Optimization
Iranian Journal of Optimization
Volume 13, Issue 1, 2021, 1-11
Research Paper

Online version is available on: www.ijo.rasht.iau.ir

Islamic Azad University
Rasht Branch
ISSN: 2588-5723
E-ISSN:2008-5427

Approximate Solution of Nonlinear Fractional Order Model of HIV Infection of $CD4^{+}T$ Via Differential Quadrature Radial Basis Functions Technique

Kokab Chalambari [1], Hamideh Ebrahimi [1*] and Zeinab Ayati [2]

[1]Department of Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran
[2]Department of Engineering sciences, Faculty of Technology and Engineering East of Guilan, University of Guilan, Rudsar-Vajargah, Iran

Revise Date: 27 September 2019
Accept Date: 12 October 2020

Abstract

In this research, differential quadrature radial basis functions Method is performed to a fractional order model of HIV infection of CD4+T. Here, Caputo fractional derivative is used and it is approximated by forward finite difference method. Results have been compared with the results of Laplace Adomian decomposition method (LADM), (Biazar & Hosseini 2016, 2017a, 2017 b; Hosseini & Biazar, 2012; Hosseini & Daneshian, 2012; Moradweysi et al., 2018), Laplace Adomian decomposition method-pade (LADM-pade), Runge-Kutta, Variational iteration method (VIM) (Biazar, Sadri & Ebrahimi , 2011; Biazar & Ebrahimi, 2009) and Variational iteration method-pade (VIM-Pade) for $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and residual functions have been plotted. In addition, approximate solutions of suggested method for different order of fractional derivatives have been shown.

Keywords:
Radial Basis Function Method(RBFM)
Differential Quadrature Radial Basis Function Method(DQ- RBFM)
Caputo Fractional Derivative
Finite Difference Method
$CD4^{+}T$ cells

***Correspondence E-mail** : ebrahimi.hamideh@gmail.com

INTRODCTION

HIV (Human Immunodeficiency Virus) is one of the crises of the present age. It is a need for human society to help the people who suffer of this virus. HIV targets $CD4^{+}T$ makes up a large portion of white blood cells, it also damages other cells and reduces immune resistance. In this work a three-compartment fractional order model of $CD4^{+}T$ cells that is infected by HIV is described as below (Kansa,1990; Senthilkumaran & Arul Joseph, 2017)

$$\begin{cases} D^{\alpha_1} T(t) = p - \alpha T(t) + rT(t)\left(1 - \dfrac{T(t)+I(t)}{T_{\max}}\right) \\ \qquad - k^{*} V(t)T(t), \\ D^{\alpha_2} I(t) = k^{*} V(t)T(t) - \beta I(t)\,, \\ D^{\alpha_3} V(t) = N\beta I(t) - \gamma\, V(t), \end{cases} \tag{1}$$

where $D^{\alpha_i} = \dfrac{\partial^{\alpha_i}}{\partial t^{\alpha_i}}$, $i = 1, 2, 3$ are fractional derivative operators and the initial conditions would be as $T(0) = T_0$, $I(0) = I_0$ and $V(0) = V_0$.

In system 1, $T(t), I(t)$ and $V(t)$ are the centralization of susceptible $CD4^{+}T$ cells, $CD4^{+}T$ cells that HIV viruses infected them and free HIV particles in the blood, respectively and α, β and γ are natural rotation rates of healthy T cells, infected T cells and virus bits respectively. $\left(1 - \dfrac{T+I}{T_{\max}}\right)$ denotes the logistic growth of healthy $CD4^{+}T$ cells and the morbidity of HIV infection of healthy $CD4^{+}T$ cells is shown by $k^{*}VT$ where $k^{*} > 0$ is the infection rate. Every infected $CD4^{+}T$ cell is supposed to generate N virus particles at its lifecycle. The body is convinced to make $CD4^{+}T$ cells from progenitors in the bone marrow and thymus at a fixed rate p .Also $T_{\max}$ is the highest level of $CD4^{+}T$ cell centralization in the body.

