Original Research

Reliability Availability Maintainability Dependability Analysis of Mirrored Distributed System

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ABSTRACT

Computer network topologies are complex systems made up of large subsystems that are arranged in seriesparallel configurations. The current paper dealt with the mathematical modeling of some reliability metrics used in determining the strength, reliability, and performance of a computer distributed system stationed in two locations A and B, all of which were configured as a series-parallel system. The system is made up of six series-parallel subsystems distributed between locations A and B. In location A, there are three subsystems: four clients running in parallel as subsystem 1, six directory servers running in parallel as subsystem 2, and two replica servers running in parallel as subsystem 3, while in location B, there are two replica servers running in parallel as subsystem 4, six directory servers running in parallel as subsystem 5, and four clients running in parallel as subsystem 6. Using the Markovian process, the goal is to build mathematical models of reliability, dependability, availability, and maintainability in order to assess the system's performance, strength, and effectiveness. The ordinary differential difference equations for each subsystem are obtained from the schematic diagrams and solved iteratively. In this work, the Ramd analysis is used to quantify the performance of a system in terms of reliability, maintainability, availability, and dependability. It is tabulated the impact of subsystem repair and failure rates on reliability, maintainability, availability, and dependability. Inspecting the network's essential subsystems and their maintenance priorities improves the system's stability, maintainability, availability, and dependability while also lowering maintenance costs.

Keywords- Mirrored server, replication, directory servers, Distributed system, repair rate, failure rate

INTRODUCTION

The demand for highly reliable systems/industrial systems/components develops substantially as expansion accelerates. In order to meet demand, the system becomes increasingly complex/sophisticated as a result of this evolution. As a system's complexity rises, it necessitates far more attention to meet the needs for reliability and availability. As a result, several researchers in fields such as networking, industry, industrial sectors, military, and others are interested in leveraging the reliability of complex systems in order to ensure that society's development is not inhibited. The strategies and procedures that can be utilized to ensure that system performance is optimized are covered by reliability theory. The RAMD analysis of complex systems can be quite useful in selecting the best design changes.

The component must be activated after the industrial system has been activated. As a result, the most vital component must be identified and specific maintenance techniques must be implemented for these components. Most businesses employ RAMD indexes to keep costs low while providing precise and timely services. If a company's systems aren't reliable, it can't use a rapid response policy. As a result, the bulk of global industrial corporations are promoting operative maintenance practices.

As a result of the foregoing assertion, researchers have devised various maintenance models and strategies aimed at improving system performance and optimizing system reliability, maintainability, availability, and dependability. To mention a few, to explore the strength of Ice cream industry, Tsarouhas (2020) applied ramd to assess the performance of the ice cream machine. Saini et al. (2020) investigated the dependability, maintainability, and availability of microprocessor systems. Kumar et al. (2020) considered ramd as one of the approaches in enhancing operational and performance improvement in some software water treatment and supply plant. Danjuma et al. (2022) investigated the impact of cold standby redundancy in enhancing the ramd of series-parallel system. Kumar and Tewari (2018) examine some methods for assessing system performance based on dependability, availability, and maintainability. Reena and Basotia (2020) created various performance models for evaluating the strength of cement plants. Saini and Kumar (2019) used the RAMD technique to examine the performance of evaporating units in the sugar sector. Sanusi and Yusuf (2021) investigated the performance of a computer-based test (CBT) at the subsystem level using the RAMD approach. Patil et al. (2021) proposed Ram optimization of Computerized Numerical Control Machine Tool through evaluation of indices such as time between failure and time to repair. Choudhary et al. (2019) investigated the cement plant's dependability, availability, and maintainability. Corvaro et al. (2017) studied reciprocating compressor reliability, availability, and maintainability. Gupta et al. (2021) look into the generator's dependability and maintainability in a steam turbine power plant. The dependability and availability of thermal power plants were studied by Jagtap et al. (2021). Tsarouhas (2018) investigated the wine packaging production system's dependability, availability, and maintainability. Khan et al. (2022) looked at how to use performance measures to make decisions about T-spherical operators. Garg and Garg (2022) investigated the profit and availability optimization of a single unit system with imperfect switchover. For a solid waste management system, Barma and Modibbo (2021) introduced a multi-objective optimization model. Pourhassan et al. (2020) proposed a simulation approach for assessing the reliability of complex systems subjected to stochastic degradation and random shock. Raissi and Ebadi (2018) looked at a computer simulation approach for estimating a complex system's reliability. Pourhassan (2021) investigated the reliability of power plants that had been subjected to fatal and non-fatal shocks. Aly et al. (2018) investigated the role of Ram in quantify the performance of 3-out-of-4 industrial system. Goyal et al. (2019) focuses on ramd approach to performance optimization of sewage treatment plant. Farahani et al. (2021) has done a deep literature review on integrated optimization of maintainability, reliability and quality.

The majority of previous research has focused on the system's reliability or availability, according to the literature. The maintainability and reliability criteria needed to identify the critical components of a mirrored distributed system have yet to be found in the literature. As a result, the current research investigates the RAMD analysis of a series parallel system. The paper is divided into four sections, each of which has its own introduction. Several important definitions and system explanations, as well as notations, are included in the second part.

