Generation Expansion Planning by Presenting a Two-Level Model to Cover Uncertainty in a Competitive Market

Abstract: This paper considers the investment in conventional and stochastic units simultaneously for a strategic producer and presents a variety of uncertainty parameters such as load forecast, rival producer actions (production investment and price bidding), and retail price bidding in generation expansion planning (GEP) in energy markets through a two-level model. Uncertainty is determined through scenarios. In this paper, the cost of production and the price bid of stochastic units and the corresponding revenues are predicted in the model. The model can be solved using branch and cutting techniques.

Keywords: generation expansion planning, Production Investment, Stochastic Optimization, Uncertainty Set

Markup

Indicators

Production capacity investment options (conventional technologies and power plants).	h
Knots	n, m
Operating conditions	o
Scenarios	ω

Collections

A set of nodes connected to the n-node	Ω_n

Parameters

The suspension of the transmission line from node n to node m.	B_{nm}
The cost of producing the candidate strategic power plant, located at node n [$/MWh].	C_n^C
The cost of generating an existing strategic power plant, located at node n [$/MWh].	C_n^E
Transmission line capacity from node n to node [MW] m	$F_{nm}^{\max}$
The bid price of the competitor unit in the n-node and in the operation state o[$/MWh].	α_{no}^O
Annual Investment Cost of Candidate Power Plant Located at Node N [$/MW]	I_n^C
Annual Investment Cost of Power Plant with Stochastic Candidate Generation Located at Node N [$/MW]	I_n^S
The maximum annual investment budget available to the strategic producer [$].	$I^{\max}$
The capacity of the conventional power plant unit of the strategic producer at the n node [MW].	$p_n^{E^{\max}}$
Maximum load located at node n [MW]	$p_n^{D^{\max}}$

The generating capacity of the rival unit located at the n node under the ω [MW] scenario.	$p_{n\omega}^{O^{\max}}$
The power capacity factor of the candidate stochastic generating unit located at the n node and in the operation condition o [p.u.]	Q_{no}^{S}
The consumer load factor located at the n-node in the exploitation state o [p.u.].	Q_{no}^{D}
The Consumer Bid price located at the n node in the exploitation state o [$/MW].	α_{no}^{D}
Option h for the candidate's conventional unit production capacity investment located at node n [MW]	X_{nh}^{C}
Maximum investment of the production capacity of the desired stochastic generating unit in the n node [MW]	$X_{n}^{S^{\max}}$
Probabilities for scenario ω [p.u.].	φ_{ω}
Weight Factor Related to Exploitation Conditions [h]	ρ_{O}
	Binary variables
The binary variable is equal to 1 if the canonical production investment option h is made in the n node.	u_{nh}^{C}
	Continuous Variables
Power generated by the strategic producing candidate unit located at the N node under the operating condition o under the Ω scenario	$p_{no\omega}^{C}$
Power consumed by the consumer located at node n in operation mode o under scenario ω [MW]	$p_{no\omega}^{D}$
Power generated by the existing strategic generating unit located at node n under operating conditions o under scenario [MW]	$p_{no\omega}^{E}$
Power generated by the rival unit located at node n under operating conditions o under scenario ω [MW]	$p_{no\omega}^{O}$
Power generated by the strategic stochastic generating unit of the strategic generator located at node n under operating conditions o under scenario ω [MW]	$p_{no\omega}^{S}$
Investment capacity of the candidate unit belonging to the strategic manufacturer located in the n node [MW]	x_{n}^{C}
Strategic Producer Candidate Stochastic Unit Investment Capacity located at Node [MW] n	x_{n}^{S}
Bid by the candidate unit of the strategic producer located at node n in operation mode o under scenario ω [$/MWh]	$\alpha_{no\omega}^{C}$
Bid by the existing conventional unit of the strategic manufacturer located at node n under operating conditions o under scenario ω [$/MWh]	$\alpha_{no\omega}^{E}$
Node voltage angle n in operation mode o under scenario ω [rad]	$\theta_{no\omega}$
Market settlement price in node n in operation condition o under scenario ω [$/MWh]	$\lambda_{no\omega}$

1. Introduction

1.1. Background & Purpose

A strategic producer in a competitive electricity market is always looking to maximize its profits by making the best production investment decisions. In this regard, due to the addition of uncertainty in the behavior of other competitors (manufacturers), the complexity of decision-making issues increases. Therefore, this paper focuses on the problem of production investment with a focus on the competitive market. [1] The uncertainty models that are used in production development planning are more common in the following cases:

1- Stochastic 2- Information Gap 3- Solid

In this paper, we use stochastic optimization to model uncertainty, in other words, we use this method to describe the uncertainty related to load forecasting and competitor production forecasting, as well as the price bid of retailers and the price bidding of rival producers through scenarios.

1.2. Review of the Literature on the Subject

One of the appropriate methods for solving problems with uncertain parameters is the use of stochastic processes through scenarios and stochastic programming. To define a stochastic process, it is important to create a sufficient number of scenarios for the stochastic process to materialize because usually a large number of scenarios complicate the solution to the stochastic programming problem. [2]

Stochastic optimization, which has been used in some articles such as [3], [4] - [7], [8], is considered as a framework for modeling problems involving uncertainty and is a useful tool for adapting the effects of uncertainty such issues. Some articles on the capacity of investment models such as [9] - [15] Considered as two-stage or multi-stage stochastic optimization issues.

[16] Apply PHA as a multi-stage investment model [17] examine the effect of the permissible coefficient used in PHA on model performance and show that by using higher coefficient values, calculation time is reduced and the quality of the solution is balanced.

The emergence of renewable energy-based generation technologies in the form of distributed generation led to a change in the goals and ideas of researchers in GEP studies. The inevitable nature of electricity demand and its growth, as renewables are considered as the most feasible strategy to deal with climate change issues in the electricity industry. Hence, generation development planning studies [18], [19], [20] and [21], consider RES-based generation options to be practically essential in power generation development planning issues. According to the works that have been analyzed in the literature review, the contribution of this article is as follows:

In other studies, in order to simplify the calculations, the price bid of stochastic generating units is considered to be zero, and also the presentation of the price offer is considered definitively so that the market settlement can be done easily at the second level of the problem, but in this paper, the cost of production of stochastic units and the related revenues have been added to the model and a simultaneous combination of the parameters of stochastic and common units has been considered. Manufacturing units owned by a GENCO and its competitor are placed in a decentralized production investment model with no market constraints, including predicting the bids of competitors and retailers.

