Original Research

Reliability Analysis of Distributed System for Enhancing Data Replication using Gumbel-Hougaard Family Copula Approach Joint Probability Distribution

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Abstract

In today's world of technology, it is impossible to see where computer system does not play an important role. The application of distributed systems is gradually becoming broad and diverse, and as a result of this, reliability prediction is a key concern. This paper, considered a distributed system with five standby subsystems A (the clients), B (two load balancers), C (two distributed database servers), D (two mirrored distributed database serves) and E (centralized database server) is considered arranged as series-parallel system. Exponential failure and repair are susceptible for all the components of this system. Each component's failure rates are constant and considered to obey an exponential distribution, and they are repaired using general repair or copula repair. The system is evaluated using first-order partial differential equations and the supplementary variable technique, Gumbel-Hougaard family of Copula, to find expressions for reliability metrics of system strength such as availability, reliability, MTTF, sensitivity, and profit function. These reliability metrics have been validated for different parametric values and the results are presented in tables and figures.

Keywords - Availability, Analysis, Distributed system, Reliability, Replication, Joint probability

1. Introduction

The copula approach is a technique for calculating joint distributions using marginal distributions in which the variables are nonnormal. Copulas can also be used to analyze pairs of random variables in a nonparametric way. Sklar is the one who first introduced Copula (1973). Since then, copula analysis has taken on new dimensions and analyses.

Numerous researchers have previously presented copula methods in the field of reliability and performance analysis of systems by examining system performance under various conditions. To name a few, Nelsen (2006) employed copula to relate a multivariate distribution to a one-dimensional marginal distribution function. The conditional copula and its application in time series analysis were introduced by Patton (2009). The application of copula in financial management is the topic of Rodriguez (2007). The application of copula in multivariate distributions was captured by Trivedi and Zimmer (2007). Ram and Singh (2008) dealt with availability and cost analysis of complex system configured in parallel subject to two types of failures and preemptive resume repair under Gumbel-Hougaard family copula. Ram and Singh (2010) analyzed the MTTF, cost and availability of a system under preemtive repair using Gumbel-Hougaard family copula. Abubakar and Singh (2019) analyzed the performance of industrial system using copula linguistics. Gulat et al. (2016) focus on performance of complex system in series configuration with different failure and repair. Gahlot et al. (2018) presented performance assessment of system in serial configuration. Tyagi et al. (2021) presented copula analysis of parallel system with fault coverage. Sha (2021) presented copula reliability analysis for hybrid systems. Chopra and Ram (2019) analyzed the reliability measures of dissimilar parallel system with two units using Gumbel-Hougaard family copula. Chopra and Ram (2021) presented reliability measures of two dissimilar units in parallel using Gumbel- Hougaard copula.

In most scenarios, systems are evaluated in terms of evaluating their performance metrics in terms of reliability, availability, and revenue generated. Reliability and availability are important in the context of evolving technology and increasing complexity in engineering systems. Distributed systems are widely utilized in most crucial domains, such as the travel and tourism business, where many applications are networked using Distributed Information Systems (DIS); however, communication between component applications, on the other hand, becomes challenging. As a result, their reliability is critical, as a system failure in this area might be costly and dangerous.

In view of the aforementioned reality, several academics have submitted good works evaluating the performance of repairable systems. Pourhassan et al. (2019) investigated the impact of fault in component reliability estimation on system designing. Raissi and Ebadi (2018) dealt with computer simulation model for reliability estimation of a complex system. Pourhassan et al. (2020) presented simulation approach to reliability assessment of complex system under stochastic degradation and random shock. Yusuf eta al. (2020) who have recently discussed the performance of a multi-computer system with three subsystems in series arrangement using the Copula repair technique. Pourhassan (2021) analyzed the reliability of power station subject fatal and non fatal shocks. Attar et al. (2016) developed a simulation-based optimization model for free distributed repairable multi-state availability-redundancy allocation problems. Potapov et al. (2019) carried out research on modeling the reliability of a client-server information system. Singh et al (2021) more recently gave a study on probabilistic assessment of CBT network system having four subsystems configured in series using Copula repair policy. Tsarouhas et al. (2009) analyzed the reliability, availability, and maintainability of a cheese (feta) production line in a Greek medium-sized company. Singh et al. (2020) discussed the reliability of a repairable network system consisting of three computer laboratories connected to a server using a 2- out-of- 3: G arrangement. Dahiya et al. (2019) used the RAMD method to evaluate the performance of the sugar industry's A-Pan crystallization system. Rahman (2018) has investigated the stationary availability factor with arbitrary topology for two-level computer networks. Chen et al (2019) used machine reliability to examine the situation of a machine, and then designed a production scheduling model that integrates machine reliability to help decision makers in determining the best scheduling strategy. Monika Gahlot et al. (2018) discussed the effectiveness of the repairable system under different types of failure and two types of fixes in series configuration using Gumbel-Hougaard family Copula. Singh and Abdul Kareem (2019) have presented the cost evaluation of complex repairable systems consisting two subsystems in series utilizing Gumbel Hougaard family copula. Harish Garg (2017) provided performance analysis of an industrial system based on a hybridized soft computing methodology. Ibrahim et al (2017) used Gumbel–Hougaard copula family to examine the reliability of a complex system having two subsystems in series configuration. Abubakar and Singh (2019) have applied a supplementary variable technique and a Copula linguistic approach to assess the effectiveness of an industrial system in the cloth industry with different failure rates of the various subsystems. Mehta et al. (2018) applied supplementary variable technique to evaluate the availability of an industrial system. Singh and Ayagi (2018) provided the performance of a complex system under a proactive resume repair strategy via Copula. Zhang (2019) reported research on computer network reliability analysis using an intelligent cloud computing technology. Based on cost-free warranty policy, Niwas and Garg (2018) proposed a method for measuring the reliability and profit of an industrial system. Yang et al. (2019) looked at how to assess the reliability of a hierarchical system with inconsistent priors and multilevel data. Yusuf et al. (2018) showed some reliability characteristics of a linear consecutive 2-out-4 system that was operated with a 2-out-4 supporting device. More recently, Sanusi et al. (2021) have used RAMD technique to study the performance of a computer-based test (CBT) at subsystem level. The study of reliability and availability analysis for a three-unit turbine power producing system with seasonal effect and FcFs was discussed by Rajesh et al. (2018).

In the light of the aforementioned empirical examination, it is quite understandable that many scholars have undertaken research in the field of reliability engineering. However, it is worth noting that distributed systems are gaining traction in the economy and across a variety of industries. As a result, sufficient information on their reliability prediction is required in order to estimate the accurate performance rate. This leaves a significant gap, which this study aims to fill.