EXPONENTIAL DISTRIBUTION

The process of disseminating objects is known as "distribution." It has a parameterized exponential distribution with density function probability as follows

$$
f(x,\beta) = \begin{cases} \beta e^{-\beta x} & \text{if } x \ge 0, \quad (1) \\ 0 & \text{otherwise} \end{cases}
$$

In engineering, it's one of the most typical failure patterns. Failures caused only by chance or random events will fall into this category.

RELIABILITY

The capacity of a system or device to fulfill its work or function in a set time and under specified conditions is referred to as "reliability." A device or system is considered reliable if it performs its function without fail for the specified period of time.

$$
R(t)=e^{-\int_0^{\infty} Z(t)dt} (2)
$$

The following is a simplified version of equation (2) for a component with a failure rate that is exponentially distributed:

R (t)= $e^{-\beta t}$ (3)

• MTBF

The average time between failures is what it's called. In the majority of cases, it is expressed in hours. As the MTBF increases, the system's reliability improves. The MTBF of an exponentially distributed system can be calculated as follows:

$$
\int_0^\infty R(t)dt = \int_0^\infty e^{-\beta t} dt = \frac{1}{\beta}(4)
$$

• MTTR

The reciprocal of the system repair rate is what this is called. Mathematically,

 $MTTR = \frac{1}{\gamma}$ (5)

• Availability

The chance that a piece of equipment will perform in a specific state for a specified amount of time is known as availability [1.] It's a different way of evaluating how well a piece of equipment, a system, or a component is kept in working order. The probability that a system will be available in a certain condition for a specific period of time, or the duration during which a system will be functional, is defined as system availability. It's the proportion of system uptime to total time spent on the system (i.e. uptime plus downtime).

$$
\\\text{Availableility} = \frac{Uptime}{Uptime + Downtime} = \frac{MTBF}{MTBF + MTRR} \tag{6}
$$

The mean time between failures is a key indicator of a system's reliability. It's similar to how long it takes for a failure to occur on average (MTBF). MTBF refers to the expected time to failure after a component or system has failed and been repaired, whereas MTTF refers to the expected time to failure of a component or system, which is also known as the mean time to failure of components or systems. The Mean Time To Failure (MTTF) is a measure that shows how long a product should last in the field based on particular testing. It's also worth noting that firms' mean time to failure estimates for certain goods or components may not have been calculated by constantly operating one unit until it failed. MTTF data is frequently generated by operating a large number of units, maybe thousands, for a specified amount of time. It's described as:

$$
MTTF = \lim_{n \to \infty} \overline{R}(s) \quad \text{or} \quad MTTF = \int_0^\infty R(t), \quad (7)
$$

R (t) denotes the system's dependability, defined as R (t)=(T>t)= $\int_t^{\infty} f(x)$ $\int_{t}^{\infty} f(x) dx$, and $\bar{R}(s)$ is Laplace transform of R (t)

Maintainability

The formula for determining the maintainability of a system is: $M(t)=1-e^{(-t)/MTTR)}=1-e^{-t\gamma}$ (8) where γ is constant repair rate.

• Dependability

To get the minimum value of dependability, use the algorithm below: $D_{min}=1 - (\frac{1}{d-1})$ $\frac{1}{(d-1)}(e^{-\ln d}/_{d-1} - e^{-d \ln d}/_{d-1})$ (10)

Where $d = \frac{\gamma}{\beta} = \frac{MTBF}{MTTR}$ $\frac{m_{I}}{MTTR}$,

SYSTEM DESCRIPTION

The system represented in Figure 1 is an example of a distributed system with server replication. Any computing device capable of sending requests to and receiving responses from a server over the internet is referred to as a client. Servers are computers that host a variety of computing resource. The clients and server differ in geographical locations A and B. Clients can request for a resource or perform operation on the resource on any of the directory servers. However, response from servers in their location is faster. Workload is distributed among the directory servers in each geographical location as they provide common service to the clients. The replication provides more reliability (multiple options) as failure of one server will not render the service unavailable. Each group of the directory servers is replicated by one server. The replication servers update changes to client's resource among them. This ensures resource consistency and integrity, and a corruption on a single resource will not affect others because backup copies are made and available. Also, the system can provide synergy among clients of the same and different location that work on the common resource.

FIGURE 1 BLOCK DIAGRAM OF THE SYSTEM

SCHEMATIC DIAGRAM OF THE SYSTEM

a) Subsystem A

Each of the four units in the series is linked to the one before it. When these units fail, the system as a whole fails.

$$
S_0 \xrightarrow{4/1} S_1 \xrightarrow{3/1} S_2 \xrightarrow{7/1} S_3 \xrightarrow{4/1} S_4
$$

FIGURE.3: BLOCK DIAGRAM OF THE SUBSYSTEM A

J I E I

b) Subsystem B

This subsystem consists of six components that are linked in a series. The entire system will fail if this unit fails.

FIGURE.4 BLOCK DIAGRAM OF THE SUBSYSTEM B

c) Subsystem C

This subsystem consists of two components linked in series. The entire system fails if this unit fails.

d) Subsystem D

This subsystem consists of two components linked in series. The entire system fails if this unit fails.