Different methods have been implemented to this model of $CD4^{+}T$ cells. For example, a multi-step differential transform method (MSDTM) is applied to approximate the solution of nonlinear fractional order ordinary differential equation (Ahmadi, Vahidi & Allahviranloo, 2017) Laplace Adomian decomposition method and Laplace Adomian decomposition method - Pade are performed to do the same (Ongun, 2011). Following (Arafa, Hanafy & Gouda, 2016) Mittage-Leffler method has been used to solve fractional order of equations. In (Biazar, Sadri & Ebrahimi, 2011) the variational iteration method and Laplace Adomian decomposition method -Pade for solving this model and comparison with results of Runge-Kutta method has been shown.

The differential quadrature radial basis function method (DQ-RBFM) has been performed in this study. Radial basis functions method was first expressed by Hardy and one of its most important features is that scattered data can be used in computing properly and these nodes dose not organize a mesh, so the information between them is not required (Hardy, 1971). For the first time, Kansa approximate differential equations by these functions (Kansa, 1990) and in recent years its combinations with other procedures are popular. Differential quadrature radial basis function method is one of these combinations that will explain in next sections. The review of this work is organized as follows:

In the first section, some common definitions of fractional calculus; radial basis functions method and differential quadrature radial basis functions method are expressed fractional derivative based on finite difference method has been applied in section 3, DQ-RBFM applied to the model of HIV infection of $CD4^{+}T$ cells in part 4 and numerical results tables and figures are shown in the last section. At the end conclusions are given in the final section.

SOM BASIC DEFINITIONS OF FRACTIONAL OPERATORS

Fractional calculus is a branch of mathematics that differentiates and integrates into non-integer order. Since fractional calculus describes natural phenomena with high precision researchers are interested in it (Chandhini, Prashanthi & Antony, 2017; Ilie, Biazar & Ayati, 2017; *Liu,* Li & Hu, 2019). The most important and most used fractional derivatives and integrals are summarized below:

Definition 1. Suppose $\Omega=[a,b]$ where $-\infty<a<b<\infty$. The left-sided Reimann-Liouville and right-sided Reimann-Liouville fractional derivatives of order α are shown as ${}^{RL}D_{a^+}^{\alpha}$ and ${}^{RL}D_{b^-}^{\alpha}$ respectively and defined as follows:

$${}^{RL}D_{a^+}^{\alpha} f(t)=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_a^t (t-\tau)^{n-\alpha-1} f(\tau)\,d\tau,$$

$${}^{RL}D_{b^-}^{\alpha} f(t)=\frac{(-1)^n}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_t^b (\tau-t)^{n-\alpha-1} f(\tau)\,d\tau,$$

$$\alpha\in\mathbb{R}^+,\quad n-1<\alpha<n.$$

Definition 2. The left-sided Reimann-Liouville and the right-sided Reimann-Liouville fractional integrals of order α are represented as ${}^{RL}I_{a^+}^{\alpha}$ and ${}^{RL}I_{b^-}^{\alpha}$ respectively and described as:

$${}^{RL}I_{a^+}^{\alpha} f(t)={}^{RL}D_{a^+}^{-\alpha} f(t)=\frac{1}{\Gamma(\alpha)}\int_a^t (t-\tau)^{\alpha-1} f(\tau)\,d\tau,$$

$${}^{RL}I_{b^-}^{\alpha} f(t)={}^{RL}D_{b^-}^{-\alpha} f(t)=\frac{1}{\Gamma(\alpha)}\int_t^b (\tau-t)^{\alpha-1} f(\tau)\,d\tau,$$

$$\alpha\in\mathbb{R}^+,\quad n-1<\alpha<n.$$

Definition 3. The left-sided Caputo and right-sided Caputo fractional derivatives of order α are defined as follows:

$${}^{C}D_{a^+}^{\alpha} f(t)=\frac{1}{\Gamma(n-\alpha)}\int_a^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\,d\tau,$$

$${}^{C}D_{b^-}^{\alpha} f(t)=\frac{1}{\Gamma(n-\alpha)}\int_t^b (\tau-t)^{n-\alpha-1} f^{(n)}(\tau)\,d\tau,$$

$$\alpha\in\mathbb{R}^+,\quad n-1<\alpha<n.$$

Definition 4. The left-sided Caputo and right-sided Caputo fractional integrals of order α are shown in this way:

$${}^{RL}I_{a^+}^{\alpha} f(t)={}^{RL}D_{a^+}^{-\alpha} f(t)=\frac{1}{\Gamma(\alpha)}\int_a^t (t-\tau)^{\alpha-1} f(\tau)\,d\tau,$$

$${}^{RL}I_{b^-}^{\alpha} f(t)={}^{RL}D_{b^-}^{-\alpha} f(t)=\frac{1}{\Gamma(\alpha)}\int_t^b (\tau-t)^{\alpha-1} f(\tau)\,d\tau,$$

$$\alpha\in\mathbb{R}^+,\quad n-1<\alpha<n.$$

EXPLANATION OF RBF AND DQ-RBF METHODS

As we know most of natural phenomena can be described by mathematical models but it is difficult to find the exact solutions and therefore, methods that yield approximate solutions have been considered. One of the important advancements in this field is finite elements method that has some weaknesses such as the amount of mesh points and decrease of step size in higher dimensions. Also, in the methods based on mesh, algorithm must be implemented in total domain that this work requires a lot of time and complicated calculations. The root of all these problems is the use of mesh which leads to the idea of meshless methods and the most important property of these methods is that does not need to know the prior information about relationships between nodes.

Radial basis functions method is a kind of meshless methods that has an efficient role in interpolating scattered points. Since an appropriate mesh does not provide on the domain it is considered for finding numerical solutions of partial differential equations (PDEs) and ordinary differential equations (ODEs). RBF and interpolation based on it are defined here.

Definition 5. Suppose $\varphi:\mathbb{R}^+ \to \mathbb{R}$ is a continuous function. Radial basis function on $\mathbb{R}^d$ is defined as $\varphi(\| t-t_0 \|_2)$ where $t \in \mathbb{R}^d$, t_0 is a fixed point on $\mathbb{R}^d$ and $\|\cdot\|_2$ is Euclidian norm.
Some famous radial basis functions are shown below in Table 1.

Table 1: Some famous radial basis functions

Name	Classic form	New form
Multiquadric	$\sqrt{c^2+r^2}$	$\sqrt{1+\varepsilon^2 r^2}$
Inverse Multiquadric	$\frac{1}{\sqrt{c^2+r^2}}$	$\frac{1}{\sqrt{1+\varepsilon^2 r^2}}$
Inverse Quadric	$\frac{1}{c^2+r^2}$	$\frac{1}{1+\varepsilon^2 r^2}$
Generalized Multiquadric	$(c^2+r^2)^\beta$	$(1+\varepsilon^2 r^2)^\beta$
Gaussian	$e^{-\frac{r^2}{c^2}}$	$e^{-\varepsilon^2 r^2}$

Definition 6. Consider the set of nodes $\{t_i\}_{i=1}^N$ and corresponding values $f_i = f(t_i)$, $i=1,...,N$. Interpolation based on radial basis functions describes as $u(t) \simeq \sum_{i=1}^N \lambda_i \varphi(\| t-\xi_i \|)$ where φ is a radial basis function, $\{\xi_i\}_{i=1}^N$ are the central points in the domain. $\{\lambda_i\}_{i=1}^N$ are the coefficients that determined by imposing interpolation conditions $u(t_i)=f_i$, $i=1,...,N$ that leads to a linear equation system as follows:

$$\begin{bmatrix} \varphi(\|t_1-\xi_1\|) & \cdots & \varphi(\|t_1-\xi_N\|) \\ \vdots & & \vdots \\ \varphi(\|t_N-\xi_1\|) & \cdots & \varphi(\|t_N-\xi_N\|) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_N \end{bmatrix} = \begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix},$$

that matrix form can be written as below
$A\Lambda = F.$
By solving the above system unknowns $\lambda_1, \lambda_2, ..., \lambda_N$ will be determined and at the end, the solution of problem will be obtained.

