_{13}(s)=\frac{4\beta_2\beta_3}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+\beta_2)}{s+\beta_2}\right\} \tag{75}$$

$$\bar{p}_{14}(s)=\frac{6\beta_1\beta_3}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+2\beta_1)}{s+2\beta_1}\right\} \tag{76}$$

$$\bar{p}_{15}(s)=\frac{12\beta_1^2\beta_3}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+\beta_1)}{s+\beta_1}\right\} \tag{77}$$

And D(s) is defined as;

$$D(s)=\left\{s+3\beta_1+2\beta_2+2\beta_3-\left(\begin{array}{l}3\beta_1\bar{s}_\phi\left(s+2\beta_1+2\beta_2+2\beta_3\right)+\\2\beta_2\bar{s}_\phi\left(s+3\beta_1+\beta_2+2\beta_3\right)+\\2\beta_3\bar{s}_\phi\left(s+3\beta_1+2\beta_2+\beta_3\right)+\\\left[\begin{array}{l}\left(6\beta_1^3+12\beta_1^3\beta_2+12\beta_1^3\beta_3\right)+\\\left(2\beta_2^2+6\beta_2^2\beta_1+4\beta_2^2\beta_3\right)+\\\left(2\beta_3^2+6\beta_3^2\beta_1+6\beta_3^2\beta_2\right)+\end{array}\right]\bar{s}_{\mu_0}(s)\end{array}\right)\right\} \tag{78}$$

INITIAL CONDITIONS

$p_0(0)=1$, and further state transition probability are zero at $t=0$ (79)

Summing all Laplace transformations of the state transition probabilities that the system is operating, are as follows:

$$\bar{p}_{up}(s)=\left[\begin{array}{l}\bar{p}_0(s)+\bar{p}_1(s)+\bar{p}_2(s)+\bar{p}_3(s)+\bar{p}_4(s)+\bar{p}_8(s)+\bar{p}_9(s)+\\\bar{p}_{10}(s)+\bar{p}_{11}(s)+\bar{p}_{12}(s)+\bar{p}_{13}(s)+\bar{p}_{14}(s)+\bar{p}_{15}(s)\end{array}\right] \tag{80}$$

$$\bar{p}_{up}(s)=\frac{1}{D(s)}\left\{\begin{array}{l}1+3\beta_1\left(\frac{1-\bar{s}_\phi\left(s+2\beta_1+2\beta_2+2\beta_3\right)}{s+2\beta_1+2\beta_2+2\beta_3}\right)+2\beta_2\left(\frac{1-\bar{s}_\phi\left(s+3\beta_1+\beta_2+2\beta_3\right)}{s+3\beta_1+\beta_2+2\beta_3}\right)+\\2\beta_3\left(\frac{1-\bar{s}_\phi\left(s+3\beta_1+2\beta_2+\beta_3\right)}{s+3\beta_1+2\beta_2+\beta_3}\right)\beta_1^2\left(\frac{1-\bar{s}_\phi\left(s+\beta_1\right)}{s+\beta_1}\right)+6\beta_1\beta_2\left(\frac{1-\bar{s}_\phi\left(s+\beta_2\right)}{s+\beta_2}\right)+\\6\beta_1\beta_3\left(\frac{1-\bar{s}_\phi\left(s+\beta_3\right)}{s+\beta_3}\right)+6\beta_2\beta_3\left(\frac{1-\bar{s}_\phi\left(s+\beta_3\right)}{s+\beta_3}\right)+6\beta_1\beta_2\left(\frac{1-\bar{s}_\phi\left(s+2\beta_1\right)}{s+2\beta_1}\right)+\\12\beta_1^2\beta_2\left(\frac{1-\bar{s}_\phi\left(s+\beta_1\right)}{s+\beta_1}\right)+4\beta_2\beta_3\left(\frac{1-\bar{s}_\phi\left(s+\beta_2\right)}{s+\beta_2}\right)+6\beta_1\beta_3\left(\frac{1-\bar{s}_\phi\left(s+2\beta_1\right)}{s+2\beta_1}\right)+\\12\beta_1^2\beta_3\left(\frac{1-\bar{s}_\phi\left(s+\beta_1\right)}{s+\beta_1}\right)\end{array}\right\}$$