FIGURE.6 BLOCK DIAGRAM OF THE SUBSYSTEM D

e) Subsystem E

This subsystem consists of five components with serial configuration. The entire system fails if this unit fails.

f) Subsystem F

This subsystem consists of four components connected in series. The entire system fails if this unit fails.

BLOCK DIAGRAM OF THE SUBSYSTEM F

The following are the RAMD indices for system subsystems: a) For Subsystem A, RAMD indices

When one of the four units fails, the complete system fails as well. The following is the transition diagram, as well as the governing differential equations that go with it:

BLOCK DIAGRAM OF THE SUBSYSTEM A

 $\dot{P}_0 = -4\beta_1 P_0(t) + \gamma_1 P_1(t),$ (12) $\dot{P}_1 = - (3\beta_1 + \gamma_1)P_1(t) + 4\beta_1P_0(t) + \gamma_1P_2(t),$ (13) $\dot{P}_2 = -(2\beta_1 + \gamma_1)P_2(t) + 3\beta_1 P_1(t)$. + $\gamma_1 P_3$ (14) $\dot{P}_3 = -(\beta_1 + \gamma_1)P_3(t) + 2\beta_1 P_2(t)$. + $\gamma_1 P_4$ (15) $\dot{P}_4 = -\gamma_1 P_4(t) + \beta_1 P_1(t).$ (16)

Under steady state, equations (1), (2) and (3) reduces and taking $t \rightarrow \infty$ $-4\beta_1 P_0(t) + \gamma_1 P_1(t)$, =0 (17) $-(3\beta_1 + \gamma_1)P_1(t) + 4\beta_1P_0(t) + \gamma_1P_2(t) = 0,$ (18) $-(2\beta_1 + \gamma_1)P_2(t) + 3\beta_1P_1(t)$. $+\gamma_1P_3(t)=0$, (19) $-(\beta_1 + \gamma_1)P_3(t) + 2\beta_1 P_2(t)$. + $\gamma_1 P_4$ $(t)=0,$ (20) $-\gamma_1 P_2 + \beta_1 P_1 = 0.$ (21)

Now, using normalization condition: $P_0+P_1+P_2+P_3+P_4=1,$ (22)

Substituting the values of P_1 , P_2 , P_3 and P_4 We can get the availability of the subsystem by solving Eqs. (17–21) in Eq. (22).

$$
AVsys1 = \frac{1 + \frac{4\beta_1}{\gamma_1} + \frac{12\beta_1^2}{\gamma_1^2} + \frac{24\beta_1^3}{\gamma_1^3}}{1 + \frac{4\beta_1}{\gamma_1} + \frac{12\beta_1^2}{\gamma_1^2} + \frac{24\beta_1^3}{\gamma_1^3} + \frac{24\beta_1^4}{\gamma_1^4}} = 0.9999
$$
 (23)

Reliability R (t)= $e^{-0.001t}$

• Maintainability $M(t)=1-e^{(-t/m_{TTR})}=1-e^{-0.6t}$. (25)

• Dependability
\n
$$
D_{min} = 1 - \left(\frac{1}{d_1 - 1}\right) \left(e^{-\ln d_1}/d_1 - 1\right) = e^{-d_1 \ln d_1}/d_1 - 1 = 0.9999.
$$
\n(26)

 (24)

Equations are used The following are some system effectiveness performance criteria for subsystem A: MTBF = 1000, $MTTR = 1.6667$, $d = 600$.

RAMD indices are used in subsystem B.

One active unit and one in cold standby make up the subsystem. Both units have the same failure rate, thus if one fails, the entire subsystem fails.

FIGURE.4 BLOCK DIAGRAM OF THE SUBSYSTEM B

 $\dot{P}_0 = -6 \beta_2 P_0(t) + \gamma_2 P_1(t),$ (27) $\dot{P}_1 = (5\beta_2 + \gamma_2)P_1(t) + 6\beta_2P_0(t) + \gamma_2P_2(t)$. (28) \dot{P}_2 = - (4 $\beta_2 + \gamma_2$) $P_2(t) + 5\beta_2 P_1(t) + \gamma_2 P_3(t)$, (29) $\dot{P}_3 = - (3\beta_2 + \gamma_2)P_3(t) + 4\beta_2 P_2(t) + \gamma_2 P_4(t),$ (30) \dot{P}_4 = - $(2\beta_2 + \gamma_2)P_4(t) + 3\beta_2P_3(t) + \gamma_2P_5(t)$, (31) $\dot{P}_5 = -(\beta_2 + \gamma_2)P_5(t) + 2\beta_2 P_4(t) + \gamma_2 P_6(t),$ (32) $\dot{P}_6 = -\gamma_2 P_6(t) + \beta_2 P_5(t),$ (33)

Under steady state, equations (23) and (24) reduces and taking t \longrightarrow ∞

