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A NEW APPROACH OF FUZZY NUMBERS WITH DIFFERENT SHAPES AND DEVIATION

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Abstract

In this paper, we propose a new method for fuzzy numbers. In this method, we assume that $A_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4})$ is to be a fuzzy number. So, the convex combination of a_{i1} and a_{i1} and also the convex combination of a_{i3} and a_{i4} are obtained separately. Then, M_{ic} and M_{is} that are to be the convex combinations and the standard deviation respectively we acquire them from these components. Finally, we can obtain ranking index (M_i) that is the convex combination of, M_{ic} and M_{is}. At the end of paper, in one example, the proposed method is compared with other methods.

Keywords: Fuzzy number, convex combination, standard deviation

1. Introduction

In many applications, ranking of fuzzy numbers is an important component of decision process. Numerous fuzzy ranking indices have been proposed since 1976. Jain [11,12] proposed a method using the concept of maximizing set to order the fuzzy numbers in 1976 and 1977. Dubbois and Prade 1978 [10] used maximizing sets to order fuzzy numbers. Chen [7], Choobineh [5], Cheng [4], Chu, Tsao [6] and Ma, Kandel and Freidman [14] have presented some methods. Wang et al [15] conducted ranking of fuzzy numbers *L*-*R* based on deviation degree. Lee and Chen [13], Wei and Chen [16], Chen and Wang [9], Abbasbandy and Hajjari [1], Asady et al [3] proposed different methods for ranking fuzzy numbers. In this paper, we proposed ranking of fuzzy numbers with different shapes and different deviation. Section 2 includes preliminaries. In section 3, we can see the main idea of this paper. In section 4, the proposed method is compared with other methods by some examples. The conclusion is to be in section 5.

2. *Preliminaries*

Fuzzy number can defined as follows [5, 6].

Definition 2.1: A fuzzy number $\tilde{A}$ is described as any fuzzy subset of the real line R with membership function $f_{\tilde{A}}$ which processes the following properties:

a. $f_{\tilde{A}}$ is a continuous mapping from R to the closed interval $[0,\omega]$, $0 \le \omega \le 1$.

b. $f_{\tilde{A}}(x) = 0$, for all $x \in (-\infty, a]$.

c. $f_{\tilde{A}}$ is strictly increasing on $[a,b]$.

d. $f_{\tilde{A}}(x) = \omega$, for all $x \in [b,c]$, where ω is a constant and $0 < \omega \le 1$.

e. $f_{\tilde{A}}$ is strictly decreasing on $[c,d]$.

f. $f_{\tilde{A}}(x) = 0$ for all $x \in [a,+\infty,)$, Where a,b,c and d are real numbers. We may let $a = -\infty$, or $a = b$, or $c = d$, or $d = +\infty$

Unless elsewhere specified, it is assumed that $\tilde{A}$ is convex and bounded, i.e. $-\infty < a, d < \infty$. If, $\omega = 1$ in (d), $\tilde{A}$ is a normal fuzzy number, and if $0 < \omega < 1$ in (d), $\tilde{A}$ is a non-normal fuzzy number. For convenience, the fuzzy number in definition 2.1 can be denoted by $\tilde{A} = (a,b,c,d;\omega)$.

The membership function $f_{\tilde{A}}$ of $\tilde{A}$ can be expressed as

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}^{l}(x) \ , & a \le x \le b \ , \\ \omega \ , & b \le x \le c \ , \\ f_{\tilde{A}}^{r}(x) \ , & c \le x \le d \ , \\ 0 \ , & \text{otherwise} \end{cases}$$

Where $f_{\tilde{A}}^{l} : [a,b] \to [0,\omega]$ and $f_{\tilde{A}}^{r} : [c,d] \to [0,\omega]$.

1. *The Main Idea*

Granted that $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n$, are fuzzy numbers n that $A_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4} \ ; \omega_i)$ and $1 \le i \le n$. ω_i is the height of defined fuzzy number A_i. Therefore, for each fuzzy number A_i we obtained ranking index(M_i) as follows:

- $R_{ic} = \lambda_1 a_{i3} + (1-\lambda_1) a_{i4}$
- $L_{ic} = \lambda_1 a_{i2} + (1-\lambda_1) a_{i1}$

where $\lambda_1 \in [0,1]$

then the convex combination of R_{ic} and L_{ic} is introduced as follow:

- $M_{ic} = \lambda_1 R_{ic} + (1-\lambda_1) L_{ic}$.