The rest of the paper is organized as follows; The notations, assumptions, and system description are included in Section 2. Section 3 contains the formulation and solutions of the model. The results of the analysis of the system for different scenario are summarized in Section 4. The outcome discussion of the study was presented and concluded in Section 5

2. Description of the System

The reference system, depicted in Figure 1, consists of two load balancers, two distributed database servers with their associated mirror database servers and a centralized database server. The primary objectives of the load balancers are to improve system performance and support green computing by ensuring proper utilization of the delivery servers. This is because, high utilization of a server results in slow response and high energy consumption. In the reference system, the load-balancing service availability is ensured with the presence of the two load-balancers (LB-1 and LB-2). It is assumed, at every point in time, that one of the load balancers is active and the other is passive. The active load balancer (LB-1) handles all the requests from the client. On failover of LB-1, the passive load balance (LB-2) assumes the active status and continue the load balancing service. The two load balancers are connected to a distributed database system involving two replicated delivery distributed database servers, DDS-1 and DDS-2, which are responsible for processing query requests from the client. Client requests are evenly distributed between the delivery database servers, which, as earlier stated, greatly improves system performance and minimizes energy consumption of the servers. Moreover, there are two mirror database servers, DDS-I and DDS-II, associated with the DDS-1 and DDS-2, respectively, which keep redundant copies of the entire primary databases. System availability is improved with the mirror servers. It is assumed a mirror server is only active when its associated primary server fails. For instance, when DDS-1 fails, DDS-I become active and continue the service that the failed DDS-1 provides. Furthermore, the centralized database server (CDS) serves as master database, to which the two delivery servers report. The delivery servers report the data processed and updated back to the CDS. Any update to the CDS by either of the DDSs is replicated in the other, which is replicated, in turn, in its associate mirror server. Therefore, the entire system fails when the CDS fails.

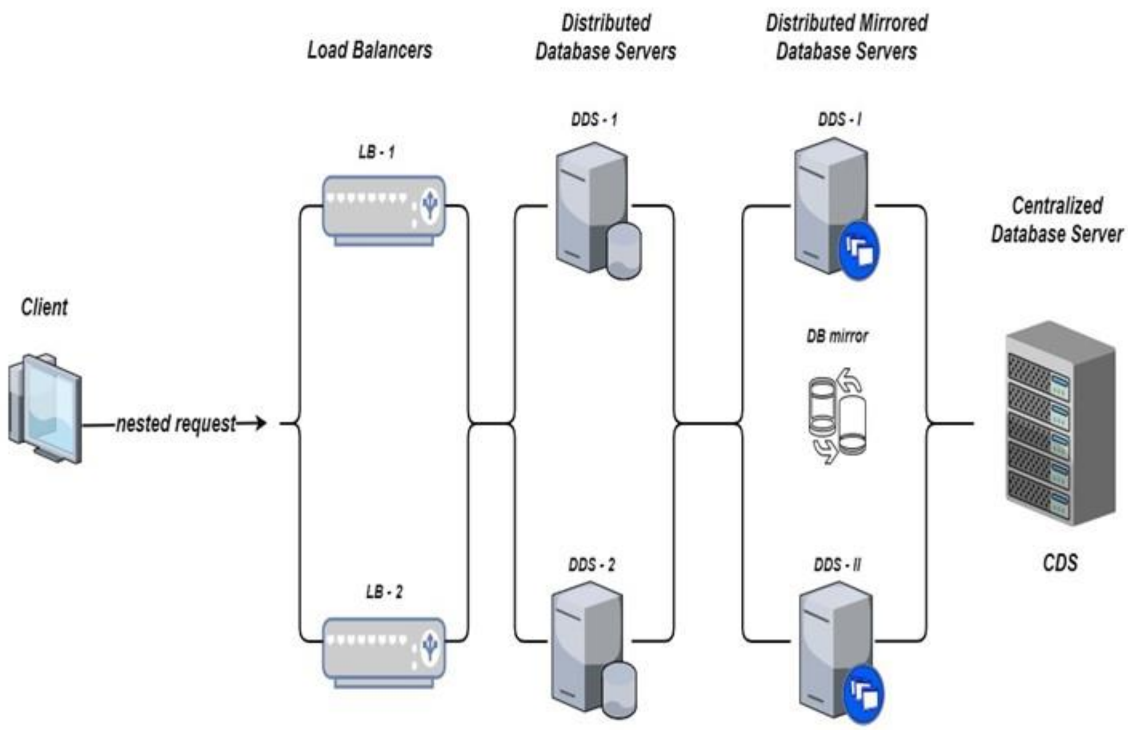

Figure 1: Reliability block diagram of the system

Notations

t: Time variable on a time scale.

s: Laplace transform variable for all expressions

η_1 : Failure rate of load balancer

η_2 : Failure rate of distributed database server

η_3 : Failure rate of distributed mirrored database server

η_4 : Failure rate of client

η_5 : Failure rate of centralized database server

$h(x)$: Repair rate of load balancer

$h(y)$: Repair rate of distributed database server

$h(r)$: Repair rate of distributed mirrored database server

$\mu_0(x)$: Copula repair rate for complete failed states of load balancer

$\mu_0(y)$: Copula repair for complete failed state of distributed database server

$\mu_0(r)$: Copula repair for complete failed state of distributed mirrored database server

$\mu_0(m)$: Copula repair for complete failed state of client

$\mu_0(n)$: Copula repair for complete failed state of centralized database server

$p_i(t)$: The probability that the system is in Si state at instants for i =0 to 14

$\overline{P}(s)$: Laplace transformation of state transition probability $p(t)$

Pi (x, t): The probability that a system is in state Si, the system under repair and elapse repair time is (x, t) with repair variable x and time variable t

Pi (y, t): The probability that a system is in state Si, the system under repair and elapse repair time is (y, t) with repair variable x and time variable t

$E_p(t)$: Expected profit during the time interval [0, t)

K1, K2: Revenue and service cost per unit time, respectively.

Gumbel-Hougaard copula is defined as

$$C_\delta(\eta_1,\eta_2) = Exp\left[-\left((-\log\eta_1)^\delta + (-\log\eta_2)\delta\right)^{\frac{1}{\delta}}\right]$$

$1 \le \delta < \infty$

The value of $\delta = 1$ corresponds to independence copula and as $\delta \to \infty$, it corresponds to the comonotonicity copula.