(81)

$$\bar{p}_{down}(s)=1-\bar{p}_{up}(s)) \tag{82}$$

STUDY OF THE SYSTEM DIFFERENT CASES

- Availability analysis of the system

Letting, $S_{\mu_0}(s)=\bar{S}_{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}(s)=\dfrac{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}$, $\bar{S}_\phi(s)=\dfrac{\phi}{s+\phi}$, and taking the values of different failure rates as $\beta_1=0.02,\beta_2=0.03,\beta_3=0.04$, $\phi=\mu=x=y=z=1$ and repair rates as $\phi(x)=\phi(y)=\phi(z)=1$ in equation (81), and carrying inverse Laplace transform, availability expression is obtained as:

$$\bar{p}_{up}(s)=\begin{Bmatrix}-0.001257e^{-1.02000t}-0.003678e^{-1.03000t}\\-0.008478e^{-1.04000t}+0.002329e^{-2.72486t}\\-0.033428e^{-1.34269t}-0.000038e^{-1.17582t}\\-0.000037e^{-1.16483t}+1.044589e^{-0.02000t}\end{Bmatrix} \tag{83}$$

Assuming different values of time variable t = 0, 2…20, units of time in equation (83), to obtain the availability in Table 1 using MAPLE package.

TABLE 1
TIME VS. AVAILABILITY

Time	0	2	4	6	8	10	12	14	16	18	20
Availability	1.0000	0.9994	0.9635	0.9259	0.8895	0.8545	0.8208	0.7885	0.7575	0.7276	0.6990

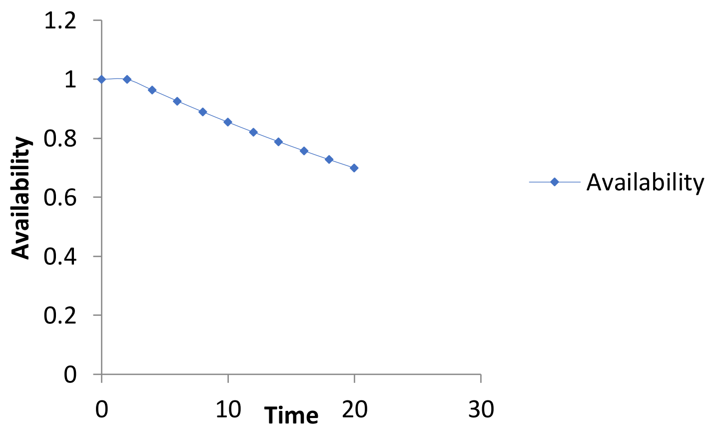

FIGURE 3:
TIME VS. AVAILABILITY

- Reliability Analysis of the system

Assuming all repair rates, $\phi(x), \phi(y), \phi(z)$, μ_0 , in equation (81) to zero for the same values of failure rates as $\beta_1 = 0.02, \beta_2 = 0.03, \beta_3 = 0.04$ and taking Laplace transformation.

$$R(t) = \left\{ \begin{aligned} &2e^{-0.17000t} + 2e^{-0.16000t} + 0.049411e^{-0.03000t} + \\ &0.015200e^{-0.02000t} + 0.127500e^{-0.04000t} - \\ &6.192111e^{-0.20000t} + 3e^{-0.18000t} \end{aligned} \right\} \quad (84)$$

TABLE 3
RELIABILITY VS TIME

Time	0	2	4	6	8	10	12	14	16	18	20
Reliability	1.0000	0.9970	0.9122	0.7957	0.6744	0.5615	0.4628	0.3796	0.3112	0.2558	0.2114