Differential quadrature radial basis functions method (DQ-RBFM)

Differential quadrature method is a numerical discretization technique to approximate derivative of order α that it can be integer or fractional number. Its main idea is deduced from numerical integration and in this method after domain discretization, the α - order of derivatives of $u(t)$ respect to t at ith node is considered as below

$$D^\alpha u(t_i) \simeq \sum_{j=1}^N w_{ij}^\alpha u(t_j), \quad i=1,2,... , \tag{2}$$

where $D^\alpha = \frac{\partial^\alpha}{\partial t^\alpha}$ and $u(t_j)$, $j=1,2,...,N$ are the values of function at the discrete points t_j , $j=1,2,...,N$.
Shu et al in (Dehghan & Abbaszadeh, 2017) introduced another type of method based on radial basis functions using differential quadrature method that called DQ-RBFM. The DQ-RBFM approximates unknowns by RBFs and the α - order of derivatives by differential quadrature method. In this method DQ method is combined with radial basis functions based on interpolation of $u(t)$. The radial basis functions have been used as a set of basic functions (Dehghan & Abbaszadeh, 2018a;Dehghan & Abbaszadeh, 2018 b; Karamali, Dehghan & Abbaszadeh

,2018; Kazem & Hatam, 2017; *Liu*, Li & Hu, 2019). Suppose that

$$u(t) \simeq \sum_{k=1}^{N} \lambda_k \varphi_k(t), \tag{3}$$

where $\varphi_k(t) = \varphi(\| t - \xi_k \|)$, $k = 1,2,...,N$ are radial basis functions and ξ_k, $k = 1,2,...,N$ are central points. By differentiating of 3 respect to t and substituting $t = t_i$, $i = 1,2,...,N$, we have

$$D^{\alpha} u(t_i) \simeq \sum_{k=1}^{N} \lambda_k D^{\alpha} \varphi_k(t_i), \quad i = 1,2,...,N. \tag{4}$$

Since 2 and 4 are equal we obtain

$$D^{\alpha} \varphi_k(t_i) = \sum_{j=1}^{N} w_{ij}^{\alpha} \varphi_k(t_j), \quad i,k = 1,2,...,N. \tag{5}$$

The matrix forms of 5 is written as following terms

$$\begin{bmatrix} D^{\alpha}\varphi_1(t_i) \\ D^{\alpha}\varphi_2(t_i) \\ \vdots \\ D^{\alpha}\varphi_N(t_i) \end{bmatrix} = \begin{bmatrix} \varphi_1(t_1) & \varphi_1(t_2) & \cdots & \varphi_1(t_N) \\ \varphi_2(t_1) & \varphi_2(t_2) & \cdots & \varphi_2(t_N) \\ \vdots & & & \\ \varphi_N(t_1) & \varphi_N(t_2) & \cdots & \varphi_N(t_N) \end{bmatrix} \begin{bmatrix} w_{i1}^{\alpha} \\ w_{i2}^{\alpha} \\ \vdots \\ w_{iN}^{\alpha} \end{bmatrix},$$

$$i = 1,2,...,N, \tag{6}$$

or it leads to the following systems

$$\overrightarrow{D^{\alpha}\Phi(t_i)} = \Phi \overrightarrow{W_i^{\alpha}}, \quad i = 1,2,...,N.$$