1.3. Structure of the article

The rest of this article is as follows:

- The second part describes and clarifies the features of the model being considered.
- The third part shows the proposed model
- The fourth part of the MPEC problem is formulated as a complex integer linear programming problem
- The fifth part is the result of a realistic case study.
- Section VI provides some relevant results of this article.

2. Model Features

2.1. Planning Horizon, Load, Network Display, Investment Model

According to the common approach in the technical literature of articles [22], [23], [240] and [25], in this study, the static model has been used, which means that the producer selects the next year, determines the optimal production sample for this target year, and makes the decision related to the investment in the field of production capacity by the producer in cooperation with other producers (competitors). This is a strategic producer, which means that it has a significant share of the production capacity in the industry and is therefore able to exert power in the market. The goal of this strategic producer is to maximize its profits, so it makes operation decisions in a short period of time in order to propose the amount of production of conventional power plants at strategic prices and capital decisions. It takes a long-term transition for the construction of new power plant units, including conventional power plants (such as gas power plants) and stochastic power plants (such as wind power plants).

It should be noted that in other studies, in order to simplify the calculations, the price suggestion of stochastic generating units is considered to be zero, but in this paper, the cost of producing stochastic units and the related revenues have been added to the model and a simultaneous combination of the parameters of stochastic and common units has been considered.

The electricity market is also based on the day-ahead market, in which an independent system operator (ISO) settles the market once a day, one day ahead and on an hourly basis.

In this paper, the DC load distribution method is used to represent the transmission network in the proposed investment model, because such a linear representation is simple and suitable for programming models.

In order to cover the demand level and production level of stochastic units during the year of the objective of the static model, the operating conditions including the demand coefficient of each consumer and the power capacity coefficient for each stochastic production unit are considered.

The above is embodied in a two-level model and is written as an Integrated-Hybrid Linear Programming (MILP) problem and solved using a direct solution approach. [26]

2.2. Uncertainty

The investment decisions of a strategic producer may be affected by various sources of uncertainty, such as demand growth, the actions of rival producers (bid and investment), the bid price of demand, investment costs, regulatory policies, and availability of production units and transmission lines, the four sources of uncertainty are modeled in this paper through a series of plausible scenarios. In other words, the uncertainty related to the growth of the load and the forecast of the production of competitors, as well as the price suggestion of competitor retailers and producers, have been shown through the scenarios.

3. Two-level model

The proposed model is a two-level model in which investment is faced with the aim of maximizing the investor's profits and considering the uncertainties of rival production and load growth at the high level of the problem, and at the lower level, market settlement is considered with the aim of maximizing social welfare, so the investment model is as follows:

$$Minimize \quad \sum_n \left(I_n^C x_n^C + I_n^S x_n^S \right) - \sum_\omega \varphi_\omega \sum_O \rho_O \begin{Bmatrix} \left(\sum_n p_{no\omega}^C \lambda_{no\omega} - \sum_n p_{no\omega}^C C_n^C \right) + \\ \left(\sum_n p_{no\omega}^E \lambda_{no\omega} - \sum_n p_{no\omega}^E C_n^E \right) + \\ \left(\sum_n p_{no\omega}^S \lambda_{no\omega} - \sum_n p_{no\omega}^S C_n^S \right) \end{Bmatrix} \tag{1}$$

$$subjectto:$$

$$\begin{Bmatrix} X_n^C = \sum_h u_{nh}^C X_{nh}^C, \sum_h u_{nh}^C = 1, \quad u_{nh}^C \in \{0,1\}, \quad \forall n, \forall h \\ 0 \le x_n^S \le X_n^{S^{\max}} : \quad \forall n \\ \sum_n \left(I_n^C x_n^C + I_n^S x_n^S \right) \le I^{\max} \\ \alpha_{no\omega}^C \ge 0, \alpha_{no\omega}^E \ge 0 \qquad \forall o, \forall n, \forall \omega \end{Bmatrix} \tag{2}$$

$$\lambda_{no\omega}, p_{no\omega}^O, p_{no\omega}^E, p_{no\omega}^C, p_{no\omega}^D \in \arg\min \left\{ \sum_n \alpha_{no\omega}^C p_{no\omega}^C + \sum_n \alpha_{no\omega}^E p_{no\omega}^E + \sum_n \alpha_{no\omega}^S p_{no\omega}^S + \sum_n \alpha_{no\omega}^O p_{no\omega}^O - \sum_n \alpha_{no\omega}^D p_{no\omega}^D \right. \tag{3}$$

$$subjectto:$$

$$\sum_n p_{no\omega}^D + \sum_{m \in \Omega_n} B_{nm} \left(\theta_{no\omega} - \theta_{mo\omega} \right) - \sum_n p_{no\omega}^C - \sum_n p_{no\omega}^S - \sum_n p_{no\omega}^E - \sum_n p_{no\omega}^O = 0 : \quad \lambda_{no\omega}, \forall n \tag{4}$$

$$0 \le p_{no\omega}^C \le x_n^C : \mu_{no\omega}^{C^{\min}}, \mu_{no\omega}^{C^{\max}}, \quad \forall n \tag{5}$$

$$0 \le p_{no\omega}^E \le p_n^{E^{\max}} : \mu_{no\omega}^{E^{\min}}, \mu_{no\omega}^{E^{\max}}, \quad \forall n \tag{6}$$

$$0 \le p_{no\omega}^S \le Q_{no}^S x_n^S : \mu_{no\omega}^{S^{\min}}, \mu_{no\omega}^{S^{\max}}, \quad \forall n \tag{7}$$

$$0 \le p_{no\omega}^O \le p_{n\omega}^{O^{\max}} : \mu_{no\omega}^{O^{\min}}, \mu_{no\omega}^{O^{\max}}, \quad \forall n \tag{8}$$

$$0 \le p_{no\omega}^D \le Q_{no}^D p_n^{D^{\max}} : \mu_{no\omega}^{D^{\min}}, \mu_{no\omega}^{D^{\max}}, \quad \forall n \tag{9}$$