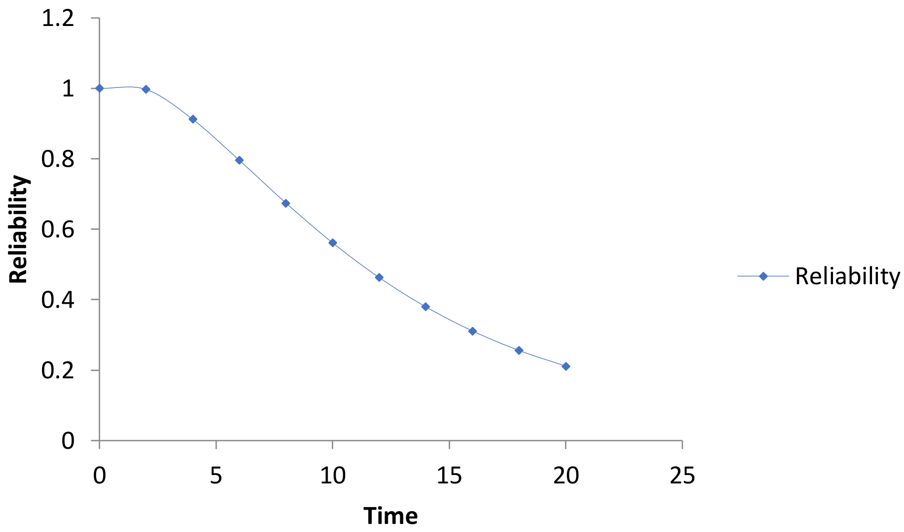

FIGURE 4
RELIABILITY VERSUS TIME

- MTTF Analysis of the system

For MTTF, supposing all repairs to zero in equation (81) and the limit as s approach zero, MTTF expression is as follows:

$$MTTF = \lim_{s \to 0} \overline{p}_{up}(s) = \frac{1}{3\beta_1 + 2\beta_2 + 2\beta_3} \left\{ \begin{array}{l} 1 + \dfrac{3\beta_1}{2\beta_1 + 2\beta_2 + 2\beta_3} + \dfrac{2\beta_2}{3\beta_1 + \beta_2 + 2\beta_3} + \\ \dfrac{2\beta_3}{3\beta_1 + 2\beta_2 + \beta_3} + 18\beta_1 + 9\beta_2 + 7\beta_3 + 12\beta_1\beta_2 + \\ 12\beta_1\beta_3 \end{array} \right\} \quad (85)$$

Taking $\beta_1 = 0.02, \beta_2 = 0.03, \beta_3 = 0.04$ and taking up $\beta_1, \beta_2, \beta_3$ one by one respectively as 0.01,0.02..., 0.09 in equation (85).

TABLE 4
MTTF VARIATION WITH RESPECT TO FAILURE RATE

Failure Rate	$MTTF$ β_1	$MTTF$ β_2	$MTTF$ β_3
0.01	18.5616	21.2183	24.2408
0.02	17.2948	18.9426	21.1273
0.03	16.4312	17.2948	18.9337
0.04	15.7889	16.0404	17.2948
0.05	15.2823	15.0483	16.0161
0.06	14.8666	14.2401	14.9880
0.07	14.5155	13.5660	14.1321
0.08	14.2130	12.9930	13.4121
0.09	13.9483	12.4984	12.7944