 $-6 \beta_2 P_0(t) + \gamma_2 P_1(t) = 0$ (34) $-(5\beta_2 + \gamma_2)P_1(t) + 6\beta_2P_0(t) + \gamma_2P_2(t) = 0. (35)$ - $(4\beta_2 + \gamma_2)P_2(t) + 5\beta_2P_1(t) + \gamma_2P_3(t) = 0$, (36) - $(3\beta_2 + \gamma_2)P_3(t) + 4\beta_2 P_2(t) + \gamma_2 P_4(t) = 0$, (37) - $(2\beta_2 + \gamma_2)P_4(t) + 3\beta_2P_3(t) + \gamma_2P_5(t) = 0$, (38) $-(\beta_2 + \gamma_2)P_5(t) + 2\beta_2P_4(t) + \gamma_2P_6(t) = 0,$ (39) $- \gamma_2 P_6(t) + \beta_2 P_5(t) = 0,$ (40)

Now, using normalization condition:

 $P_0+P_1+P_2+P_3+P_4+P_5+P_6=1$ (41)

Substituting the values of P_i in Eq. (27) and solving Eqs. (25–26) yields the availability of the subsystem.

$$
AVsys2 = \frac{1 + \frac{6\beta_2}{\gamma_2} + \frac{30\beta_2^2}{\gamma_2^2} + \frac{120\beta_2^3}{\gamma_2^3} + \frac{360\beta_2^4}{\gamma_2^4} + \frac{720\beta_2^5}{\gamma_2^5}}{1 + \frac{6\beta_2}{\gamma_2} + \frac{30\beta_2^2}{\gamma_2^2} + \frac{120\beta_2^3}{\gamma_2^3} + \frac{360\beta_2^4}{\gamma_2^4} + \frac{720\beta_2^5}{\gamma_2^5} + \frac{720\beta_2^6}{\gamma_2^6}} = 0.9999
$$
\n
$$
(42)
$$

Reliability

R (t)= $e^{-0.002t}$. (43)

Maintainability

 $M(t)=1-e^{(-t/MTTR)}=1-e^{-0.8t}$. (44)

• Dependability

9

$$
D_{min} = 1 - \left(\frac{1}{d_2 - 1}\right) \left(e^{-\ln d_2} / d_2 - 1\right) = e^{-d_2 \ln d_2} / d_2 - 1 = 0.9999. \tag{45}
$$

Using Eqs. $(4-6, 8-9)$, the following are some performance indicators of subsystem B's system effectiveness: $d = 400$, $MTBF = 5000$, $MTTR = 1.2500$,

RAMD indices are used for subsystem C.

This subsystem is made up entirely of units. When one of the two components fails, the system as a whole fails. The govering differential equations that go with it, as well as the transition diagram, are as follows:

BLOCK DIAGRAM OF THE SUBSYSTEM C

 $\dot{P}_0 = -2\beta_3 P_0(t) + \gamma_3 P_1(t),$ (46) $\dot{P}_1 = - (2\beta_3 + \gamma_3) P_1(t) + 2\beta_3 P_0(t) + \gamma_3 P_2(t)$, (47) \dot{P}_2 = - (2 $\beta_3 + \gamma_3$) $P_2(t) + 2\beta_3 P_1(t) + \gamma_3 P_3(t)$, (48) \dot{P}_2 = - $\gamma_3 P_3(t) + 2\beta_3 P_3(t)$. (49)

Under steady state, equations (32), (33), (34) and (35) reduces and taking t \longrightarrow ∞

 $-2\beta_3 P_0 + \gamma_3 P_1 = 0,$ (50) - $(2\beta_3 + \gamma_3)P_1 + 2\beta_3P_0 + \gamma_3P_2 = 0$, (51) $-(2\beta_3 + \gamma_3)P_2 + 2\beta_3P_1 + \gamma_3P_3 = 0$, (52) $-\gamma_3 P_3 + 2\beta_3 P_3 = 0$. (53)

Now, using normalization condition:

 $P_0+P_1+P_2+P_3 = 1.$ (54)

By solving Eqs. (50–53) and inserting the values of P_1 , P_2 and P_3 in Eq. (54), we may obtain the availability of the subsystem.

$$
AVsys3 = \frac{1 + \frac{2\beta_3}{\gamma_3} + \frac{4\beta_3^2}{\gamma_3^2}}{1 + \frac{2\beta_3}{\gamma_3} + \frac{4\beta_3^2}{\gamma_3^2} + \frac{8\beta_3^3}{\gamma_3^3}} = 0.9999. (55)
$$

• Reliability

R (t)= $e^{-0.003t}$. (56)

- **Maintainability** $M(t)=1-e^{(-t/MTTR)}=1-e^{-1.0t}$. (57)
- Dependability $D_{min}=1 - (\frac{1}{d_{min}})$ $rac{1}{a_3-1}$) $(e^{-\ln a_3}/a_3-1 - e^{-a_3 \ln a_3}/a_3-1) = (58)$

Other system efficacy indicators for subsystem C based on Eqs. $(4-6, 8-9)$ include: MTBF = 333.3333, MTTR = 1.0000, $d = 333.3333$

For subsystem D, RAMD indices

This subsystem consist only of two units only. The governing differential equations that go with it, as well as the transition diagram, are as follows:

FIGURE.5 BLOCK DIAGRAM OF THE SUBSYSTEM D

 $\dot{P}_0 = -2\beta_4 P_0(t) + \gamma_4 P_1$ (59) $\dot{P}_1 = - (2\beta_4 + \gamma_4)P_1(t) + 2\beta_4 P_0(t) + \gamma_4 P_2(t)$, (60) \dot{P}_2 = - $(2\beta_4 + \gamma_4)P_2(t) + 2\beta_4P_1(t) + \gamma_4P_3(t)$ (61) $\dot{P}_2 = -\gamma_4 P_3(t) + 2\beta_4 P_3(t)$. (62)

Under steady state, equations (59 - 62) reduces and taking t \longrightarrow ∞

 $-2\beta_4 P_0 + \gamma_4 P_1 = 0,$ (63) - $(2\beta_4 + \gamma_4)P_1 + 2\beta_4 P_0 + \gamma_4 P_2 = 0$, (64) - $(2\beta_4 + \gamma_4)P_2 + 2\beta_4P_1 + \gamma_4P_3 = 0$, (65) $-\gamma_4 P_3 + 2\beta_4 P_3 = 0$. (66)

Now, using normalization condition:

 $P_0+P_1+P_2+P_3 = 1. (67)$

By solving Eqs. (36–39) and substituting the values of P_1 , P_2 , and P_3 in Eq. (67), we can obtain the availability of the subsystem.

$$
AVsys4 = \frac{1 + \frac{2\beta_4}{\gamma_4} + \frac{4\beta_4^2}{\gamma_4^2}}{1 + \frac{2\beta_4}{\gamma_4} + \frac{4\beta_4^2}{\gamma_4^2} + \frac{8\beta_4^2}{\gamma_4^2}} = 0.9999. (68)
$$

Reliability

R (t)= $e^{-0.004t}$. (69)

• Maintainability

 $M(t)=1-e^{(-t/MTTR)}=1-e^{-1.2t}$. (70)

Dependability

$$
D_{min} = 1 - \left(\frac{1}{d_4 - 1}\right) \left(e^{-\ln d_4} / d_4 - 1\right) = e^{-d_4 \ln d_4} / d_4 - 1 = 0.9999\tag{71}
$$

Other system efficacy indicators for subsystem C based on Eqs. $(4-6, 8-9)$ are as follows: MTBF = 250, MTTR = 0.8333 , $d = 300$,

e) For Subsystem E, RAMD indices

$$
\frac{\xi_{5}}{6\rho_{5}} \frac{\xi_{5}^{1/5}}{\xi_{5}^{2}} \frac{\xi_{5}^{2}}{\xi_{5}^{2}} \xi_{5} \frac{\xi_{5}^{2}}{\xi_{5}^{2}} \xi_{5
$$

 $\dot{P}_0 = -6 \beta_5 P_0(t) + \gamma_5 P_1(t),$ (72) $\dot{P}_1 = (5\beta_5 + \gamma_5)P_1(t) + 6\beta_5P_0(t) + \gamma_5P_2(t)$.(73) \dot{P}_2 = - (4 $\beta_5 + \gamma_5$) $P_2(t) + 5\beta_5 P_1(t) + \gamma_5 P_3(t)$, (74) $\dot{P}_3 = - (3\beta_5 + \gamma_5)P_3(t) + 4\beta_5P_2(t) + \gamma_5P_4(t),$ (75) \dot{P}_4 = - (2 $\beta_5 + \gamma_5$) P_4 (t) + 3 $\beta_5 P_3$ (t) + $\gamma_5 P_5$ (t), (76) $\dot{P}_5 = -(\beta_5 + \gamma_5)P_5(t) + 2\beta_5P_4(t) + \gamma_5P_6(t),$ (77) $\dot{P}_6 = -\gamma_5 P_6(t) + \beta_5 P_5(t),$ (78)

Under steady state, equations (23) and (24) reduces and taking t $\rightarrow \infty$

 $-6 \beta_5 P_0(t) + \gamma_5 P_1(t) = 0$ (79) $-(5\beta_5 + \gamma_5)P_1(t) + 6\beta_5P_0(t) + \gamma_5P_2(t) = 0.$ (80) - $(4\beta_5 + \gamma_5)P_2(t) + 5\beta_5P_1(t) + \gamma_5P_3(t) = 0$, (81) $(3\beta_5 + \gamma_5)P_3$ - $(2\beta_5 + \gamma_5)P_4(t) + 3\beta_5P_3(t) + \gamma_5P_5(t) = 0$, (83) - $(\beta_5 + \gamma_5)P_5(t) + 2\beta_5P_4(t) + \gamma_5P_6(t) = 0$, (84) $- \gamma_5 P_6(t) + \beta_5 P_5(t) = 0$, (85)

$$
-(3\beta_5 + \gamma_5)P_3(t) + 4\beta_5 P_2(t) + \gamma_5 P_4(t) = 0, \qquad (82)
$$

Now, using normalization condition:

 $P_0+P_1+P_2+P_3+P_4+P_5+P_6=1$ (86)