$$-F_{nm}^{\max} \le B_{nm} \left(\theta_{no\omega} - \theta_{mow} \right) \le F_{nm}^{\max} : \quad \nu_{nmo\omega}^{\min}, \nu_{nmo\omega}^{\max} \quad \forall n, \forall m \in \Omega_n \tag{10}$$

$$-\pi \le \theta_{no\omega} \le \pi : \xi_{no\omega}^{\min}, \xi_{no\omega}^{\max}, \quad \forall n \tag{11}$$

$$\theta_{now} = 0 : \xi_t^1, n = ref \} \quad \forall n \tag{12}$$

The variables of exploitation of low-level problems are:

$$\left\{ \begin{array}{l} \lambda_{no\omega}, p_{no\omega}^C, p_{no\omega}^S, p_{no\omega}^E, p_{no\omega}^O, p_{no\omega}^D, \theta_{no\omega}, \\ \mu_{no\omega}^{C^{\min}}, \mu_{no\omega}^{C^{\max}}, \mu_{no\omega}^{S^{\min}}, \mu_{no\omega}^{S^{\max}}, \mu_{no\omega}^{E^{\min}}, \mu_{no\omega}^{E^{\max}}, \mu_{no\omega}^{O^{\min}}, \\ \mu_{no\omega}^{O^{\max}}, \mu_{no\omega}^{D^{\min}}, \mu_{no\omega}^{D^{\max}}, \nu_{nmo\omega}^{\min}, \nu_{nmo\omega}^{\max}, \xi_{no\omega}^{\min}, \xi_{no\omega}^{\max} \end{array} \right\} \tag{13}$$

In addition to the above optimization variables, the level (1) and (2) above problem also includes the following optimization variables:

$$\left\{ \alpha_{no\omega}^C, \alpha_{no\omega}^E, \alpha_{no\omega}^S, \alpha_{no\omega}^O, x_n^C, x_n^S, u_{nh}^C \right\} \tag{14}$$

Equations (1)-(2) are the upper level of the problem and (3)-(12) are the lower level of the problem. The objective function (1) is the negative profit of the strategic producer, which minimizes the investment and operation costs of conventional and renewable candidate production units for the strategic producer. Constraint (2) It includes development constraints. Clause (3) shows the settlement of the market. Maximizing social welfare at any lower level of the problem is expressed by adverb (4). Equations (5)-(9) apply capacity limits to new and existing units of strategic producer, units of other producers, and demand. Limits (10) apply the transmission capacity limits of each line. Note that the buses connected to the M bus specify the connection to the n bus. Constraints (11) apply angular constraints to each node, and constraints (12) apply 1 bus as the reference bus. The dual variables in the respective equations are represented after the two-point sign.

4. Converting a Two-Level Problem to MPEC

The steps of MPEC related to Problem (1) - (11) are given below Initially, the KKT conditions related to low-level problems (3) - (11) are obtained. Thus, the relationships are categorized as follows:

$$(1)-(2) \tag{15}$$

$$\frac{\partial \ell}{\partial p_{no\omega}^C} = \alpha_{no\omega}^C - \lambda_{no\omega} + \mu_{no\omega}^{C^{\max}} - \mu_{no\omega}^{C^{\min}} = 0 \qquad \forall \text{n}, \forall \text{o}, \forall \omega \tag{16}$$

$$\frac{\partial \ell}{\partial p_{no\omega}^E} = \alpha_{no\omega}^E - \lambda_{no\omega} + \mu_{no\omega}^{E^{\max}} - \mu_{no\omega}^{E^{\min}} = 0 \qquad \forall \text{n}, \forall \text{o}, \forall \omega \tag{17}$$

$$\frac{\partial \ell}{\partial p_{no\omega}^S} = \alpha_{no\omega}^S - \lambda_{no\omega} + \mu_{no\omega}^{S^{\max}} - \mu_{no\omega}^{S^{\min}} = 0 \qquad \forall \text{n}, \forall \text{o}, \forall \omega \tag{18}$$

$$\frac{\partial \ell}{\partial p_{no\omega}^O} = \alpha_{no\omega}^O - \lambda_{no\omega} + \mu_{no\omega}^{O^{\max}} - \mu_{no\omega}^{O^{\min}} = 0 \qquad \forall \text{n}, \forall \text{o}, \forall \omega \tag{19}$$

$$\frac{\partial \ell}{\partial p_{no\omega}^D} = \alpha_{no\omega}^D + \lambda_{no\omega} + \mu_{no\omega}^{D^{\max}} - \mu_{no\omega}^{D^{\min}} = 0 \qquad \forall \text{n}, \forall \text{o}, \forall \omega \tag{20}$$

$$\frac{\partial \ell}{\partial \theta} = \sum_{m \in \Omega_n} B_{nm} \left(\lambda_{no\omega} - \lambda_{mo\omega} \right) + \sum_{m \in \Omega_n} B_{nm} \left(\nu_{nmo\omega} - \nu_{mno\omega} \right) + \xi_{no\omega}^{\max} - \xi_{no\omega}^{\min} + \left(\xi_{o\omega}^{\theta^{ref}} \right)_{n=ref} = 0 \quad \forall n, \forall o, \forall \omega \tag{21}$$

$$(4),(12) \tag{22}$$

$$0 \le p_{no\omega}^{C} \perp \mu_{no\omega}^{C^{\min}} \ge 0 \quad \forall n \tag{23}$$

$$0 \le p_{no\omega}^{E} \perp \mu_{no\omega}^{E^{\min}} \ge 0 \quad \forall n \tag{24}$$

$$0 \le p_{no\omega}^{O} \perp \mu_{no\omega}^{O^{\min}} \ge 0 \quad \forall n \tag{25}$$

$$0 \le p_{no\omega}^{D} \perp \mu_{no\omega}^{D^{\min}} \ge 0 \quad \forall n \tag{26}$$

$$0 \le \left(x_n^C - p_{no\omega}^{C} \right) \perp \mu_{no\omega}^{C^{\max}} \ge 0 \quad \forall n \tag{27}$$

$$0 \le \left(p_{no\omega}^{S^{\max}} - p_{no\omega}^{S} \right) \perp \mu_{no\omega}^{S^{\max}} \ge 0 \quad \forall n \tag{28}$$

$$0 \le \left(p_{no\omega}^{O^{\max}} - p_{no\omega}^{O} \right) \perp \mu_{no\omega}^{O^{\max}} \ge 0 \quad \forall n \tag{29}$$

$$0 \le \left(p_{no\omega}^{D^{\max}} - p_{no\omega}^{D} \right) \perp \mu_{no\omega}^{D^{\max}} \ge 0 \quad \forall n \tag{30}$$