The values of radial basis functions and their derivatives are given and if coefficient matrix Φ be non-singular, then unknowns in 6 are determined by solving the below system

$$\overrightarrow{W_i^{\alpha}} = \Phi^{-1} \overrightarrow{D^{\alpha}\Phi(t_i)}, \quad i = 1,2,...,N.$$

APPROXIMATING CAPUTO FRACTIONAL DERIVATIVE BY FORWARD FINITE DIFFERENCE METHOD

Consider the following Caputo fractional

$${}^{C}D^{\alpha}u(t) = \frac{1}{\Gamma(1-\alpha)} \int_a^t \frac{1}{(t-\tau)^{\alpha}} u'(\tau) d\tau,$$

where $0 < \alpha < 1$. Suppose that $t = t_i$, $i = 1,2,...,N$, therefore

$${}^{C}D^{\alpha}u(t_i) = \frac{1}{\Gamma(1-\alpha)} \int_a^{t_i} \frac{1}{(t_i-\tau)^{\alpha}} u'(\tau) d\tau,$$

$$i = 1,2,...,N.$$

Then, consider the partition $a = \zeta_1 < \zeta_2 < ... < \zeta_{M_i} = t_i$, $i = 1,2,...,N$. for interval $[a, t_i]$ that leads to the following form

$${}^{C}D^{\alpha}u(t_i) = \frac{1}{\Gamma(1-\alpha)} \sum_{k=1}^{M_i - 1} \int_{\zeta_k}^{\zeta_{k+1}} \frac{1}{(t_i-\tau)^{\alpha}} u'(\tau) d\tau,$$

$$0 < \alpha < 1, \quad i = 1,2,...,N.$$

Approximating $u'(\tau)$ in every subinterval by forward finite difference definition as shown bellow

$${}^{C}D^{\alpha}u(t_i) \simeq \frac{1}{\Gamma(1-\alpha)} \sum_{k=1}^{M_i - 1} \int_{\zeta_k}^{\zeta_{k+1}} \frac{1}{(t_i-\tau)^{\alpha}} \frac{u(\zeta_{k+1}) - u(\zeta_k)}{\zeta_{k+1} - \zeta_k} d\tau,$$

$$0 < \alpha < 1, \quad i = 1,2,...,N. \tag{7}$$

Now, by calculating the integral of 7, we obtain

$${}^{C}D^{\alpha}u(t_i) \simeq \sum_{k=1}^{M_i - 1} b_{i,k}^{\alpha} \, u(\zeta_{k+1}) - u(\zeta_k),$$

$$i = 1,2,...,N, \tag{8}$$

where

$$b_{i,k}^{\alpha} = \frac{(t_i - \zeta_k)^{1-\alpha} - (t_i - \zeta_{k+1})^{1-\alpha}}{(\zeta_{k+1} - \zeta_k)\Gamma(2-\alpha)},$$

$$k = 1,2,...,M_i, \quad i = 1,2,...,N.$$

The equation 8 can be used as an approximation for Caputo fractional derivative.

APPLYING THE DQ-RBFM FOR SOLVING THE MODEL OF HIV INFECTION OF CD4⁺T CELLS

In general, the approximate solutions by using RBF method will be considered as follows

$$T(t) \simeq \sum_{k=1}^{N} \lambda_k^1 \varphi_k(t),$$

$$I(t) \simeq \sum_{k=1}^{N} \lambda_k^2 \varphi_k(t),$$

$$V(t) \simeq \sum_{k=1}^{N} \lambda_k^3 \varphi_k(t),$$

where λ_k^1, λ_k^2 and λ_k^3 are unknown coefficients and $\varphi_k(t)=\varphi(\|t-\xi_k\|)$, $k=1,2,...,N$.
Using the differential quadrature radial basis functions definition at the *i*th point t_i, we have