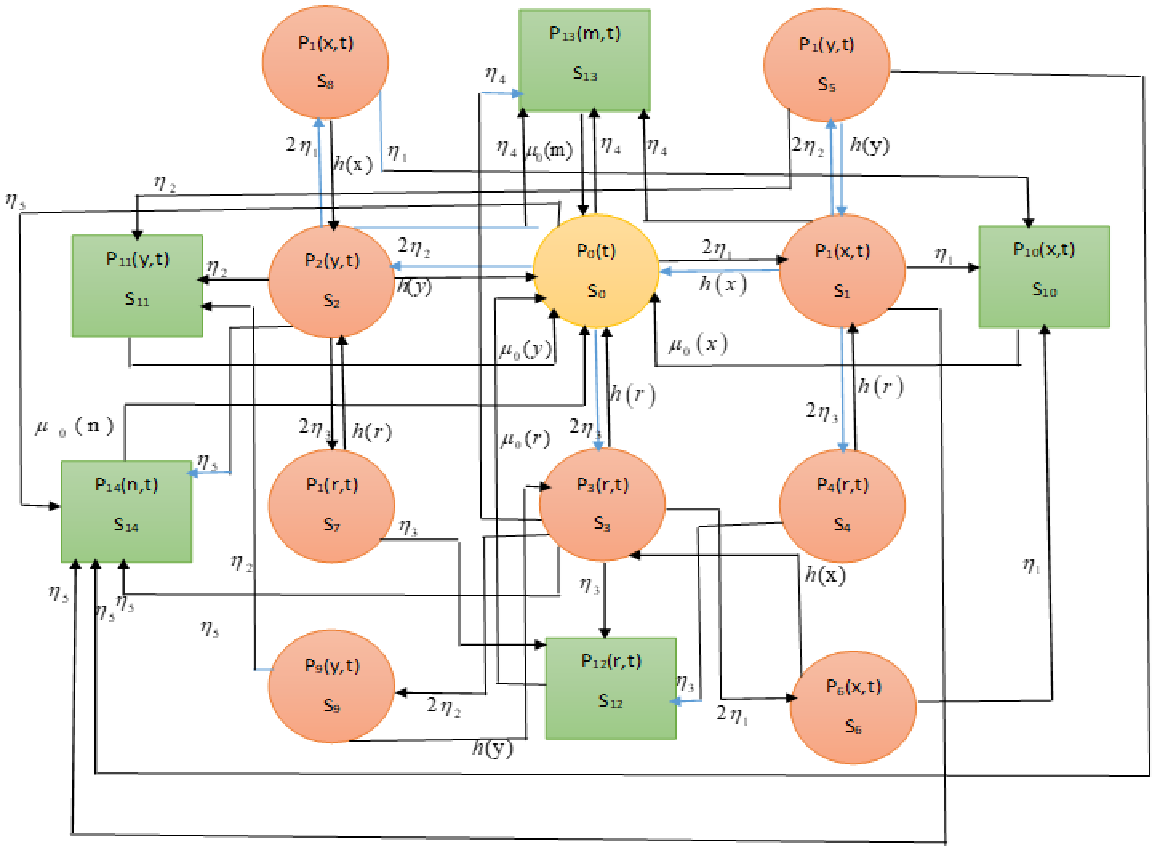

FIGURE 2: TRANSITION DIAGRAM OF THE SYSTEM

TABLE 1: STATE OF THE SYSTEM;
W COMES FOR WORKING, F (FAILED), FF (FAILED FIRST), FL (FAILED LAST), I (IDLE), D (DOWN)

State	Client	Load Balancer		DDS		Mirrored DDS		CDS	Status
		I	II	I	II	I	II		
S_0	W	W	W	W	W	W	W	W	W
S_1	W	F	W	W	W	W	W	W	W
S_2	W	W	W	F	W	W	W	W	W
S_3	W	W	W	W	W	F	W	W	W
S_4	W	FF	W	W	W	FL	W	W	W
S_5	W	FF	W	FL	W	W	W	W	W
S_6	W	FF	W	W	W	FF	W	W	W
S_7	W	W	W	FF	W	FF	W	W	W
S_8	W	FL	W	FF	W	W	W	W	W
S_9	W	W	W	FL	W	FF	W	W	W
S_{10}	I	FF	FL	I	I	I	I	I	D
S_{11}	I	I	I	FF	FL	I	I	I	D
S_{12}	I	I	I	I	I	FF	FL	I	D
S_{13}	F	I	I	I	I	I	I	I	D
S_{14}	I	I	I	I	I	I	I	F	D

3. Formulation of Reliability Models

By the probability of considerations and continuity of arguments as in Nelson (2006) as in Nelson (2006), Ram and Singh (2008), Ram and Singh (2010) and Chopra and Ram (2019), the system of differential difference equations obtained from Figure 2 are presented below:

$$\left(\frac{\partial}{\partial t}+2\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5\right)p_0(t)=\int_0^\infty h(x)\,p_1(x,t)dx+\int_0^\infty h(y)\,p_2(y,t)dy+\int_0^\infty h(r)\,p_3(r,t)dr+$$

$$\int_0^\infty \mu_0(x)\,p_{10}(x,t)dx+\int_0^\infty \mu_0(y)\,p_{11}(y,t)dy+\int_0^\infty \mu_0(r)\,p_{12}(r,t)dr+$$

$$\int_0^\infty \mu_0(m)\,p_{13}(m,t)dm+\int_0^\infty \mu_0(n)\,p_{14}(n,t)dn \qquad (1)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5+h(x)\right)p_1(x,t)=0 \qquad (2)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5+h(y)\right)p_2(y,t)=0 \qquad (3)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+2\eta_1+2\eta_2+\eta_3+\eta_4+\eta_5+h(r)\right)p_3(r,t)=0 \qquad (4)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\eta_3+h(r)\right)p_4(r,t)=0 \qquad (5)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\eta_2+h(y)\right)p_5(y,t)=0 \qquad (6)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\eta_1+h(x)\right)p_6(x,t)=0 \qquad (7)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\eta_3+h(r)\right)p_7(r,t)=0 \qquad (8)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\eta_1+h(x)\right)p_8(x,t)=0 \qquad (9)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\eta_2+h(y)\right)p_9(y,t)=0 \qquad (10)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)p_{10}(x,t)=0 \qquad (11)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)p_{11}(y,t)=0 \qquad (12)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\mu_0(r)\right)p_{12}(r,t)=0 \qquad (13)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+\mu_0(m)\right)p_{13}(m,t)=0 \qquad (14)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+\mu_0(n)\right)p_{14}(n,t)=0 \qquad (15)$$

and the initial Condition $p_0(0)=1$, all other transition probabilities are zero at t=0.