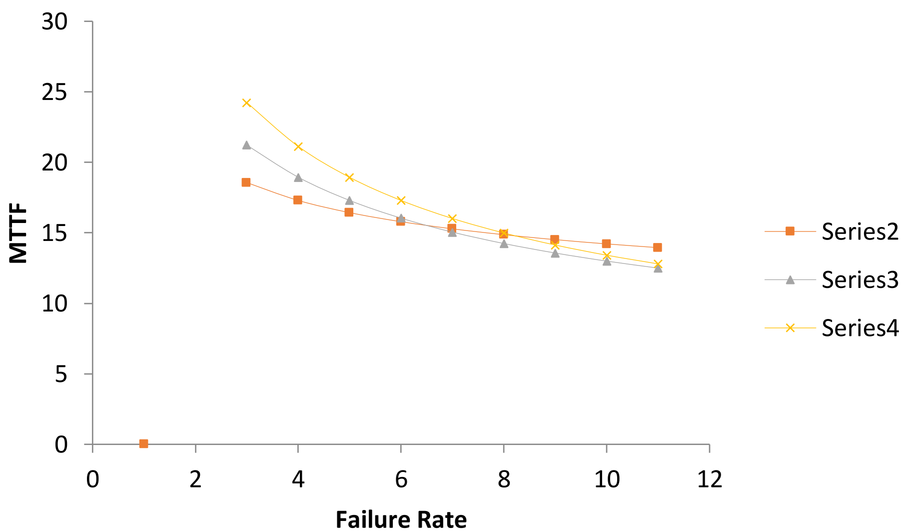

FIGURE 5
RATES OF FAILURE VS. MTTF

- Sensitivity Analysis of the system

Sensitivity is computed by taking the partial differentiation of MTTF with respect to the failure rates, and assuming the values of parameters as, $\beta_1 = 0.02, \beta_2 = 0.03, \beta_3 = 0.04$.

TABLE 5
SYSTEM FAILURE RATE SENSITIVITY ANALYSIS

Failure Rate	$\frac{\partial(MTTF)}{\beta_1}$	$\frac{\partial(MTTF)}{\beta_2}$	$\frac{\partial(MTTF)}{\beta_3}$
0.01	-157.8862	-271.3275	-377.4265
0.02	-102.1673	-190.9913	-257.0912
0.03	-73.3536	-142.3046	-187.3854
0.04	-56.4611	-110.6859	-143.5037
0.05	-45.5658	-88.9993	-114.0592
0.06	-38.0066	-73.4559	-93.2867
0.07	-32.4615	-61.9068	-78.0326
0.08	-28.2166	-53.0658	-66.4597
0.09	-24.8584	-46.1265	-57.4409

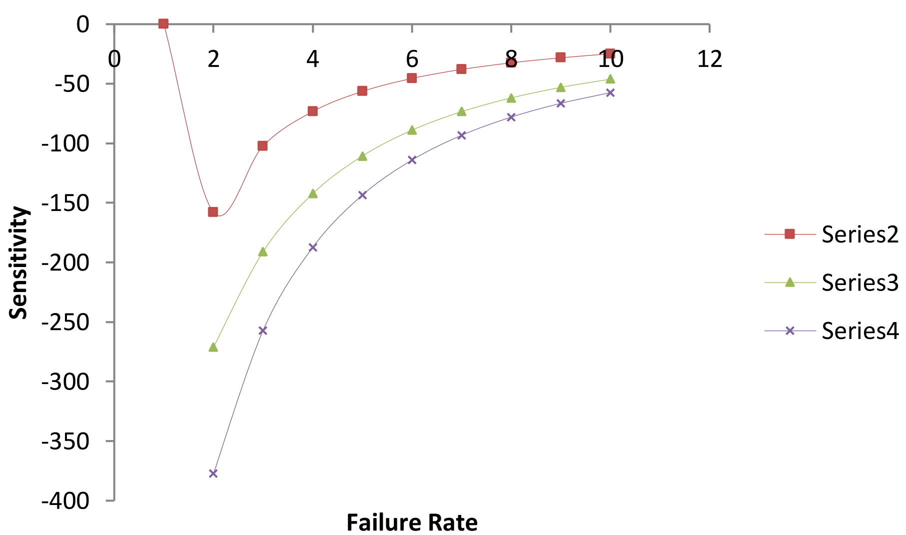

FIGURE 6
SYSTEM FAILURE RATE SENSITIVITY ANALYSIS

- Cost Analysis of the system

The formula below will determine the estimated profit in the interval if the service facility is always open $(0, t)$.

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{86}$$

Where K_1 and K_2 are the sales rate and service cost per unit time, respectively.

Taking fixed values of parameters of equation (81), equation (87) is follows as

$$E_p(t) = k_1 \begin{Bmatrix} 0.001232e^{-1.02000t} + 0.003571e^{-1.03000t} + 0.008152e^{-1.04000t} - \\ 0.000854e^{-2.72486t} + 0.024896e^{-1.34269t} + 0.000032e^{-1.17582t} + \\ 0.000031e^{-1.16483t} - 52.011062e^{-0.02008t} 51.973 \end{Bmatrix} - k_2(t) \qquad (87)$$

Let K_1= 1and K_2= 0.6, 0.5..., 0.1, respectively and changing t = 0, 1, 2...10. Units of time, the expected profit is obtained as:

TABLE 6
TIME VERSUS EXPECTED PROFIT

Time	$E_p(t)$ K_2=0.6	$E_p(t)$ K_2=0.5	$E_p(t)$ K_2=0.4	$E_p(t)$ K_2=0.3	$E_p(t)$ K_2=0.2	$E_p(t)$ K_2=0.1
0	0	0	0	0	0	0
1	0.4071	0.5071	0.6071	0.7071	0.8071	0.9071
2	0.8130	1.0130	1.2130	1.4130	1.6130	1.8130
3	1.2041	1.5041	1.8041	2.1041	2.4041	2.7041
4	1.5771	1.9771	2.3771	2.7771	3.1771	3.5771
5	1.9313	2.4313	2.9313	3.4313	3.9313	4.4313
6	2.2666	2.8666	3.4666	4.0666	4.6666	5.2666
7	2.5833	3.2833	3.9833	4.6833	5.3833	6.0833
8	2.8818	3.6818	4.4818	5.2818	6.0818	6.8818
9	3.1625	4.0625	4.9625	5.8625	6.7625	7.6625
10	3.4256	4.4256	5.4256	6.4256	7.4256	8.4256

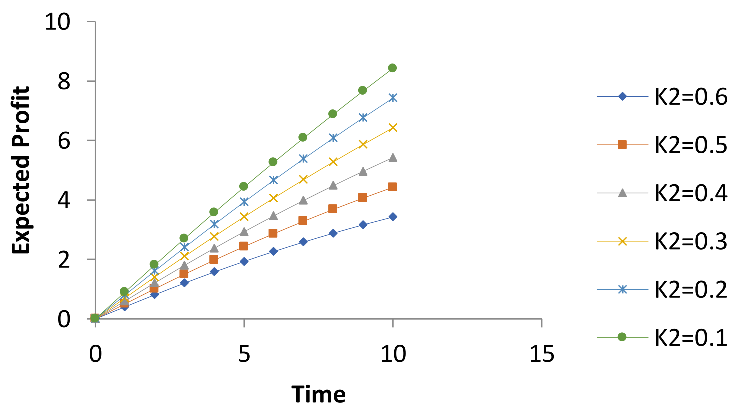

FIGURE 7
TIME VERSUS EXPECTED PROFIT

RESULT ANALYSIS

On the basis of the analysis above, table 1 and figure 3 reflect that the system's availability decreases rapidly as time passes. At any time, the model's graphical representation will predict the future behavior of a complex system for a given set of parameters. The gap in reliability over time is depicted in table 2 and figure 4. When system reliability is compared to system availability, it is clear that system reliability suffers due to lack of maintenance to the system. This exemplifies the consequences of failing to maintain the system. This called for adequate maintenance action which will lead to maximum system reliability. Maintenance action such as regular and adequate inspection preventive maintenance, replacement of worn part with new fault tolerant units, etc should be practice to allow the system work without interruption, leading increase in production output and revenue.

Table 3 and Figure 5 show the variations in the values of the system's mean time to failure (MTTF) for failure rates β_1, β_2 and β_3, respectively, while other parameters are kept constant. With regard to failure rates β_1, β_2 and β_3, the MTTF values decrease. It's also worth noting from Table 3 and Figure 5 that the MTFF is in order $MTTF\ w.r.t\ \beta_1 < MTTF\ w.r.t\ \beta_2 < MTTF\ w.r.t\ \beta_3$ which shows that β_3 is more responsible for successful system functioning. From Table 3 and Figure 5 is enough to conclude that system MTTF with respect to the unit is less compared to the system MTTF with respect to conveyor and processor. Overcome this problem, the result presented in Table 3 and Figure 5 is suggesting the used or adding fault tolerant units in order to prolong the life span and MTTF of the system. Table 4 and figure 6 contain details on the sensitivity analysis discussed in this chapter.