Substituting the values of P_i in Eq. (86) and solving Eqs. (79–85) yields the availability of the subsystem

$$
AVsys5 = \frac{1 + \frac{6\beta_5}{\gamma_5} + \frac{30\beta_5^2}{\gamma_5^2} + \frac{120\beta_5^3}{\gamma_5^2} + \frac{360\beta_5^4}{\gamma_5^4} + \frac{720\beta_5^5}{\gamma_5^5}}{1 + \frac{6\beta_5}{\gamma_5} + \frac{30\beta_5^2}{\gamma_5^2} + \frac{120\beta_5^3}{\gamma_5^3} + \frac{360\beta_5^4}{\gamma_5^4} + \frac{720\beta_5^5}{\gamma_5^5} + \frac{720\beta_5^6}{\gamma_5^5}} = 0.9999
$$
\n
$$
(87)
$$

Reliability

R (t)= $e^{-0.005t}$. (88)

Maintainability

 $M(t)=1-e^{(-t/MTTR)}=1-e^{-1.4t}$. (89)

• Dependability

$$
D_{min} = 1 - \left(\frac{1}{d_5 - 1}\right) \left(e^{-\ln d_5}/d_5 - 1\right) = e^{-d_5 \ln d_5}/d_5 - 1 = 0.9999. \tag{90}
$$

Using Eqs. $(4-6, 8-9)$, the following are performance indicators of subsystem B: $d = 280$, MTBF = 200, $MTTR = 0.7143$,

f) For Subsystem F, RAMD indices

FIGURE.3 BLOCK DIAGRAM OF THE SUBSYSTEM A

 $\dot{P}_0 = -4\beta_6 P_0(t) + \gamma_6 P_1(t),$ (91) \dot{P}_1 = - (3 $\beta_6 + \gamma_6$) P_1 (t) + 4 $\beta_6 P_0$ (t) + $\gamma_6 P_2$ (t), (92) $\dot{P}_2 = -(2\beta_6 + \gamma_6)P_2(t) + 3\beta_6P_1(t)$. + $\gamma_6P_3(t)$ (93) $\dot{P}_3 = -(\beta_6 + \gamma_6)P_3(t) + 2\beta_6P_2(t)$. + $\gamma_6P_4(t)$ (94) $\dot{P}_4 = -\gamma_6 P_4(t) + \beta_6 P_1(t)$. (95)

Under steady state, equations (91 - 95), reduces and taking t $\rightarrow \infty$

 $-4\beta_6 P_0(t) + \gamma_6 P_1(t)$, =0 (96) - $(3\beta_6 + \gamma_6)P_1(t) + 4\beta_6P_0(t) + \gamma_6P_2(t) = 0$, (97) $-(2\beta_6 + \gamma_6)P_2(t) + 3\beta_6P_1(t)$. $+\gamma_6P_3(t)=0$, (98) $-(\beta_6+\gamma_6)P_3(t) + 2\beta_6P_2(t)$. + $\gamma_6P_4(t)$ =0, (99) $-\gamma_6 P_2 + \beta_6 P_1 = 0.$ (100)

Now, using normalization condition: $P_0+P_1+P_2+P_3+P_4=1$, (101)

Substituting the values of P_1 , P_2 , P_3 and P_4 We get the availability of the subsystem by solving Eqs. (16–19) in Eq. (18).

$$
AVsys6 = \frac{1 + \frac{4\beta_6}{\gamma_6} + \frac{12\beta_6^2}{\gamma_6^2} + \frac{24\beta_6^3}{\gamma_6^3}}{1 + \frac{4\beta_6}{\gamma_6} + \frac{12\beta_6^2}{\gamma_6^2} + \frac{24\beta_6^3}{\gamma_6^3} + \frac{24\beta_6^5}{\gamma_6^4}} = 0.9999 (102)
$$

Reliability

R (t)= $e^{-0.006t}$ (103)

Maintainability

 $M(t)=1-e^{(-t/MTTR)}=1-e^{-1.6t}$. (104)

• Dependability

$$
D_{min} = 1 - \left(\frac{1}{d_6 - 1}\right) \left(e^{-\ln d_6}/d_6 - 1\right) = e^{-d_6 \ln d_6}/d_6 - 1 = 0.9999. \tag{106}
$$

Equations are used The following are other system effectiveness performance parameters for subsystem A: MTBF = 166.7, MTTR = 0.625 , d = 266.7

System reliability

Because all three subsystems are interconnected, if one fails, the entire system will fail. The system's overall dependability is determined by a number of factors:

$$
R_{sys}(t) = R_{sys1}(t) \times R_{sys2}(t) \times R_{sys3}(t) \times R_{sys4}(t) \times R_{sys5}(t) \times R_{sys6}(t)
$$

$$
R_{sys}(t) = e^{-0.001t} \times e^{-0.002t} \times e^{-0.003t} \times e^{-0.004t} \times e^{-0.005t} \times e^{-0.006t}
$$

 $R_{sys}(t) = e^{-0.0000000000072t}$. (107) Table 2 displays the findings of Eq. (106), which is used to examine the fluctuation in dependability over time.