Boundary condition

$$p_1(0,t)=2\eta_1 p_0(t) \qquad (16)$$

$$p_2(0,t)=2\eta_2 p_0(t) \qquad (17)$$

$$p_3(0,t)=2\eta_3 p_0(t) \qquad (18)$$

$$p_4(0,t)=2\eta_3 p_1(0,t) \qquad (19)$$

$$p_5(0,t)=2\eta_2 p_1(0,t) \qquad (20)$$

$$p_6(0,t)=2\eta_1 p_3(0,t) \qquad (21)$$

$$p_7(0,t)=2\eta_3 p_2(0,t) \qquad (22)$$

$$p_8(0,t)=2\eta_1 p_2(0,t) \qquad (23)$$

$$p_9(0,t)=2\eta_2 p_3(0,t) \qquad (24)$$

$$p_{10}(0,t)=\eta_1\left(p_1(0,t)+p_6(0,t)+p_8(0,t)\right) \qquad (25)$$

$$p_{11}(0,t)=\eta_2\left(p_2(0,t)+p_5(0,t)+p_9(0,t)\right) \qquad (26)$$

$$p_{12}(0,t)=\eta_3\left(p_3(0,t)+p_4(0,t)+p_7(0,t)\right) \qquad (27)$$

$$p_{13}(0,t)=\eta_4\left(p_0(0,t)+p_1(0,t)+p_2(0,t)+p_3(0,t)\right) \qquad (28)$$

$$p_{14}(0,t)=\eta_5\left(p_0(0,t)+p_1(0,t)+p_2(0,t)+p_3(0,t)\right) \qquad (29)$$

Applying Laplace transformations to equation (1) - (28) with help of boundary conditions to give

$$\left(s+2\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5\right)\overline{p}_0(\mathrm{s})=1+\int_0^\infty h(x)\overline{p}_1(x,\mathrm{s})dx+\int_0^\infty h(y)\overline{p}_2(y,s)dy+\int_0^\infty h(r)\overline{p}_3(r,s)dr+$$

$$\int_0^\infty \mu_0(x)\overline{p}_{10}(x,s)dx+\int_0^\infty \mu_0(y)\overline{p}_{11}(y,s)dy\ \int_0^\infty \mu_0(r)\overline{p}_{12}(r,s)dr+$$

$$\int_0^\infty \mu_0(m)\overline{p}_{13}(m,s)dm+\int_0^\infty \mu_0(n)\overline{p}_{14}(n,s)dn \qquad (30)$$

$$\left(s+\frac{\partial}{\partial x}+\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5+h(x)\right)\overline{p}_1(x,s)=0 \quad (31)$$

$$\left(s+\frac{\partial}{\partial y}+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5+h(y)\right)\overline{p}_2(y,s)=0 \quad (32)$$

$$\left(s+\frac{\partial}{\partial r}+2\eta_1+2\eta_2+\eta_3+\eta_4+\eta_5+h(r)\right)\overline{p}_3(r,s)=0 \quad (33)$$

$$\left(s+\frac{\partial}{\partial r}+\eta_3+h(r)\right)\overline{p}_4(r,t)=0 \quad (34)$$

$$\left(s+\frac{\partial}{\partial y}+\eta_2+h(y)\right)\overline{p}_5(y,t)=0 \quad (35)$$

$$\left(s+\frac{\partial}{\partial x}+\eta_1+h(x)\right)\overline{p}_6(x,t)=0 \quad (36)$$

$$\left(s+\frac{\partial}{\partial r}+\eta_3+h(r)\right)\overline{p}_7(r,t)=0 \quad (37)$$

$$\left(s+\frac{\partial}{\partial x}+\eta_1+h(x)\right)\overline{p}_8(x,t)=0 \quad (38)$$

$$\left(s+\frac{\partial}{\partial y}+\eta_2+h(y)\right)\overline{p}_9(y,t)=0 \quad (39)$$

$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\overline{p}_{10}(x,s)=0 \quad (40)$$

$$\left(s+\frac{\partial}{\partial y}+\mu_0(y)\right)\overline{p}_{11}(y,s)=0 \quad (41)$$

$$\left(s+\frac{\partial}{\partial r}+\mu_0(r)\right)\overline{p}_{12}(r,s)=0 \quad (42)$$

$$\left(s+\frac{\partial}{\partial m}+\mu_0(m)\right)\overline{p}_{13}(m,s)=0 \quad (43)$$

$$\left(s+\frac{\partial}{\partial n}+\mu_0(n)\right)\overline{p}_{14}(n,s)=0 \quad (44)$$

Boundary conditions

$$\overline{p}_1(0,s)=2\eta_1\overline{p}_0(s) \quad (45)$$

$$\overline{p}_2(0,s)=2\eta_2\overline{p}_0(s) \quad (46)$$

$$\overline{p}_3(0,s)=2\eta_3\overline{p}_0(s) \quad (47)$$

$$\bar{p}_4(0,s)=2\eta_3\bar{p}_1(0,s) \tag{48}$$

$$\bar{p}_5(0,s)=2\eta_2\bar{p}_1(0,s) \tag{49}$$

$$\bar{p}_6(0,s)=2\eta_1\bar{p}_3(0,s) \tag{50}$$

$$\bar{p}_7(0,s)=2\eta_3\bar{p}_2(0,s) \tag{51}$$

$$\bar{p}_8(0,s)=2\eta_1\bar{p}_2(0,s) \tag{52}$$

$$\bar{p}_9(0,s)=2\eta_2\bar{p}_3(0,s) \tag{53}$$

$$\bar{p}_{10}(0,s)=\eta_1\left(\bar{p}_1(0,s)+\bar{p}_6(0,s)+\bar{p}_8(0,s)\right) \tag{54}$$

$$\bar{p}_{11}(0,s)=\eta_2\left(\bar{p}_2(0,s)+\bar{p}_5(0,s)+\bar{p}_9(0,s)\right) \tag{55}$$

$$\bar{p}_{12}(0,s)=\eta_3\left(\bar{p}_3(0,s)+\bar{p}_4(0,s)+\bar{p}_7(0,s)\right) \tag{56}$$

$$\bar{p}_{13}(0,s)=\eta_4\left(\bar{p}_0(s)+\bar{p}_1(0,s)+\bar{p}_2(0,s)+\bar{p}_3(0,s)\right) \tag{57}$$

$$\bar{p}_{14}(0,s)=\eta_5\left(\bar{p}_0(s)+\bar{p}_1(0,s)+\bar{p}_2(0,s)+\bar{p}_3(0,s)\right) \tag{58}$$

$p_0(0)=1$ and other state transition probabilities are zero at $t=0$ (59)