Table 5 and figure 7 are obtained by setting the revenue per unit time to one and varying the service cost to $0.1, 0.2, 0.3, 0.4, 0.5,$ and 0.6. The profit decreases as the service cost rises, as seen from this figure and table. The expected profit is highest when the service cost is low and lowest when the service cost is high. Manufacturing companies exist to make profit, and none of them will thrive if the cost of maintaining them is too high. From this analysis it clear that the management should invoke means of reducing the failures and service cost to allow the profit to flourish. This can be done by invoking maintenance actions such as condition monitoring, inspection, control limit policy, etc.

CONCLUSION

Failures of systems in various manufacturing systems can result in a variety of issues, including unsatisfactory usage and a loss in profitability. To prevent these situations, we need to have enough knowledge about system failures as well as certain maintenance strategies. For illustration, the present paper used the manufacturing system, where the repair is performed utilizing Copula and General distributions. The paper carried out an investigation of the performance models used in testing the effectiveness and strength of manufacturing system whose components such units, conveyors, and processors are viewed as subsystems A, B and C arranged in series-parallel. The system is analyzed using Markovian process, Laplace transformation, and supplementary variable techniques are used to calculate the system's transient probabilities and reliability measures of system performance and strength such as availability, cost, reliability and mean time to failure.

Through numerical experiments, expressions of reliability metrics for testing the strength and performance of the system, such as availability, cost, reliability and mean time to failure are validated. MATLAB was used to simulate the effect of time and various system parameters on reliability metrics. The impact of time, variation of failures and repairs on system performance is investigated. Based on numerical results obtained for a specific case, Figure 3-7 and Table 1-6 show that Copula repair is a better repair policy for improving the performance of the systems. This paper will provide the basis for production scheduling and preventive maintenance to the manufacturer from the perspective of reliability assessment of repairable systems. Where the system's reliability strength is strong, it may help the system to withstand some of the obstacles hindering the system's performance and increasing the system's life span. These are the paper's main contributions. The future research direction of the present study will incorporate the role of condition monitoring, inspection, online and offline preventive maintenance, general and copula repairs in enhancing the system's performance which may lead to increase in production output, revenue mobilization and decrease in maintenance cost. It will also the application of methods of solving reliability problems and determination the optimal method.