$$D^{\alpha_1}T(t_i)\simeq\sum_{k=1}^{N} w_{ik}^{\alpha_1}T(t_k),$$
$$D^{\alpha_2}I(t_i)\simeq\sum_{k=1}^{N} w_{ik}^{\alpha_2}I(t_k), \qquad (9)$$
$$D^{\alpha_3}V(t_i)\simeq\sum_{k=1}^{N} w_{ik}^{\alpha_3}V(t_k),$$

where $w_{ik}^{\alpha_1}$, $w_{ik}^{\alpha_2}$ and $w_{ik}^{\alpha_3}$ are the weighting coefficients and can be determined by using equations 6, 8, t_i, $i=1,2,...,N$ and following equations for equidistance nodes
$t_i=t_1+(t_N-t_1)(i-1)/(N-1),\ \ i=1,2,...,N,$

Or non-equidistance nodes

$$t_i=\frac{t_N}{2}\left(1-\cos\left(\frac{i-1}{N-1}\right)\pi\right),\ \ i=1,2,...,N.$$

According to initial values of system 1,

$$\begin{cases}T(t_1)=T_0,\\ I(t_1)=I_0,\\ V(t_1)=V_0.\end{cases}$$

By replacing 9 in system 1, the equations will be as follows

$$\mathrm{Res}(t_i)=\sum_{k=1}^{N} w_{ik}^{\alpha_1}T(t_k)-p+\alpha T(t_i)+$$
$$rT(t_i)\left(1-\frac{T(t_i)+I(t_i)}{T_{\max}}\right)-k^*V(t_i)T(t_i)\simeq 0,$$
$$\mathrm{Res}(t_i)=\sum_{k=1}^{N} w_{ik}^{\alpha_2}I(t_k)-k^*V(t_i)T(t_i)+\beta I(t_i)\simeq 0,$$
$$\mathrm{Res}(t_i)=\sum_{k=1}^{N} w_{ik}^{\alpha_3}V(t_k)-N\beta I(t_i)+\gamma V(t_i)\simeq 0,$$
$$i=2,....,N.$$

(10)

Unknown values $T(t_i)$, $I(t_i)$ and $V(t_i)$, $i=2,...,N$ .can achieved by solving non-linear system 10. Then by solving the following systems, the unknown coefficients λ_k^1, λ_k^2 and λ_k^3 can be derived that leads to the desired results for approximate solutions of system 1.

$$T(t_i)\simeq\sum_{k=1}^{N}\lambda_k^1\varphi_k(t_i),\qquad i=1,2,...,N,$$
$$I(t_i)\simeq\sum_{k=1}^{N}\lambda_k^2\varphi_k(t_i),\qquad i=1,2,...,N,$$
$$V(t_i)\simeq\sum_{k=1}^{N}\lambda_k^3\varphi_k(t_i),\qquad i=1,2,...,N.$$

COMPUTATIONAL RESULTS

Assume that
$\alpha=0.2$, $\beta=0.3$, $\gamma=2.4$, $r=3$,
$p=0.1$, $k^*=0.0027$, $T_{\max}=1500$.

Number of points that consider in this method is $N=30$. Here, Gaussian radial basis function $\varphi(t)=e^{-t^2/2c}$ is considered and $c=\mathrm{var}(t_i)$ $i=1,2,...,N$ used as shape parameter. The systems of 10 are solved by Newton method for non-linear systems and the results are compared with some methods in Tables 2-4 and the residual functions of the suggested method for $T(t)$, $I(t)$ and $V(t)$ are shown in Fig 1-3, respectively for $\alpha_1=\alpha_2=\alpha_3=1$. Also, for the different α's plots of approximate solutions are compared in Fig 4 to 6.

Table 2: Numerical comparison for $T(t)$.