As explained in Appendix A, the set of equation constraints (16)-(21) is obtained according to the Lagrange function. The complementary constraints (23)-(30) and (15) related to MPEC are converted into a complex integer linear programming problem. The MPEC problem includes all of the optimization variables of problem (1)-(11) plus the binary variables used to linearize the complementary constraints described in Appendix B and covers the inherent uncertainties related to load forecasting and competitor production forecasting, as well as the price bidding of rival retailers and manufacturers. Therefore, the set of non-deterministic parameters is displayed as follows:

$$U = \left\{ p_{no\omega}^{O}, p_{no\omega}^{D}, \alpha_{no\omega}^{O}, \alpha_{no\omega}^{D} \right\}$$

The nonlinear part of the objective function (1) has been replaced by its linear equivalent given in Appendix A in relation to (50), and as a result, the formulation with uncertainty parameters is as follows:

$$Minimize \quad \sum_{n} \left(I_n^C x_n^C + I_n^S x_n^S \right) -$$

$$\sum_{\omega} \varphi_{\omega} \sum_{O} \rho_O \left\{ \sum_{n} C_{no\omega}^{D} p_{no\omega}^{D} - \sum_{n} \alpha_{no\omega}^{O} p_{no\omega}^{O} - \sum_{n} p_{no\omega}^{C} C_n^C - \sum_{n} p_{no\omega}^{E} C_n^E - \sum_{n} p_{no\omega}^{S} C_n^S - y \right\} \tag{31}$$

$$subjectto:$$

$$(4)\text{-}(12) \tag{32}$$

5. Case Study

In this example, as shown in Figure 1, a power system with two nodes (n1 and n2) is considered. The two nodes are connected by the N1 – N2 transmission line with a capacity of 200 MW and a capacity of 1000 S. Stochastic units are not considered as investment options in this example.

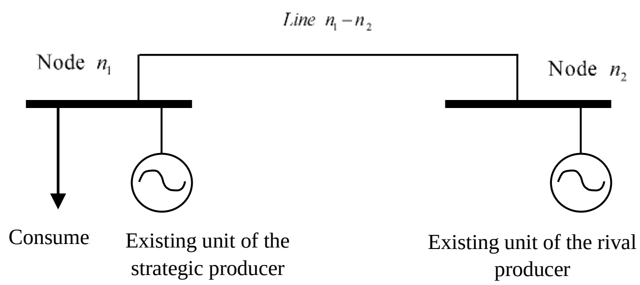

Figure 1: **Grid with two nodes**

Table 1 provides data for existing (conventional) strategic generating units and other rival units in this example. Each row refers to a specific type of production unit. The second column contains the power capacity of each unit, column 3 is the production cost, and column 4 is the production bus.

Table 1: **Type and Location of Existing Power Plant Units**

Unit Type	**Power [MW]**	**Cost [MW]**	**green**
Typical	150	10	1
Competitor	100	15	2

Table 2 presents investment options involving two technologies:

1) Basic technology with lower investment cost and low production cost
2) Renewable technology with stochastic generation with high investment cost and high production cost.

The second pillar contains the maximum investment capacity of each technology. The third column contains the maximum investment capacity and the last column contains the cost of production of each technology.

Table 2: **Type and Data Option for Investment**

Type of Power Plant	Annual Investment Cost [€/MW]	Available Capacity [MW]	Production Cost ($/MWh)
Typical	55000	100	12
Renewable	66000	200	15

The uncertainty of demand growth and the decisions of rival producers (investment and supply) are presented through four scenarios in Table 3:

1. To describe the uncertainty of demand growth, the maximum load in bus 1 is considered and modeled through four different levels.
2. The investment uncertainty of rival producers is modeled through the two options of no investment and investment in the typical unit located in a bus 1
3. The uncertainty of the decisions proposed by the rival manufacturers for the new unit is modeled

Table 3: Type and Data for Investment Option

Scenario	Maximum load at bus 1 (MW)	Retailer Price Quote ($/MWh)	Competitor Manufacturing Investment (MW)	Bid Price of Rival Production Units ($/MWh)		Scenario Probabilities
				Available Unit	New Unit	
1	360	35	0	15	0	0.4
2	310	32	10	0	12	0.3
3	300	30	50	5	10	0.2
4	250	25	100	7	2	0.1

Two operating conditions (O1 and O2) are considered with the characteristics of Table 4:

Table 4: Data to Consider Operation Operations

Operating conditions	Load Factor	Wind Power Capacity Factor	Weight coefficient
1	1	1	5060
2	0.6	0.5	3700

As can be seen in Tables 5 to 8. According to the results of the market settlement, the production/consumption levels of scenarios 1 to 4 are displayed. In scenario 1, which is without rival investment, the strategic producer has entered the market with its maximum amount of investment. In scenario 2, the rival producer builds a conventional unit of 10 MW, so the production of the existing unit of the strategic producer in operation condition 2 will be reduced from 150 MW to 126 MW. In scenario 3 and 4, the rival producer builds a typical unit of 50 and 100 MW, so the production of the existing unit of the strategic producer will be reduced to 80 MW and zero MW in operation conditions 2, in which case the annual profit will decrease due to the uncertainty of the competitor's investment. The expected annual profit of this strategic producer is $41.41 million.