Determining of equation (31) - (44) with help of Laplace transform of boundary conditions

$$\bar{p}_0(s)=\frac{1}{D(s)} \tag{60}$$

$$\bar{p}_1(s)=\frac{2\eta_1}{D(s)}\left\{\frac{1-\bar{s}_h(s+\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5)}{s+\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5}\right\} \tag{61}$$

$$\bar{p}_{11}(s)=\left(\frac{2\eta_2^2+4\eta_2^2\eta_1+4\eta_2^2\eta_3}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{73}$$

$$\bar{p}_2(s)=\frac{2\eta_2}{D(s)}\left\{\frac{1-\bar{s}_h(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5)}{s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5}\right\} \tag{62}$$

$$\bar{p}_3(s)=\frac{2\eta_3}{D(s)}\left\{\frac{1-\bar{s}_h(s+2\eta_1+2\eta_2+\eta_3+\eta_4+\eta_5)}{s+2\eta_1+2\eta_2+\eta_3+\eta_4+\eta_5}\right\} \tag{63}$$

$$\bar{p}_4(s)=\frac{4\eta_1\eta_3}{D(s)}\left\{\frac{1-\bar{s}_h(s+\eta_3)}{s+\eta_3}\right\} \tag{66}$$

$$\bar{p}_5(s)=\frac{4\eta_1\eta_2}{D(s)}\left\{\frac{1-\bar{s}_h(s+\eta_2)}{s+\eta_2}\right\} \tag{67}$$

$$\bar{p}_6(s)=\frac{4\eta_1\eta_3}{D(s)}\left\{\frac{1-\bar{s}_h(s+\eta_1)}{s+\eta_1}\right\} \tag{68}$$

$$\bar{p}_7(s)=\frac{4\eta_2\eta_3}{D(s)}\left\{\frac{1-\bar{s}_h(s+\eta_3)}{s+\eta_3}\right\} \tag{69}$$

$$\bar{p}_8(s)=\frac{4\eta_1\eta_2}{D(s)}\left\{\frac{1-\bar{s}_h(s+\eta_1)}{s+\eta_1}\right\} \tag{70}$$

$$\bar{p}_9(s)=\frac{4\eta_2\eta_3}{D(s)}\left\{\frac{1-\bar{s}_h(s+\eta_2)}{s+\eta_2}\right\} \tag{71}$$

$$\bar{p}_{10}(s)=\left(\frac{2\eta_1^2+4\eta_1^2\eta_3+4\eta_1^2\eta_2}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{72}$$

$$\bar{p}_{12}(s)=\left(\frac{2\eta_3^2+4\eta_3^2\eta_1+4\eta_3^2\eta_2}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{74}$$

$$\bar{p}_{13}(s)=\left(\frac{\eta_4+2\eta_1\eta_4+2\eta_2\eta_4+2\eta_3\eta_4}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{75}$$

$$\bar{p}_{14}(s)=\left(\frac{\eta_5+2\eta_1\eta_5+2\eta_2\eta_5+2\eta_3\eta_5}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{76}$$

Where D(s) is defined as;

$$D(s)=\left\{s+2\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5-\left(\begin{array}{l}2\eta_1\bar{s}_h(s+\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5)+\\ 2\eta_2\bar{s}_h(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5)+\\ 2\eta_3\bar{s}_h(s+2\eta_1+2\eta_2+\eta_3+\eta_4+\eta_5)+\\ \left[\begin{array}{l}(2\eta_1^2+4\eta_1^2\eta_3+4\eta_1^2\eta_2)+\\ (2\eta_2^2+4\eta_2^2\eta_1+4\eta_2^2\eta_3)+\\ (2\eta_3^2+4\eta_3^2\eta_1+4\eta_3^2\eta_2)+\\ (\eta_4+2\eta_1\eta_4+2\eta_2\eta_4+2\eta_3\eta_4)+\\ (\eta_5+2\eta_1\eta_5+2\eta_2\eta_5+2\eta_3\eta_5)\end{array}\right]\bar{s}_{\mu_0}(s)\end{array}\right)\right\} \tag{77}$$

Summing all Laplace transformations of the state transition probabilities that the system is operating, are as follows:

$$\bar{p}_{up}(s)=\left[\begin{array}{l}\bar{p}_0(s)+\bar{p}_1(s)+\bar{p}_2(s)+\bar{p}_3(s)+\bar{p}_4(s)+\bar{p}_5(s)+\bar{p}_6(s)+\\ \bar{p}_7(s)+\bar{p}_8(s)+\bar{p}_9(s)\end{array}\right] \tag{78}$$

$$\bar{p}_{up}(s)=\frac{1}{D(s)}\left\{\begin{array}{l}1+2\eta_1\left(\frac{1-\bar{s}_h(s+\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5)}{s+\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5}\right)+2\eta_2\left(\frac{1-\bar{s}_h(s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5)}{s+2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5}\right)\\ +2\eta_3\left(\frac{1-\bar{s}_h(s+2\eta_1+2\eta_2+\eta_3+\eta_4+\eta_5)}{s+2\eta_1+2\eta_2+\eta_3+\eta_4+\eta_5}\right)+4\eta_1\eta_3\left(\frac{1-\bar{s}_h(s+\eta_3)}{s+\eta_3}\right)+\\ 4\eta_1\eta_2\left(\frac{1-\bar{s}_h(s+\eta_2)}{s+\eta_2}\right)+4\eta_1\eta_3\left(\frac{1-\bar{s}_h(s+\eta_1)}{s+\eta_1}\right)+4\eta_2\eta_3\left(\frac{1-\bar{s}_h(s+\eta_3)}{s+\eta_3}\right)\\ +4\eta_1\eta_2\left(\frac{1-\bar{s}_h(s+\eta_1)}{s+\eta_1}\right)+4\eta_2\eta_3\left(\frac{1-\bar{s}_h(s+\eta_2)}{s+\eta_2}\right)\end{array}\right\}$$

(79)

$$\overline{p}_{down}(s) = 1 - \overline{p}_{up}(s)) \tag{80}$$

1. Numerical Analysis of the Study

4.1 Formulation and Analysis of Availability Analysis

Setting

$$S_{\mu_0}(s) = \overline{S}_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) = \frac{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}$$

, $\overline{S}_h(s) = \dfrac{h}{s+h}$, and taking the values of different parameters as $\eta_1 = 0.01, \eta_2 = 0.02, \eta_3 = 0.03, \eta_4 = 0.04 and \eta_5 = 0.05$ and $h(x) = h(y) = 1$ in equation (79), then taking inverse Laplace transform, one may obtain the availability expression as:

$$\overline{p}_{up}(t) = \begin{bmatrix} -0.000102e^{-1.00200t} - 0.000070e^{-1.03000t} \\ +0.004334e^{-2.73016t} - 0.003436e^{-1.11169t} \\ -0.000001e^{-1.07370t} - 0.000059e^{-1.06919t} \\ +0.999388e^{-0.00054t} - 0.000053e^{-1.00100t} \end{bmatrix} \tag{81}$$

For different values of time variable t = 0, 2, 4, …20, units of time, one may get different values of Availability from equation (81) as shown in Table 2 below.