REFERENCES

[1] Abubakar, M.I. and Singh, V.V. (2019) 'Performance assessment of an industrial system (African Textile Manufactures Ltd.) through copula linguistic approach', Operation Research and Decisions, 29(4),1–18.
[2] Chang, P.C., Lin, Y.K and Chen, J.C. (2017). System reliability for a multi-state manufacturing network with joint buffer stations, J Manuf Syst; 42: 170–178
[3] Chen Z., He Y., Zhao Y., Han X., He Z., Xu Y and Zhang A. (2019) Mission reliability evaluation based on operational quality data for multistate manufacturing systems, International Journal of Production Research, 57:6, 1840-1856, DOI: 10.1080/00207543.2018.1508906
[4] Chen, Y and Jin, J. (2005). Quality-reliability chain modeling for system-reliability analysis of complex manufacturing processes. IEEE Trans. Reliab. ;54:475–488. doi: 10.1109/TR.2005.853441.
[5] Chopra, G and Ram, M.(2019). Reliability measures of two dissimilar units parallel system using Gumbel-Hougaard family copula, International Journal of Mathematical, Engineering and Management Sciences Vol. 4, No. 1, 116–130. DOI: 10.33889/IJMEMS.2019.4.1-011
[6] Duan, J., Xie, N and Li, L. (2019). Modelling and evaluation of multi-state reliability of repairable non-series manufacturing system with finite buffers, Advances in Mechanical Engineering, Vol. 11(6) 1–13.DOI: 10.1177/1687814019855483
[7] Farahani, A., Tohidi, H., (2020). Integrated optimization of quality and maintenance: A literature review, Computers & Industrial Engineering, doi: https://doi.org/10.1016/j.cie.2020.106924
[8] Farahani, A., Tohidi, H., and Shoja, A. (2019). An integrated optimization of quality control chart parameters and preventive maintenance using Markov chain. Advances in Production Engineering & Management. 14(1), 5-14, https://doi.org/10.14743/apem2019.1.307
[9] Gahlot, M., Singh, V.V., Ayagi, H.I. and Goel, C.K. (2018) 'Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics', International Journal of Reliability and Safety, 12(4),348–374.
[10] Garg, S., Singh, J and Singh, D. V. (2010). Availability analysis of crank-case manufacturing in a two-wheeler automobile industry. Applied Mathematical Modelling, 34, 1672–1683
[11] Gulati, J., Singh, V.V., Rawal, D.K. and Goel, C.K. (2016) 'Performance analysis of complex system in series configuration under different failure and repair discipline using copula', International Journal of Reliability, Quality and Safety Engineering, Vol. 23, No. 2, pp.812–832.
[12] Han X., Wang Z., He Y., Zhao Y., Chen Z., Zhou D. A Mission Reliability-Driven Manufacturing System Health State Evaluation Method Based on Fusion of Operational Data. Sensors. 2019;19:442. doi: 10.3390/s19030442
[13] He,Y., He,Z., Wang, L and Gu, C.(2015). Reliability modeling and optimization strategy for manufacturing system based on RQR chain. Mathematical Problems in Engineering, 2015:1-13.
[14] He, Y., Han, X., Gu, C and Chen, Z. (2018). Cost-oriented predictive maintenance based on mission reliability state for cyber manufacturing systems. Adv Mech Eng; 10(1),1–15.
[15] He, Y., Gu, C., Han, X., Cui, J and Chen, Z. (2017). Mission reliability modeling for multi-station manufacturing system based on Quality State Task Network. Proc. Inst. Mech. Eng. Part O J. Risk Reliab. ;231:701–715. doi: 10.1177/1748006X17728599
[16] Kumar, K and Kumar, P. (2010). Mathematical Modeling and Analysis of Stainless-Steel Utensil Manufacturing Unit Using Fuzzy Reliability, International Journal of Engineering Science and Technology, Vol. 2(6), 2370-2376.
[17] Lado, A and Singh, V.V. (2019), Cost assessment of complex repairable system consisting of two subsystems in the series configuration using Gumbel–Hougaard family copula, International Journal of Quality and Reliability Management, 36(10), 1683–1698.
[18] Lin, L and Jun, N. (2008). Reliability estimation based on operational data of manufacturing systems. Quality and Reliability Engineering International, 24:843-854.
[19] Li, G.D., Masuda,S., Yamaguchi, D and Nagai, M. (2010). A New Reliability Prediction Model in Manufacturing Systems. IEEE Transactions on Reliability, 59:170-177.
[20] Li, L and Ni, J. (2008). Reliability estimation based on operational data of manufacturing systems. Quality and Reliability Engineering International, 24, 843–854
[21] Lu., B and Zhou., X. (2019). Quality and reliability oriented maintenance for multistage manufacturing systems subject to condition monitoring. J. Manuf. Syst.;52:76–85. doi: 10.1016/j.jmsy.2019.04.003
[22] Maihulla, A. S., Yusuf, I., and Bala, S. I. (2021). Reliability and Performance Analysis of a Series-Parallel System using Gumbel–Hougaard Family Copula. Journal of Computational and Cognitive Engineering, 01–10, doi: https://doi.org/10.47852/bonviewJCCE2022010101
[23] Nelson, R.B. (2006) An Introduction to Copulas, 2nd ed., Springer Publisher, New York.
Abubakar and Singh (2019)
[24] Patton, A.J. (2009). Copula–based models for financial time series. In Handbook of Financial Time Series (pp. 767-785). Springer, Berlin, Heidelberg.
[25] Ram, M and Singh, S. B. (2008). Availability and cost analysis of a parallel redundant complex system with two types of failure under preemptive-resume repair discipline using Gumbel-Hougaard family copula in repair. International Journal of Reliability, Quality and Safety Engineering, 15(04), 341-365.
[26] Ram, M and Singh, S. B. (2010). Availability, MTTF and cost analysis of complex system under preemptive-repeat repair discipline using Gumbel-Hougaard family copula. International Journal of Quality & Reliability Management, 27(5), 576-595.
[27] Rodriguez, J.C. (2007). Measuring financial contagion: a copula approach. Journal of Empirical Finance, 14(3), 401-423.
[28] Sha, N. (2021). A Copula Approach of Reliability Analysis for Hybrid Systems. Reliability: Theory & Applications, 16(1), 231-242.
[29] Savsar, M. (2000). Reliability analysis of a flexible manufacturing cell. Reliability Engineering & System Safety, 67(2), 147-152.