Table 5: Market settlement results, production/consumption levels in the scenario ω_1 (without competitor investment)

Market Participant	Production/Consumption Levels in *O1* Operation [MW]	Production/consumption levels in *O2* operation [MW]
Competitor Unit	$p^{O}_{n_2 o_1} = 0$	$p^{O}_{n_2 o_2} = 0$

New Conventional Strategic Manufacturing Unit	$p^{E}_{n_1o_1} = 150$	$p^{E}_{n_1o_2} = 150$
New Wind Unit Strategic Manufacturer	$p^{S}_{n_2o_1} = 100$	$p^{S}_{n_2o_2} = 50$
Consumer	$p^{D}_{n_1o_1} = 250$	$p^{D}_{n_1o_2} = 200$

Table 6: Market settlement results, production / consumption level in the scenario ω_2 (with competitor investment)

Market Participant	Production/Consumption Levels in *O1* Operation [MW]	Production/consumption levels in *O2* operation [MW]
Competitor Unit	$p^{O}_{n_2o_1} = 10$	$p^{O}_{n_2o_2} = 10$
New Conventional Strategic Manufacturing Unit	$p^{E}_{n_1o_1} = 150$	$p^{E}_{n_1o_2} = 126$
New Wind Unit Strategic Manufacturer	$p^{S}_{n_2o_1} = 100$	$p^{S}_{n_2o_2} = 50$
Consumer	$p^{D}_{n_1o_1} = 260$	$p^{D}_{n_1o_2} = 186$

Table 7: Market settlement results, production levels/Consumption in the scenario (ω_3No competitor investment)

Market Participant	Production/Consumption Levels in O1 Operation [MW]	Production/consumption levels in O2 operation [MW]
Competitor Unit	$p^{O}_{n_2o_1} = 50$	$p^{O}_{n_2o_2} = 50$
New Conventional Strategic Manufacturing Unit	$p^{E}_{n_1o_1} = 150$	$p^{E}_{n_1o_2} = 80$
New Wind Unit Strategic Manufacturer	$p^{S}_{n_2o_1} = 100$	$p^{S}_{n_2o_2} = 50$
Consumer	$p^{D}_{n_1o_1} = 300$	$p^{D}_{n_1o_2} = 180$

Table 8: Market Settlement Results, Production / Consumption Level in the Scenario ω_4 (with Competitor's Investment)

Market Participant	Production/Consumption Levels in *O1* Operation [MW]	Production/consumption levels in *O2* operation [MW]
Competitor Unit	$p^{O}_{n_2o_1} = 100$	$p^{O}_{n_2o_2} = 100$
New Conventional Strategic Manufacturing Unit	$p^{E}_{n_1o_1} = 50$	$p^{E}_{n_1o_2} = 0$
New Wind Unit Strategic Manufacturer	$p^{S}_{n_2o_1} = 100$	$p^{S}_{n_2o_2} = 50$
Consumer	$p^{D}_{n_1o_1} = 250$	$p^{D}_{n_1o_2} = 150$

From the above tables, it can be concluded that the lower the probability of the scenario horizon, the greater the uncertainty leading to a decrease in profits and, as a result, investment in new units is faced with a decrease in capacity. This was done by examining separate scenario building for both the competitor's offer, the retailer's offer and the simultaneous offer, and the results are shown in Figures 2 to 4.