Table 2: Computed availability with respect of time

Time	0	2	4	6	8	10	12	14	16	18	20
Availability	1.0000	0.9979	0.9971	0.9961	0.9950	0.9939	0.9928	0.9918	0.9907	0.9896	0.9885

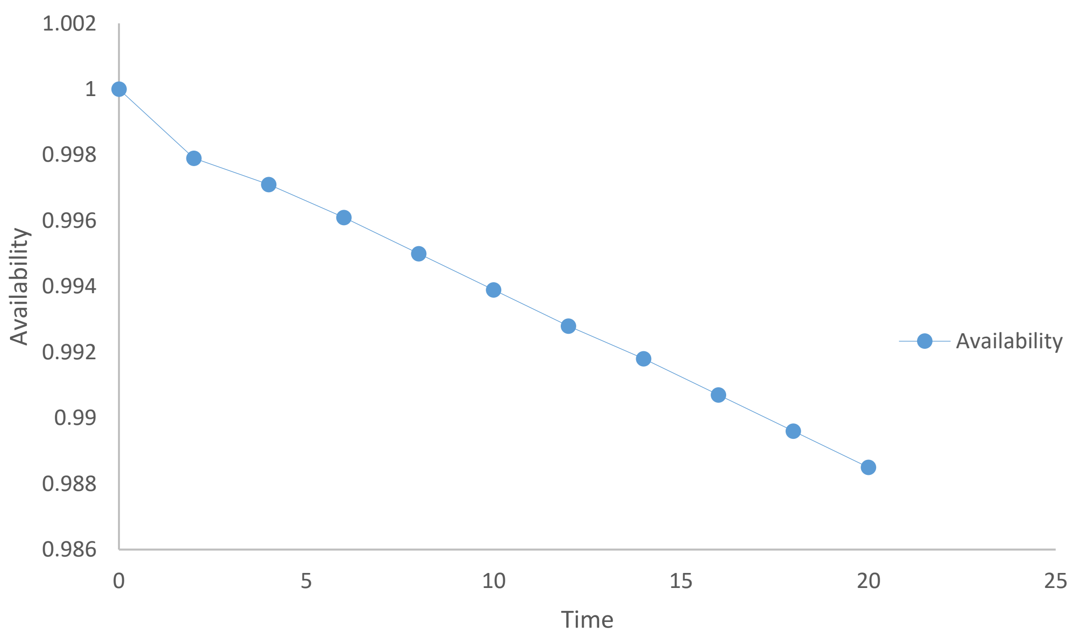

Figure 3: Availability against time

4.
2 Formulation and Analysis of Reliability Analysis

Taking all repair rates, h, μ in equation (79) to zero for the same values of failure rates as $\eta_1 = 0.01, \eta_2 = 0.02, \eta_3 = 0.03, \eta_4 = 0.04 and \eta_5 = 0.05$ and then taking Laplace transformation, one gets reliability expression as:

$$R(t) = \begin{bmatrix} 0.020000e^{-0.03000t} + 0.016842e^{-0.02000t} + \\ 2e^{-0.20000t} + 0.10000e^{-0.01000t} + 2e^{-0.19000t} \\ +2e^{-0.18000t} - 5.046842e^{-0.21000t} \end{bmatrix} \tag{82}$$

Table 3: Computed of Reliability for different values of time (t)

Time	0	2	4	6	8	10	12	14	16	18	20
Reliability	1.0000	0.8325	0.6716	0.5307	0.4137	0.3200	0.2466	0.1901	0.1472	0.1149	0.0907

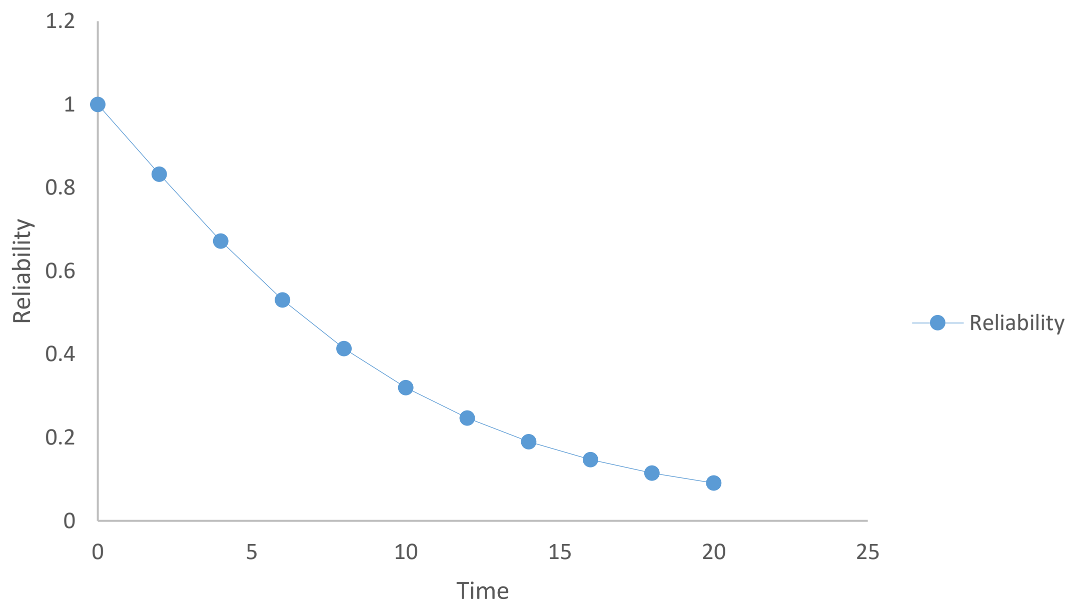

Figure 4: Reliability against time

4.3 Formulation and Analysis of Mean Time to Failure

Letting all repairs to zero and s approach zero in equation (79), MTTF is defined as follows:

$$MTTF=\lim_{s\to 0}\overline{p}_{up}\left(s\right)=\frac{1}{2\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5}\left\{\begin{array}{l}1+\dfrac{2\eta_1}{\eta_1+2\eta_2+2\eta_3+\eta_4+\eta_5}+\\ \dfrac{2\eta_2}{2\eta_1+\eta_2+2\eta_3+\eta_4+\eta_5}\\ +\dfrac{2\eta_3}{2\eta_1+2\eta_2+\eta_3+\eta_4+\eta_5}\\ +8\eta_1+12\eta_2+4\eta_3\end{array}\right\} \quad (83)$$

Assuming $\eta_1=0.01,\eta_2=0.02,\eta_3=0.03,\eta_4=0.04 and \eta_5=0.05$ and changing $\eta_1,\eta_2,\eta_3,\eta_4 and \eta_5$ one by one respectively as 0.01, 0.02...0.09 in equation (83), MTTF is computed with respect to failure as presented in the subsequent Table 4.