In the follow, R_{is} and L_{is} that is the right and the left deviation standard respectively obtained as follows:

- $R_{is} = \sqrt{\frac{\lambda_2 (a_{i3} - R_{ic})^2 + (1-\lambda_2)(a_{i4} - R_{ic})^2}{\delta_1 + \delta_2}}$
- $L_{is} = \sqrt{\frac{\lambda_2 (a_{i2} - L_{ic})^2 + (1-\lambda_2)(a_{i1} - L_{ic})^2}{\delta_1 + \delta_2}}$

that δ_i for $i = 1,2$ is defined as follows:

$$\delta_i = \begin{cases} \delta_1 = 1, \delta_2 = 0, & \text{if} \quad \lambda_2 = 1 \\ \delta_1 = 0, \delta_2 = 1, & \text{if} \quad \lambda_2 = 0 \\ \delta_1 = 1, \delta_2 = 1, & \text{if} \quad \lambda_2 \in (0,1) \end{cases}$$

and $\lambda_2 \in [0,1]$

Therefore, M_{is} that is the convex combination of the left and right standard deviation is defined as follows:

- $M_{is} = \lambda_2 R_{is} + (1 - \lambda_2) L_{is}$

So, by using M_{is} and M_{ic} we can define the ranking index M_i as follow:

- $M_i = \alpha M_{ic} + (1-\alpha) M_{is}, \quad \alpha \in [0,1]$

Then, for ranking of $\tilde{A}_i, \tilde{A}_j$, we have,

$\tilde{A}_i \succ \tilde{A}_j$ iff $M_i > M_j$

$\tilde{A}_i \prec \tilde{A}_j$ iff $M_i < M_j$

$\tilde{A}_i \sim \tilde{A}_j$ iff $M_i = M_j$

3. Comparison with Other Methods

- Example : Consider the following sets:

Set 1: $\tilde{A} = (0.1, 0.3, 0.5; 1)$, $\tilde{B} = (0.3, 0.5, 0.7; 1)$.

Set 2: $\tilde{A} = (0.1, 0.3, 0.5; 0.8)$, $\tilde{B} = (0.1, 0.3, 0.5; 1)$.

Set 3: $\tilde{A} = (0.3, 0.5, 1; 1)$, $\tilde{B} = (0.1, 0.6, 0.8; 1)$.

Set 4: $\tilde{A} = (0.3, 0.5, 1; 1)$, $\tilde{B} = (0.2, 0.5, 0.9; 1)$, $\tilde{C} = (0.1, 0.6, 0.7, 0.8, 1)$.

Which are shown in figure 1.

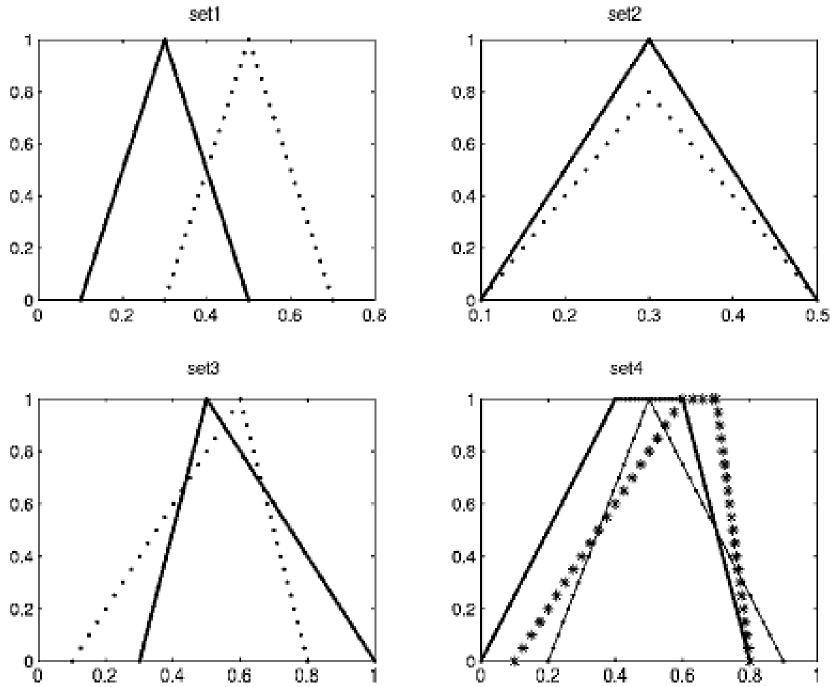

Fig. 1. Four sets of normal and non-normal fuzzy numbers

Table 1: (***comparative results of example***)

Methods	$fuzzy \quad number$	$set1$	$set2$	$set3$	$set4$
Cheng's method (1998)	$\tilde{A}$	0.5831	0.461	0.7673	0.68
	$\tilde{B}$	0.7071	0.5831	0.7241	0.7257
	$\tilde{C}$	–	–	–	0.7462
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Chu's method (2002)	$\tilde{A}$	0.15	0.12	0.287	0.2281
	$\tilde{B}$	0.25	0.15	0.2619	0.2624
	$\tilde{C}$	–	–	–	0.2784
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Abbasbandy and Asady's method (2005)	$\tilde{A}$	0.4546	-	0.8641	0.7394
	$\tilde{B}$	0.7275	0.4546	0.7979	0.7958
	$\tilde{C}$	–	–	–	0.8406
Results		$\tilde{A} \prec \tilde{B}$	–	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Asady et al' method (2007)	$\tilde{A}$	0.3	-	0.575	0.45
	$\tilde{B}$	0.5	0.3	0.525	0.525
	$\tilde{C}$	–	–	–	0.55
Results		$\tilde{A} \prec \tilde{B}$	–	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Chen and Chen's method(2007)	$\tilde{A}$	0.4456	0.3565	0.4128	0.3719
	$\tilde{B}$	0.4884	0.4456	0.4005	0.4155
	$\tilde{C}$	–	–	–	0.3979
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Abbasbandy and Hajjari's method (2009)	$\tilde{A}$	0.3	-	0.525	0.483
	$\tilde{B}$	0.5	0.3	0.575	0.508
	$\tilde{C}$	–	–	–	0.617
Results		$\tilde{A} \prec \tilde{B}$	–	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Chen and Chen's method(2009)	$\tilde{A}$	0.2579	0.2063	0.4428	0.3354
	$\tilde{B}$	0.4298	0.2579	0.4043	0.4079
	$\tilde{C}$	–	–	–	0.4196
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$