• System availability

Because all three subsystems are interconnected, if one fails, the entire system will fail. The system availability is calculated using the formula below:

$$
AV_{sys}(t) = = \left(\frac{1 + \frac{4\beta_1}{\gamma_1} + \frac{12\beta_1^2}{\gamma_1^2} + \frac{24\beta_1^3}{\gamma_1^3}}{1 + \frac{4\beta_1}{\gamma_1} + \frac{12\beta_1^2}{\gamma_1^2} + \frac{24\beta_1^3}{\gamma_1^3} + \frac{24\beta_1^5}{\gamma_1^4}}\right) \left(\frac{1 + \frac{6\beta_2}{\gamma_2} + \frac{30\beta_2^2}{\gamma_2^2} + \frac{120\beta_2^3}{\gamma_2^3} + \frac{360\beta_2^4}{\gamma_2^4} + \frac{720\beta_2^5}{\gamma_2^5}}{1 + \frac{2\beta_3}{\gamma_3} + \frac{4\beta_3^2}{\gamma_3^2} + \frac{12\beta_4^2}{\gamma_4^4} + \frac{4\beta_4^2}{\gamma_4^2} + \frac{12\beta_4^2}{\gamma_1^2} + \frac{24\beta_1^3}{\gamma_1^4} + \frac{24\beta_1^2}{\gamma_1^4} + \frac{24\beta_1^2}{\gamma_1^4} + \frac{4\beta_1^2}{\gamma_1^2} + \frac{24\beta_1^3}{\gamma_1^4} + \frac{4\beta_1^2}{\gamma_1^2} + \frac{24\beta_1^3}{\gamma_1^2} + \frac{4\beta_1^2}{\gamma_1^2} + \frac{8\beta_1^2}{\gamma_1^2} + \frac{24\beta_1^2}{\gamma_1^2} + \frac{12\beta_1^2}{\gamma_1^2} + \frac{4\beta_1^2}{\gamma_1^2} + \frac{8\beta_1^2}{\gamma_1^2} + \frac{24\beta_1^2}{\gamma_1^2} + \frac{8\beta_1^2}{\gamma_1^
$$

Analysis of System Maintainability

The system maintainability is :

M (t) =
$$
(1 - e^{-0.6t}) X (1 - e^{-0.8t}) X (1 - e^{-1.0t}) X (1 - e^{-1.2t}) X (1 - e^{-1.4t}) X (1 - e^{-1.6t})
$$

M (t) = $(1 - e^{-1.2902t}).$ (108)

Eq. (108) is used to assess the variance in maintainability over time, and Table 3 shows the results.

Dependability of the system

Because all three subsystems are linked, if one fails, the system as a whole will fail. The following factors have an impact on overall system reliability:

$$
D_{min} = D_{min} = 1 - \left(\frac{1}{d-1}\right)(e^{-\ln d}/d - 1 - e^{-d \ln d}/d - 1)
$$

 D_{min} = 0.9999 X 0.9999 X 0.9999 X 0.9999 X 0.9999 X 0.9999 = 0.9993. (109)

A summary of all RAMD indices is presented in Table 4.

The failure and repair rates of various subsystems in the system are shown in Table 1.

SHUWS HUW SUBSTSTEM NELIADILI I TIAS CHAINHEIZUVEN TIME								
	$R_{\gamma}(t)$	$R_{2}(t)$	$R_{\scriptscriptstyle 4}$ (t	$R_{5}(t)$		R(t)		
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
0.9802	0.9608	0.9418	0.9231	0.9048	0.8869	0.9999		
0.9608	0.9231	0.8869	0.8521	0.8187	0.7866	0.9999		
0.9418	0.8869	0.8353	0.7866	0.7408	0.6977	0.9999		
0.9231	0.8521	0.7866	0.7262	0.6703	0.6188	0.9999		
0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.9999		
0.8869	0.7866	0.6977	0.6188	0.5488	0.4868	0.9999		
0.8694	0.7558	0.6571	0.5712	0.4966	0.4317	0.9999		
0.8521	0.7262	0.6188	0.5273	0.4493	0.3829	0.9999		

TABLE 2 SHOWS HOW SUBSYSTEM RELIABILITY HAS CHANGED OVER TIME

TABLE 4 FOR SYSTEM RAMD INDICES

Indexes RAMD	Subsystem A	Subsystem B	Subsystem C	Subsystem D	Subsystem E	Subsystem	System
RELIBILITY	$e^{-0.00\overline{1t}}$	$e^{-0.002t}$	$e^{-0.003t}$	$e^{-0.004t}$	$e^{-0.005t}$	$e^{-0.006t}$	$e^{-0.0000000000072t}$
AVAILABILITY	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
MAINTAINABILITY	$-e^{-0.6t}$	$-e^{-0.8t}$	$-e^{-1.0t}$	$-e^{-1.2t}$	$1 - e^{-1.4t}$	$1 - e^{-1.6t}$	$1 - e^{-1.2902t}$
DEPENDABILITY	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
MTBF	1000	500	333	250	200	167	
MTTR	1.6667	1.2500	1.0000	0.8333	0.7143	0.6250	
DEPENDABILITY RATIO	600	400	333	300	280	267	