Table 4: Calculated MTTF for different failure rates

Failure Rate	$MTTF\ \eta_1$	$MTTF\ \eta_2$	$MTTF\ \eta_3$	$MTTF\ \eta_4$	$MTTF\ \eta_5$
0.01	9.9231	10.0906	11.0392	12.2647	13.2955
0.02	9.5693	9.9231	10.4299	11.3758	12.2647
0.03	9.2774	9.7865	9.9231	10.6021	11.3758
0.04	9.0289	9.6680	9.4888	9.9231	10.6021
0.05	8.8123	9.5609	9.1089	9.3228	9.9231
0.06	8.6200	9.4618	8.7716	8.7886	9.3228
0.07	8.4469	9.3686	8.4687	8.3103	8.7886
0.08	8.2894	9.2803	8.1943	7.8798	8.3103
0.09	8.1449	9.1960	7.9440	7.4905	7.8798

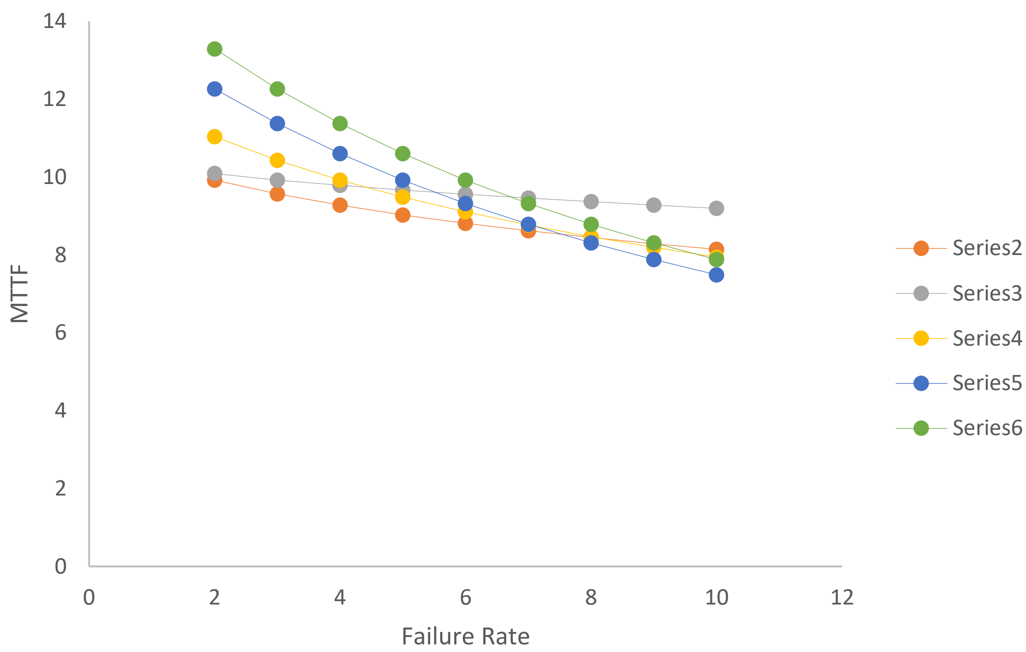

Figure 5: MTTF against Failure Rate

4.4 Sensitivity Analysis

Taking partial differential of the MTTF with respect to failure rate gives sensitivity of the system. Considering same values of failure rates as, $\eta_1 = 0.01, \eta_2 = 0.02, \eta_3 = 0.03, \eta_4 = 0.04 and \eta_5 = 0.05$ in the partial differential equation of the MTTF; the result is presented in Table 5 below.

Table 5: Computed Sensitivity with respect to Failure Rate

Failure Rate	$\frac{\partial(MTTF)}{\eta_1}$	$\frac{\partial(MTTF)}{\eta_2}$	$\frac{\partial(MTTF)}{\eta_3}$	$\frac{\partial(MTTF)}{\eta_4}$	$\frac{\partial(MTTF)}{\eta_5}$
0.01	-39.3621	-18.9975	-67.5160	-95.4773	-111.2692
0.02	-31.8956	-14.9128	-55.1689	-82.7417	-95.4773
0.03	-26.7813	-12.6068	-46.6813	-72.3342	-82.7417
0.04	-23.1034	-11.2005	-40.4694	-63.7286	-72.3342
0.05	-20.3449	-10.2675	-35.6988	-56.5382	-63.7286
0.06	-18.1998	-9.5925	-31.8975	-50.4734	-56.5382
0.07	-16.4799	-9.0632	-28.7819	-45.3143	-50.4734
0.08	-15.0650	-8.6198	-26.1720	-40.8917	-45.3143
0.09	-13.8756	-8.2299	-23.9479	-37.0737	-40.8917

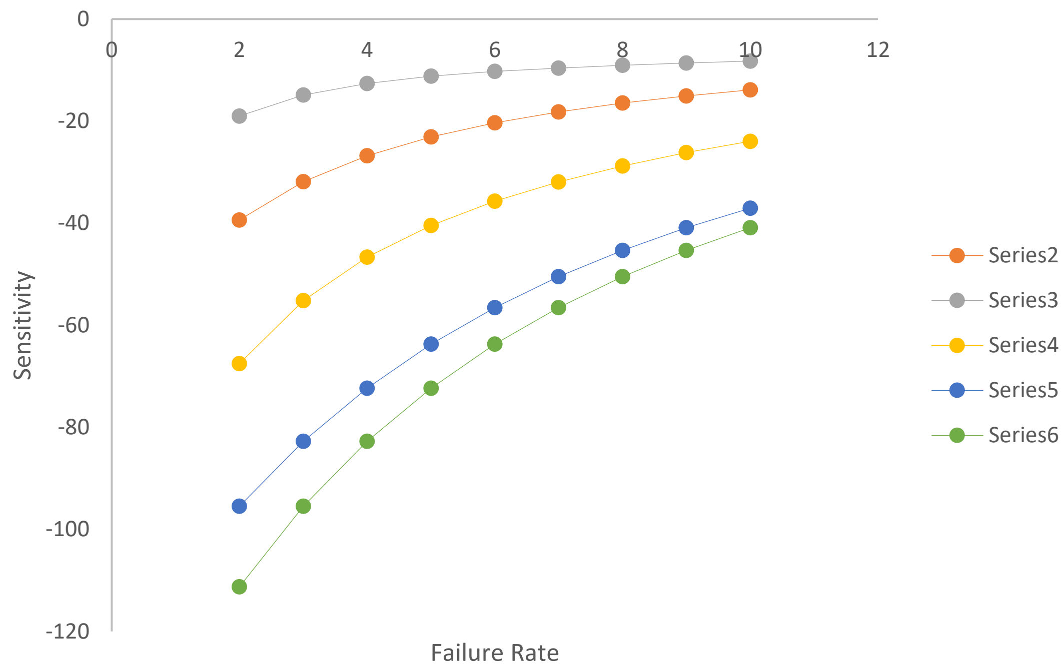

Figure 6: Sensitivity against Failure Rate

4.5 Formulation and Analysis of Cost

The expression for the expected profit incurred in $[0,t)$ is

$$E_p(t) = K_1\int_0^t P_{up}(t)dt - K_2 t \qquad (84)$$

$$E_p\left(t\right)=k_1\begin{Bmatrix} 0.000102e^{-1.00200t}+0.000068e^{-1.03000t} \\ -0.001587e^{-2.73016t}+0.003090e^{-1.11169t} \\ +9.333550e^{-1.07370t}+0.000055e^{-1.06919t} \\ -1839.587880e^{-0.0054t}+0.000053e^{-1.00100t} \\ +1839.5861 \end{Bmatrix}-k_2\left(t\right) \qquad (84)$$

Taking fixed values of parameters of equation (81), the subsequent equation (84) follows;

Assuming K_1= 1and K_2= 0.5, 0.4..., 0.1, respectively and varying t = 0, 2, ...20, units of time, the calculations of expected profit is presented in Table 6 below.