In set 1, other scientists´ method and the proposed method result in $\tilde{A} \prec \tilde{B}$. In set 2, with other methods we have $\tilde{A} \prec \tilde{B}$ but with other method we have $\tilde{A} \sim \tilde{B}$. In set 3, we can see the $\tilde{A} \succ \tilde{B}$ but for $\lambda_i = 0.8,\ i = 1, 2$ and $\alpha = 1,\ \lambda_i = 1$ with $i = 1, 2$ and $\alpha = 0.5, 1$ in the proposed method similar to Abbasbandy and Hajjari´method we have $\tilde{A} \prec \tilde{B}$. In set 4, it is logical that $\tilde{A} \prec \tilde{B} \prec \tilde{C}$. Cheng´ method (1998), Chu´ method (2002), Chen and Chen´ method (2009), Abbasbandy et al [2], Asady et al [3], our proposed method for $\lambda_i = 0.8$, $\alpha = 0.5$ we acquire $\tilde{A} \prec \tilde{B} \prec \tilde{C}$. But Chen and Chen´ method (2007), Abbasbandy and Hajjari [1] our proposed method for $\lambda_i = 0.0, 0.4,\ \alpha = 0.5, 1$ $\tilde{A} \prec \tilde{C} \prec \tilde{B}$, and also for $\lambda_i = 0.8,\ i = 1, 2,\ \alpha = 1$, and $\lambda_i = 1,\ i = 1, 2$ and $\alpha = 0.5, 1$ we have $\tilde{B} \prec \tilde{A} \prec \tilde{C}$.

Table 2

	fuzzy number	$\lambda_1 = \lambda_2 = 0.0$	$\lambda_1 = \lambda_2 = 0.4$	$\lambda_1 = \lambda_2 = 0.8$	$\lambda_1 = \lambda_2 = 1$
set(1)	$\tilde{A}$	0.05	0.1726	0.1903	0.15
$\alpha = 0.5$	$\tilde{B}$	0.15	0.2726	0.2903	0.25
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$
set(1)	$\tilde{A}$	0.1	0.2760	0.3240	0.3
$\alpha = 1.0$	$\tilde{B}$	0.3	0.4760	0.5240	0.5
Results		$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$
set(2)	$\tilde{A}$	0.05	0.1726	0.1903	0.15
$\alpha = 0.5$	$\tilde{B}$	0.05	0.1726	0.1903	0.15
Results		$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$
set(2)	$\tilde{A}$	0.1	0.2760	0.3240	0.3
$\alpha = 1.0$	$\tilde{B}$	0.1	0.2760	0.3240	0.3
Results		$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$
set(3)	$\tilde{A}$	0.15	0.3294	0.3482	0.25
$\alpha = 0.5$	$\tilde{B}$	0.05	0.2998	0.3428	0.3
Results		$\tilde{A} \succ \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B}$
set(3)	$\tilde{A}$	0.3	0.5480	0.5720	0.5
$\alpha = 1.0$	$\tilde{B}$	0.1	0.4680	0.6120	0.6
Results		$\tilde{A} \succ \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$
set(4)	$\tilde{A}$	0.0	0.2474	0.3219	0.3
$\alpha = 0.5$	$\tilde{B}$	0.1	0.3029	0.3297	0.25
	$\tilde{C}$	0.05	0.3009	0.3635	0.35
Results		$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{B} \prec \tilde{A} \prec \tilde{C}$
set(4)	$\tilde{A}$	0.0	0.3840	0.5760	0.6
$\alpha = 1.0$	$\tilde{B}$	0.2	0.4880	0.5520	0.5
	$\tilde{C}$	0.1	0.4840	0.6760	0.7
Results		$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{B} \prec \tilde{A} \prec \tilde{C}$	$\tilde{B} \prec \tilde{A} \prec \tilde{C}$

4.Conclusion

In this paper, a new approach is introduced for ranking of fuzzy numbers. The advantage of this method over other methods introduced by scientists is that different ranking indices are obtained with both λ_i for $i = 1, 2$ and different α. In a few part of the example we observe that ranking indices are different with each other. This is natural event due to fuzzy number is.

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