$p = 0.1, \alpha = 0.2, \beta = 0.3, \gamma = 2.4, r = 3, k^* = 0.0027, \alpha_1 = \alpha_2 = \alpha_3 = 1, T_{max} = 1500$.

t	*LADM-Pade*	*LADM*	*Runge-Kutta*	*VIM*	*VIM-Pade*	*RBF-DQ*
0.0	0.1	0.1	0.1	0.1	0.1	0.097
0.2	0.2088072731	0.2088073298	0.2088080833	0.2088073294	0.2088082795	0.208
0.4	0.4061052625	0.4061358315	0.4062405393	0.4061358276	0.4072641833	0.40777
0.6	0.7611467713	0.7624762220	0.7644238890	0.7624762056	0.7650980592	0.7647
0.8	1.37731959	1.398082863	1.414046831	1.398082810	1.423501776	1.416
1	2.329169761	2.507874151	2.591594802	2.507874011	2.682537702	2.5918

Table 3: Numerical comparison for $I(t)$,

$p = 0.1, \alpha = 0.2, \beta = 0.3, \gamma = 2.4, r = 3, k^* = 0.0027, \alpha_1 = \alpha_2 = \alpha_3 = 1, T_{max} = 1500$

t	*LADM-Pade*	*LADM*	*Runge-Kutta*	*VIM*	*VIM-Pade*	*RBF-DQ*
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2	0.6032707289e-	0.6032706956e-	0.6032702150e-	0.603270695e-5	0.6032703383e-	0.64e-5
0.4	0.1315916175e-	0.1315891002e-	0.1315834073e-	0.131589099e-4	0.1315843257e-	0.1339e-4
0.6	0.2126836882e-	0.2123298178e-	0.2122378506e-	0.2122329815e-	0.2122492821e-	0.2163e-4
0.8	0.3006918678e-	0.3024270157e-	0.3017741955e-	0.302427010e-4	0.3018391897e-	0.306e-4
1	0.3987365427e-	0.4033321858e-	0.4003781468e-	0.403332174e-4	0.4006166602e-	0.402e-4

Table 4: Numerical comparison for $V(t)$,

$p = 0.1, \alpha = 0.2, \beta = 0.3, \gamma = 2.4, r = 3, k^* = 0.0027, \alpha_1 = \alpha_2 = \alpha_3 = 1, T_{max} = 1500$

t	*LADM-Pade*	*LADM*	*Runge-Kutta*	*VIM*	*VIM-Pade*	*RBF-DQ*
0.0	0.1	0.1	0.1	0.1	0.1	0.09999
0.2	0.06187996025	0.06187995305	0.06187984331	0.06187995306	0.06187984119	0.06192
0.4	0.03831324883	0.03830818047	0.3829488788	0.03830818047	0.0382946866	0.038313
0.6	0.02439174349	0.02391981608	0.2370455014	0.02391981608	0.0237019975	0.023685
0.8	0.009967218934	0.01621234343	0.01468036377	0.01621234343	0.01466705316	0.01465
1	0.003305076447	0.01605502238	0.009100845043	0.016055224	0.009056965718	0.00910

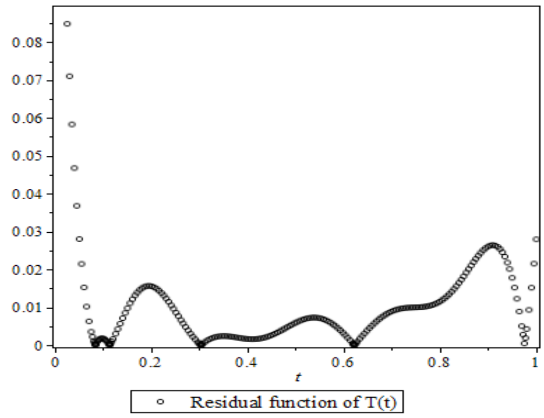

Fig.1. Plot of residual function for $T(t)$.