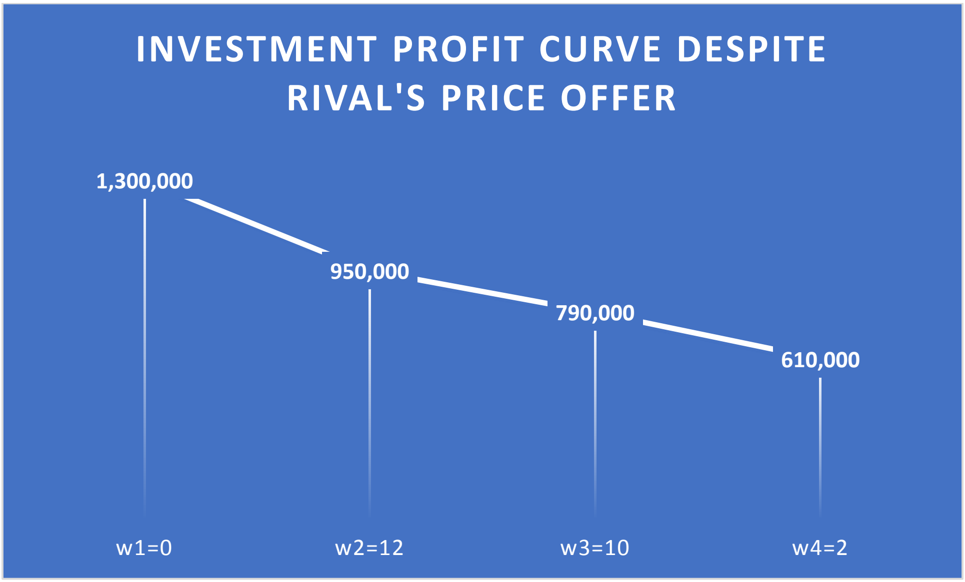

Figure 2: **Grid with two nodes**

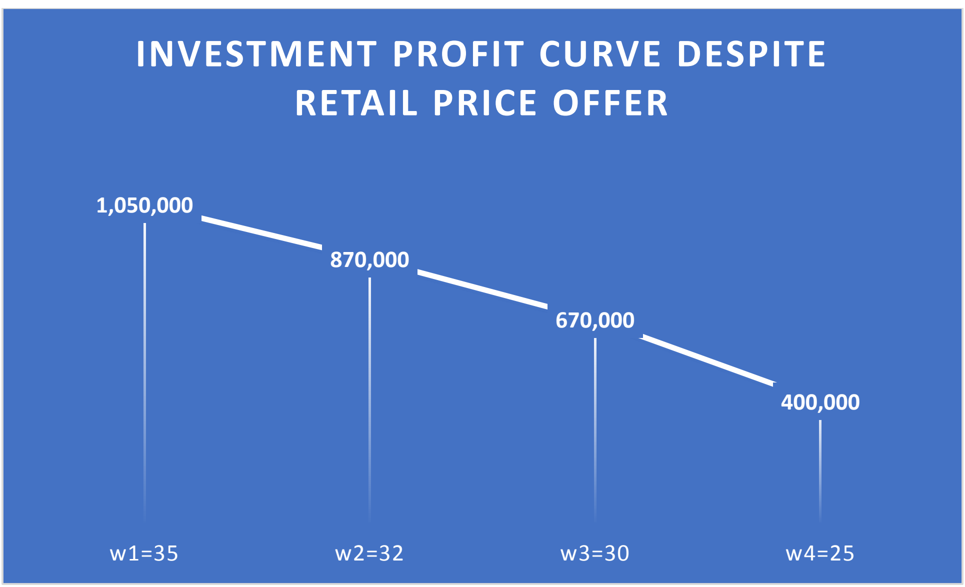

Figure 3: **Profit curve with retailer offer scenario**

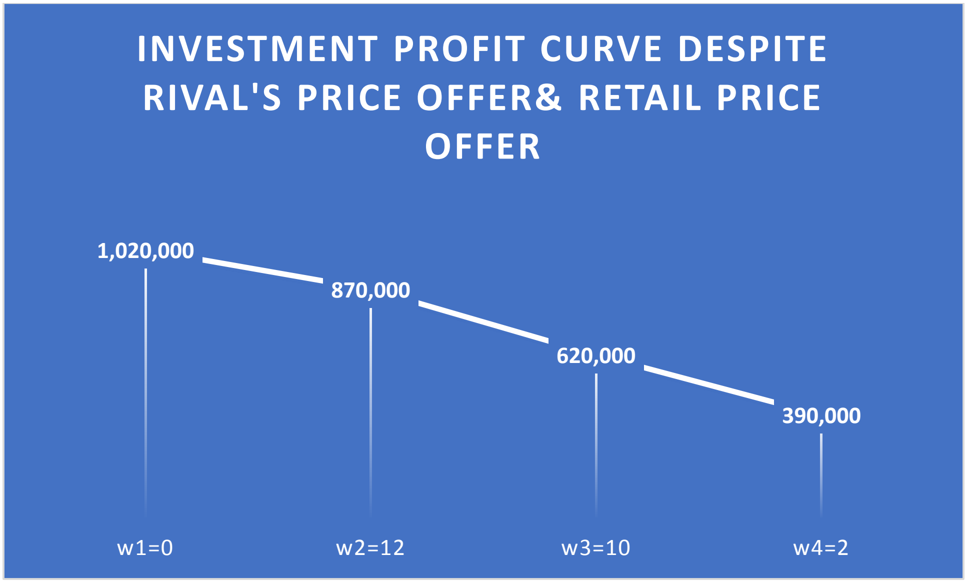

Figure 4: **Profit curve with rival's offer & retailer offer scenario**

6. Conclusion

This paper presents the decision-making approach of the strategic producer participating in the competitive electricity market to invest in conventional and stochastic generating units. The proposed method is a stochastic two-level model that can be solved as a MILP problem, using a direct solution approach, in which all practical conditions and scenarios are considered simultaneously.

Using the proposed model, the strategic producer will be able to make decisions in the following areas:

1. What is the size of the capacity of the new production units that are built in the electrical energy system?

2. What should be the optimal presentation strategy in the market?

This model is based on a static approach with a focus on a future target year and the uncertainties, demand conditions, and stochastic production conditions during this target year are modeled using a set of operating conditions and scenarios. The features of the proposed model and the simulation carried out in a nutshell have the following achievements.

1. The proposed model appropriately represents the level of production capacity of new units that is determined in the strategic producer's investment decision-making, allowing us to easily demonstrate market uncertainties, including demand growth and the actions of rival producers.
2. Using the scenarios considered for the values of uncertainty parameters in the decentralized market that lead to the nonlinearity of the formulation, the proposed model can be easily solved.

Appendix A: Lagrange Function

$$\begin{aligned}
\ell = & \sum_i \alpha^C_{no\omega} p^C_{no\omega} + \sum_k \alpha^E_{no\omega} p^E_{no\omega} + \sum_n \alpha^S_{no\omega} p^S_{no\omega} + \sum_j \alpha^O_{no\omega} p^O_{no\omega} - \sum_d C^D_{no} p^D_{no\omega} + \\
& \lambda_{no\omega} \left(\begin{array}{l} \sum_n p^D_{no\omega} + \sum_{m\in\Omega_n} B_{nm}\left(\theta_{no\omega} - \theta_{mo\omega}\right) - \\ \sum_n p^C_{no\omega} - \sum_n p^S_{no\omega} - \sum_n p^E_{no\omega} - \sum_n p^O_{no\omega} \end{array} \right) + \mu^{C^{\max}}_{no\omega}\left(p^C_{no\omega} - x^C_n\right) - \mu^{C^{\min}}_{no\omega} p^C_{no\omega} + \\
& \mu^{S^{\max}}_{no\omega}\left(p^S_{no\omega} - Q^S_{no} x^S_n\right) - \mu^{S^{\min}}_{no\omega} p^S_{no\omega} - \mu^{O^{\max}}_{no\omega}\left(p^O_{no\omega} - p^{O^{\max}}_{no\omega}\right) - \mu^{O^{\min}}_{no\omega} p^O_{no\omega} + \\
& \mu^{D^{\max}}_{no\omega}\left(p^D_{no\omega} - p^{D^{\max}}_{no\omega}\right) - \mu^{D^{\min}}_{no\omega} p^D_{no\omega} + \sum_{m\in\Omega_n} \nu^{\max}_{no\omega}\left(B_{nm}\left(\theta_{no\omega} - \theta_{mo\omega}\right) - F^{\max}_{nm}\right) + \\
& \xi^{\max}_{no\omega}\left(\theta_{no\omega} - \pi\right) - \xi^{\min}_{no\omega}\left(\theta_{no\omega} + \pi\right) + \xi^{\theta^{ref}}_{o\omega} \theta_{(n=ref)o\omega} \quad \forall o, \forall \omega
\end{aligned} \tag{33}$$

Appendix B: Linearization

In the GEP problem, write nonlinear statements as follows:

$$\left(\sum_n p^C_{no\omega}\lambda_{no\omega} + \sum_n p^E_{no\omega}\lambda_{no\omega} + \sum_n p^S_{no\omega}\lambda_{no\omega} \right) \tag{34}$$