[30] Singh, V.V and Ayagi, H.I. (2018). Stochastic analysis of a complex system under preemptive resume repair policy using Gumbel-Hougaard family copula, International Journal of Mathematics in Operational Research, 12(2), 273-291

[31] Singh, V.V., Gahlot, M., Ayagi, H.I and Goel, C.K. (2018). Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics, International Journal of Reliability and Safety, 12(4), 348-374.

[32] Singh, V. V and Ayagi, H. I. (2017). Study of reliability measures of system consisting of two subsystems in series configuration using copula. Palestinian Journal of Mathematics, 6(1), 1-10.

[33] Singh, V.V., Gulati, J., Rawal, D.K and Goel, C.K. (2016). Performance analysis of complex system in series configuration under different failure and repair discipline using copula, International Journal of Reliability Quality and Safety Engineering, 23 (2). https://doi.org/10.1142/S0218539316500108

[34] Sklar, A. (1973). Random variables, joint distribution functions, and copulas. Kybernetika, 9(6), 449-460.

[35] Sun J., Xi L., Du S., Ju B. (2008). Reliability modeling and analysis of serial-parallel hybrid multi-operational manufacturing system considering dimensional quality, tool degradation and system configuration. Int. J. Prod. Econ. 114:149–164. doi: 10.1016/j.ijpe.2008.01.002

[36] Trivedi, P.K., & Zimmer, D.M. (2007). Copula modeling: an introduction for practitioners. Now Publishers Inc, U.S.A.

[37] Tyagi, V., Arora, R and Ram, M. (2021). Copula based measures of repairable parallel system with fault coverage, International Journal of Mathematical, Engineering and Management Sciences Vol. 6, No. 1, 322-344

[38] Xu, X., Jianlin, L., Gong, J., Deng, H and Wan, L. (2016). Copula-based slope reliability analysis using the failure domain defined by the *g*-Line, Mathematical Problems in Engineering Volume 2016, Article ID 6141838. http://dx.doi.org/10.1155/2016/6141838.

[39] Ye, Z., Cai, Z., Si, S., Zhang, S and Yang,H.(2020). Competing Failure Modeling for Performance Analysis of Automated Manufacturing Systems With Serial Structures and Imperfect Quality Inspection, *IEEE Transactions on Industrial Informatics*, vol. 16, no. 10, pp. 6476-6486, Oct. 2020, doi: 10.1109/TII.2020.2967030.

[40] Ye, Z., Cai, Z., Zhou, F., Zhao, J and Zhang, P. (2019). Reliability analysis for series manufacturing system with imperfect inspection considering the interaction between quality and degradation. Reliab. Eng. Syst. Saf. 189:345–356. doi: 10.1016/j.ress.2019.05.005

[41] Ye, Z., Cai, Z and Yang, H. (2020). "Reliability Analysis of Manufacturing Machine with Degradation and Low-quality Feedstocks," *2020 Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM)*, 2020, pp. 1-5, doi: 10.1109/APARM49247.2020.9209452.

[42] Youssef, A.M.A., Mohib, A and Eimaraghy.H.A. (2006). Availability assessment of multi-state manufacturing systems using universal generating function, CIRP annals, 55(1): 445–448.

[43] Youssef, A.M.A., Mohib, A and Eimaraghy.H.A. (2008). Performance analysis of manufacturing systems composed of modular machines using the universal generating function, J Manuf Syst, 27(2), 55–69.

[44] Zhou, X and Lu, B. (2018). Preventive maintenance scheduling for serial multi-station manufacturing systems with interaction between station reliability and product quality. Comput. Ind. Eng. 122:283–291. doi: 10.1016/j.cie.2018.06.009

[45] Zhang, H, Li., A.P and Liu., X.M. (2017). Modeling and performance evaluation of a multistage serial manufacturing system considering rework and product polymorphism. Chin J Mech Eng; 53: 191–201.