TABLE 5 EFFECTS OF FAILURE RATE ON SUBSYSTEM A RELIABILITY

						ы гестр от тишете китер оп рерототел р керикрен г	
Time	$=$						
	0.01	0.02	0.03	0.04	0.05	0.06	0.000000000072
20	0.8187	0.6703	0.5488	0.5488	0.5488	0.5488	0.9881
40	0.6703	0.4493	0.3012	0.3012	0.3012	0.3012	0.9763
60	0.5488	0.3012	0.1653	0.1653	0.1653	0.1653	0.9646
80	0.4493	0.2019	0.0907	0.0907	0.0907	0.0907	0.9531
100	0.3679	0.1353	0.0498	0.0498	0.0498	0.0498	0.9418
120	0.3012	0.0907	0.0273	0.0273	0.0273	0.0273	0.9305
140	0.2466	0.0608	0.0149	0.0149	0.0149	0.0149	0.9194
160	0.2019	0.0408	0.0083	0.0083	0.0083	0.0083	0.9085

TABLE 6 EFFECTS OF FAILURE RATES ON SUBSYSTEM B RELIABILITY

TABLE 7 FAILURE RATE EFFECTS ON SUBSYSTEM C RELIABILITY

Time								
	0.1	0.2	0.3	0.4	0.5	0.6	0.000000000072	
20	0.1353	0.0183	0.0025	0.0025	0.0025	0.0025	0.8869	
40	0.0183	0.0003	0.0000	0.0000	0.0000	0.0000	0.7866	
60	0.0024	0.0000	0.0000	0.0000	0.0000	0.0000	0.6977	
80	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.6188	
100	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.5488	
120	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4868	
140	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4317	
160	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3829	

TABLE 8 FAILURE RATE EFFECTS ON SYSTEM D RELIABILITY

Time							
	0.1	0.2	0.3	0.4	0.5	0.6	0.000000000072
20	0.1353	0.0183	0.0025	0.0025	0.0025	0.0025	0.8869
40	0.0183	0.0003	0.0000	0.0000	0.0000	0.0000	0.7866
60	0.0024	0.0000	0.0000	0.0000	0.0000	0.0000	0.6977
80	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.6188
100	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.5488
120	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4868
140	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4317
160	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3829

TABLE 9 FAILURE RATE EFFECTS ON SYSTEM E RELIABILITY

TABLE 10 FAILURE RATE EFFECTS ON SYSTEM F RELIABILITY

Time	$\beta_{5} =$							
	0.1	0.2	0.3	0.4	0.5	0.6	0.000000000072	
20	0.1353	0.0183	0.0025	0.0025	0.0025	0.0025	0.8869	
40	0.0183	0.0003	0.0000	0.0000	0.0000	0.0000	0.7866	
60	0.0024	0.0000	0.0000	0.0000	0.0000	0.0000	0.6977	
80	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.6188	
100	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.5488	
120	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4868	
140	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4317	
160	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3829	

SENSITIVITY ANALYSIS AGAINST TIME

SENSITIVITY ANALYSIS

Under a set of assumptions, sensitivity analysis is a technique for assessing how the values of independent variables will affect a certain dependent variable. It's used to see how responsive a model is to changes in its structure and parameters. With respect to various failure rate parameters 1, 2, 3,4,5, and 6, the system reliability has been subjected to a sensitivity analysis. The system reliability is slightly separated in relation to Eq. (106). β_1 , β_2 , β_3 , β_4 , β_5 , and β_6 , In the same method, the expressions that follow are derived. By filling in the values for the parameters shown below. $\beta_1 = 0.001$, $\beta_2 = 0.002$, $\beta_3 = 0.003$, $\beta_4 = 0.004$, $\beta_5 = 0.005$, $\beta_6 = 0.006$ The graphical results of a sensitivity analysis of system dependability are shown in Eq. 106. (109), as seen in Fig. 9, from time $t = 0$ to time $t = 160$.

CONCLUSION AND DISCUSSION

Table 2 shows that for period $t = 40$ days, the probability of fault-free operation of the system is 0.9999, and for subsystems is 0.7866, with RSSA = 0.9608, RSSB = 0.9231,RSSC = 0.8869, RSSD=0.8521, RSSE=0.8187, and RSSF=0.7866.

With MSSA = 1.00000, MSSB = 1.0000, MSSC = 1.0000, MSSD = 1.0000, MSSE = 1.0000, and MSSF = 1.0000 Table 3 shows that when the applicable subsystem maintainability values are used, the likelihood of successful maintenance and repair accomplished within 40 days is 0.9999. The SSF subsystem's reliability values are quite poor at various time points, needing extra attention and precise maintenance procedures. The failure rate has a significant impact on the reliability of Subsystem F, as shown in Tables 5, 6, 7, 8, 9, and 10. Subsystem dependability varies, but the system's overall reliability is low due to its series architecture. The failure rate (SSF failure rate), as shown in Fig. 9, has a substantial impact on system dependability, making it necessary to pay special attention to subsystem FAs a result, effective maintenance procedures should be established, and various redundancy mechanisms should be implemented into the system's architecture to improve system reliability.

DECISION MAKING INFERENCES

The majority of dependability evaluation approaches necessitate large mathematical computations, and not everyone is gifted in this area. This method can be used by managers, system designers, and engineers to accurately measure system performance. Managers can use the RAMD analysis of the system at the outer layer to design maintenance policies that control reliability parameters such as MTBF, MTTR, and availability.

Determine the type of failure, how frequently it occurs, the mode of failure, and the repair procedures. Find the weakest link in the system. Determine the likelihood of repair and failure. Sync the system's RAMD requirements.

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