In order to find one, we use a strong duality theorem and some KKT equivalences. The strong duality theorem says that if a problem is convex, then the objective functions of the primary and dual problems are the same at the optimal level. So, after applying the strong dichotomy theorem to each low-level problem (3)– (11), we get the following formula for each time as follows:

$$\begin{aligned}
& \sum_n \alpha^C_{no\omega} p^C_{no\omega} + \sum_n \alpha^E_{no\omega} p^E_{no\omega} + \sum_n \alpha^S_{no\omega} p^S_{no\omega} + \sum_n \alpha^O_{no\omega} p^O_{no\omega} - \sum_n C^D_{no} p^D_{no\omega} = \\
& \qquad - \sum_n \mu^{C^{\max}}_{no\omega} x^C_n - \sum_n \mu^{E^{\max}}_{no\omega} p^{E^{\max}}_{no\omega} - \sum_n \mu^{S^{\max}}_{no\omega} Q^S_{no} x^S_n - y
\end{aligned} \tag{35}$$

$$y = -\sum_n \mu^{O^{\min}}_{no\omega} p^{O^{\max}}_{no\omega} - \sum_n \mu^{D^{\max}}_{no\omega} Q^D_{no} p^{D^{\max}}_{no\omega} - \sum_{n(m\in\Omega_n)} \nu^{\min}_{nmo\omega} F^{\max}_{nm} - \sum_{n(m\in\Omega_n)} \nu^{\max}_{nmo\omega} F^{\max}_{nm} - \sum_n \xi^{\min}_{no\omega}\pi - \sum_n \xi^{\max}_{no\omega}\pi \tag{36}$$

On the other hand, with regard to (5), (6) and (7), we have:

$$\sum_n \mu^{C^{\max}}_{no\omega} x^C_n = \sum_n \mu^{C^{\max}}_{no\omega} p^C_{no\omega} \tag{37}$$

$$\sum_n \mu^{E^{\max}}_{no\omega} p^{E^{\max}}_n = \sum_n \mu^{E^{\max}}_{no\omega} p^E_{no\omega} \tag{38}$$

$$\sum_n \mu_{no\omega}^{S^{\max}} Q_{no}^S x_n^S = \sum_n \mu_{no\omega}^{S^{\max}} p_{no\omega}^S \tag{39}$$

By placing (37), (38) and (39) in (35), we will have:

$$\begin{aligned} &\sum_n \alpha_{no\omega}^C p_{no\omega}^C + \sum_n \alpha_{no\omega}^E p_{no\omega}^E + \sum_n \alpha_{no\omega}^S p_{no\omega}^S + \sum_n \alpha_{no\omega}^O p_{no\omega}^O - \sum_n C_{no}^D p_{no\omega}^D = \\ &\qquad - \sum_n \mu_{no\omega}^{C^{\max}} p_{no\omega}^C - \sum_n \mu_{no\omega}^{E^{\max}} p_{no\omega}^E - \sum_n \mu_{no\omega}^{S^{\max}} p_{no\omega}^S - y \end{aligned} \tag{40}$$

By sorting (40) we have:

$$\begin{aligned} &\sum_n p_{no\omega}^C \left(\alpha_{no\omega}^C + \mu_{no\omega}^{C^{\max}} \right) + \sum_n p_{no\omega}^E \left(\alpha_{no\omega}^E + \mu_{no\omega}^{E^{\max}} \right) + \sum_n p_{no\omega}^S \left(\alpha_{no\omega}^S + \mu_{no\omega}^{S^{\max}} \right) = \\ &\qquad - \sum_n \alpha_{no\omega}^O p_{no\omega}^O + \sum_n C_{no}^D p_{no\omega}^D - y \end{aligned} \tag{41}$$

On the other hand, with regard to (16), (17) and (18), we have:

$$\lambda_{no\omega} = \alpha_{no\omega}^C + \mu_{no\omega}^{C^{\max}} - \mu_{no\omega}^{C^{\min}}, \forall n \tag{42}$$

$$\lambda_{no\omega} = \alpha_{no\omega}^E + \mu_{no\omega}^{E^{\max}} - \mu_{no\omega}^{E^{\min}}, \forall n \tag{43}$$

$$\lambda_{no\omega} = \alpha_{no\omega}^S + \mu_{no\omega}^{S^{\max}} - \mu_{no\omega}^{S^{\min}}, \forall n \tag{44}$$

Therefore, by placing the values of (42), (43) and (44) in (34), we have:

$$\sum_n p_{no\omega}^C \lambda_{no\omega} = \sum_n \alpha_{no\omega}^S p_{no\omega}^C + \sum_n \mu_{no\omega}^{C^{\max}} p_{no\omega}^C - \sum_n \mu_{no\omega}^{C^{\min}} p_{no\omega}^C \tag{45}$$

$$\sum_n p_{no\omega}^E \lambda_{no\omega} = \sum_n \alpha_{no\omega}^E p_{no\omega}^E + \sum_n \mu_{no\omega}^{E^{\max}} p_{no\omega}^E - \sum_n \mu_{no\omega}^{E^{\min}} p_{no\omega}^E \tag{46}$$

$$\sum_n p_{no\omega}^S \lambda_{no\omega} = \sum_n \alpha_{no\omega}^S p_{no\omega}^S + \sum_n \mu_{no\omega}^{S^{\max}} p_{no\omega}^S - \sum_n \mu_{no\omega}^{S^{\min}} p_{no\omega}^S \tag{47}$$

In addition, we have (23) and (24):

$$\sum_i \mu_{ti}^{C^{\min}} p_{ti}^C = 0 \;\; , \;\; \sum_k \mu_{ti}^{E^{\min}} p_{ti}^E = 0 \;\; , \;\; \sum_k \mu_{ti}^{S^{\min}} p_{ti}^S = 0 \tag{48}$$

Using simplifications (45), (46), (47) and (48), we have:

$$\begin{aligned} &\sum_n p_{no\omega}^C \lambda_{no\omega} + \sum_n p_{no\omega}^E \lambda_{no\omega} + \sum_n p_{no\omega}^S \lambda_{no\omega} = \sum_n p_{no\omega}^C \left(\alpha_{no\omega}^C + \mu_{no\omega}^{C^{\max}} \right) + \sum_n p_{no\omega}^E \left(\alpha_{no\omega}^E + \mu_{no\omega}^{E^{\max}} \right) \\ &\qquad + \sum_n p_{no\omega}^S \left(\alpha_{no\omega}^S + \mu_{no\omega}^{S^{\max}} \right) \end{aligned} \tag{49}$$