Table 6: Computed Profit with respect to time

Time	$E_p(t)$ K2 = 0.5	$E_p(t)$ K2 = 0.4	$E_p(t)$ K2 = 0.3	$E_p(t)$ K2 = 0.2	$E_p(t)$ K2 = 0.1
0	0	0	0	0	0
2	0.9962	1.1962	1.3962	1.5962	1.7962
4	1.9914	2.3914	2.7914	3.1914	3.5914
6	2.9847	3.5847	4.1847	4.7847	5.3847
8	3.9759	4.7759	5.5759	6.3759	7.1759
10	4.9649	5.9649	6.9649	7.9649	8.9649
12	5.9518	7.1518	8.3518	9.5518	10.7518
14	6.9365	8.3365	9.7365	11.1365	12.5365
16	7.9191	9.5191	11.1191	12.7191	14.3191
18	8.8995	10.6995	12.4995	14.2995	16.0995
20	9.8777	11.8777	13.8777	15.8777	17.8777

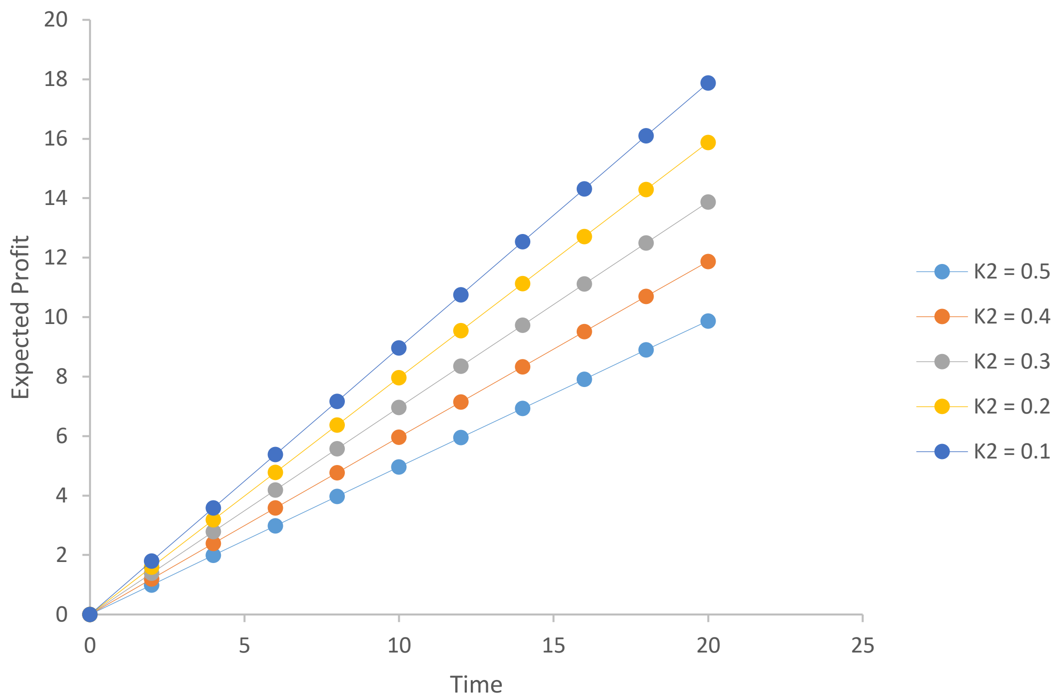

Figure 7: Expected Profit against time

2. Discussion and Concluding Remark

From Table 2 and simulation presented in Figure 3 shows that availability decreases slightly as time passes. From this analysis, one can predict the future behavior of the system at any given time. Table 3 and Figure 4 depicts the system's reliability over time. The graph shows that reliability decline drastically as time t goes from 0 to 20. From the analysis above, it is evident that the system availability and reliability can be improved by incorporating more units on standby, invoking perfect repair in the event of an incomplete failure, replacing the affected subsystem with a new one in the event of a complete failure, regular inspection and preventive maintenance, employing more repair personnel, and so on. Table 4 and chart in Figure 5 depict the mean time to failure against failure rate. The table and the figure show that as failure rate increases, the corresponding MTTF decreases resulting in reduction of life span of the system. To improve the MTTF, it is worthwhile to utilize fault tolerant components to boost the life span of the system. Results presented in Table 5 and Figure 6 depict the impact of failure rates on sensitivity. It is evident from table and figure that with increase in the value of failure rate, the system sensitivity decreases slightly. It can be seen from the table and figure that the strength of each failure rate is not enough to weaken the sensitivity. Table 6 and Figure 7 shows profit against time for different values of K_2. From the table and graph, the predicted profit decreases with increase in time for any value of K_2. However, the predicted profit increases as the value of K_2 increases. The expected profit can be increased by implementing the above-mentioned replacement and redundancy suggestions. In this paper, a distributed system with five standby subsystems A (the clients), B (two load balancers), C (two distributed database servers), D (two mirrored distributed database serves) and E (centralized database server) is considered. Expressions for reliability metrics of testing strength of the system such reliability, availability, mean time to failure (MTTF), sensitivity as well as cost function are derived and validated by performing numerical experiments. Analysis of the effect of various system parameters was performed through MATLAB package and excel. On the basis of the tables and figures, it is evident that the strength of the system, can be enhanced through of replacement worn out unit/subsystem, regular inspection, using fault tolerant units, etc. Thus, higher system strength and performance can be achieved through repair of early failure of units, individual subsystem replacement, and proper maintenance planting to avoid the occurrence of catastrophic failure, and by adding fault tolerant units/subsystems there by keeping the system strength at the highest order leading to product quality, and production output and revenue generation. This work can be extended further to a honeynet computer system used in detecting, controlling and preventing attackers from attacking server using various techniques such genetic algorithm, particle swamp optimization, Grey Wolf, etc. The present study will be useful to production, manufacturing and industrial settings requiring the application of distributed systems.

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