And finally, the results (35) and (49) will be:

$$\sum_n p_{no\omega}^C \lambda_{no\omega} + \sum_n p_{no\omega}^E \lambda_{no\omega} + \sum_n p_{no\omega}^S \lambda_{no\omega} = -\sum_n \alpha_{no\omega}^O p_{no\omega}^O + \sum_n C_{no\omega}^D p_{no\omega}^D - y \tag{50}$$

References

[1] Antonio J. Conejo, Luis Baringo S. Jalal Kazempour • Afzal S. Siddiqui, “Investment in Electricity Generation and Transmission Decision Making under Uncertainty,” Springer International Publishing Switzerland 2016

[2]. Antonio J. Conejo, Juan M. Morales and M. Carrión, “Decision Making Under Uncertainty in Electricity Markets,” Springer Science+Business Media, LLC 2010

[3] J. ´Alvarez L´opez, K. Ponnambalam, and V. H. Quintana, “Generation and Transmission Expansion Under Risk Using Stochastic Programming,”IEEE Transactions on Power Systems, vol. 22, pp. 1369–1378, August 2007.

[4] R. Dom´ınguez, A. J. Conejo, and M. Carri ´on, “Toward Fully Renewable Electric Energy Systems,” IEEE Transactions on Power Systems, vol. 30, pp. 316–326, January 2015.

[5] S. J. Kazempour, “Strategic Generation Investment and Equilibria in Oligopolistic Electricity Markets,” Ph.D. dissertation, Universidad de Castilla-La Mancha, Ciudad Real, Spain, May 2013.

[6] L. Baringo, “Stochastic Complementarity Models for Investment in Wind-Power and Transmission Facilities,” Ph.D. dissertation, Universidad de Castilla-La Mancha, Ciudad Real, Spain, October 2013.

[7] S. Wogrin, “Generation Expansion Planning in Electricity Markets with Bilevel Mathematical Programming Techniques,” Ph.D. dissertation,Universidad Pontificia Comillas, Madrid, Spain, June 2013.

[8] J. H. Roh, M. Shahidehpour, and L. Wu, “Market-Based Generation and Transmission Planning with Uncertainties,” IEEE Transactions on Power Systems, vol. 24, pp. 1587–1598, August 2009.

[9] S. Wogrin, E. Centeno, and J. Barquin, “Generation Capacity Expansion in Liberalized Electricity Markets: A Stochastic MPEC Approach,” IEEE Transactions on Power Systems, vol. 26, pp. 2526–2532, November 2011.

[10] A. Khodaei, M. Shahidehpour, L.Wu, and Z. Li, “Coordination of Short-Term Operation Constraints in Multi-Area Expansion Planning,” IEEETransactions on Power Systems, vol. 27, pp. 2242–2250, November2012.

[11] D. Mej´ıa-Giraldo and J. D. McCalley, “Maximizing Future Flexibility in Electric Generation Portfolios,” IEEE Transactions on Power Systems,vol. 29, pp. 279–288, January 2014.

[12] S. Jin, A. Botterud, and S. M. Ryan, “Temporal Versus Stochastic Granularity in Thermal Generation Capacity Planning with Wind Power,” IEEE Transactions on Power Systems, vol. 29, pp. 2033–2041, September 2014.

[13] L. Tang and M. C. Ferris, “A Hierarchical Framework for Long-Term Power Planning Models,” IEEE Transactions on Power Systems, vol. 30, pp. 46–56, January 2015.

[14] I. Konstantelos and G. Strbac, “Valuation of Flexible Transmission Investment Options Under Uncertainty,” IEEE Transactions on Power Systems, vol. 30, pp. 1047–1055, March 2015.

[15] H. Ergun, B. Rawn, R. Belmans, and D. Van Hertem, “Stepwise Investment Plan Optimization for Large Scale and Multi-Zonal Transmission System Expansion,” in Power Engineering Society General Meeting. Boston, MA, USA: Institute of Electrical and Electronics Engineers, 17-21 July 2016.

[16] F. D. Munoz and J.-P. Watson, “A scalable solution framework for stochastic transmission and generation planning problems,” Computational Management Science, vol. 12, pp. 491–518, October 2015.

[17] Yixian Liu, Ramteen Sioshansi, and Antonio J. Conejo, “Multistage Stochastic Investment Planning withMultiscale Representation of Uncertainties andDecisions ,” 2017 IEEE

[18] A. PAPAZIS, George C. BAKOS “Generalized Model of Economic Dispatch Stylianos Optimization as an Educational Tool for Management of Energy Systems,” Advances in Electrical and Computer Engineering, Volume 21, Number 2, 2021

[19] Sandeep KAUR1, 2, Ganesh Balu KUMBHAR1 “Incentive Driven Distributed Generation Planning with Renewable Energy Resources,” Advances in Electrical and Computer Engineering, Volume 14, Number 4, 2014

[20] Zoran MILJANIC, Vladan RADULOVIC, Budimir LUTOVAC “Efficient Placement of Electric Vehicles Charging Stations using Integer Linear Programming,” Advances in Electrical and Computer Engineering, Volume 18, Number 2, 2018

[21] Zhou Y, Wang L, McCalley JD. Designing effective and efficient incentive policies for renewable energy in generation expansion planning. Appl Energy2011; 88:2201–9.

[22] P. Kamyaz, J. Valenzuela, and C. S. Park, “Transmission congestion and competition on power generation expansion,” IEEE Trans. Power Syst., vol. 22, no. 1, pp. 156–163, Feb. 2007.

[23] A. S. Chuang, F.Wu, and P. Varaiya, "A game-theoretic model for generation expansion planning: Problem formulation and numerical comparisons," IEEE Trans. Power Syst., vol. 16, no. 4, pp. 885–891, Nov.2001.

[24] M. Ventosa, R. Denis, and C. Rodendo, "Expansion planning in electricity markets. Two different approaches," in Proc. 14th Power System Computation Conf. (PSCC), Sevilla, Spain, Jun. 2002.

[25] F. Oliveira, "The value of information in electricity investment games,"Energy Pol., vol. 36, no. 7, pp. 2364–2375, Jul. 2008.

[26] The ILOG CPLEX, 2008. [Online]. Available: http://www.ilog.com/products/cplex/.