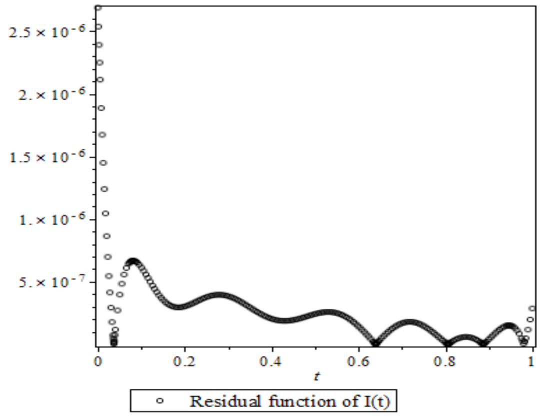

Fig.2. Plot of residual function for $I(t)$.

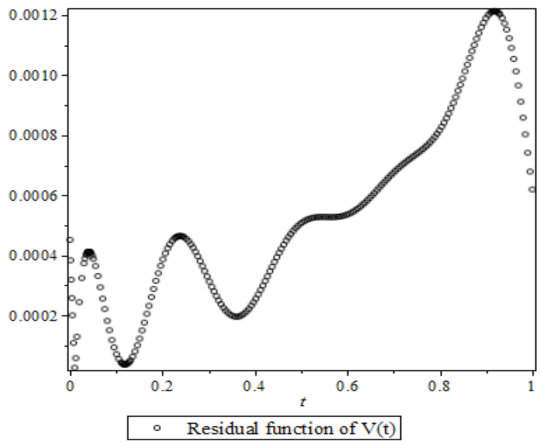

Fig.3. Plot of residual function for $V(t)$.

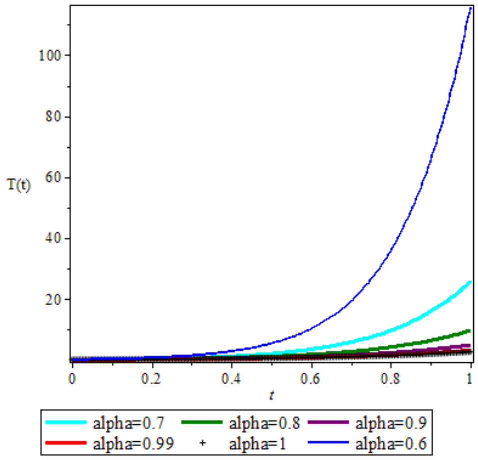

Fig. 4. Plots of approximate solutions for some values of $\alpha_1=\alpha_2=\alpha_3=\alpha$ for $T(t)$.

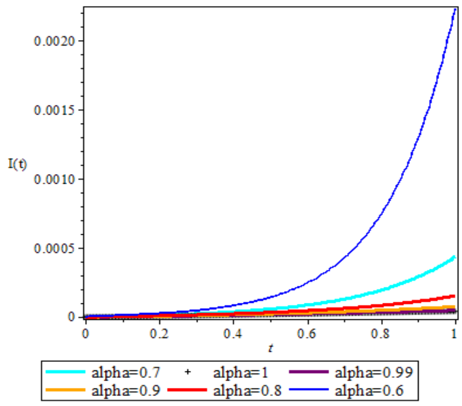

Fig. 5. Plots of approximate solutions for some values of $\alpha_1=\alpha_2=\alpha_3=\alpha$ for $I(t)$.

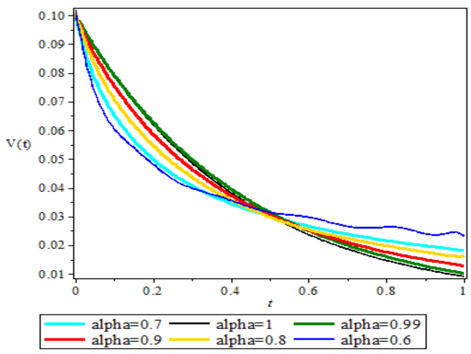

Fig. 6. Plots of approximate solutions for some values of $\alpha_1=\alpha_2=\alpha_3=\alpha$ for $V(t)$.

CONCLUSION

The purpose of this work is to apply DQ-RBFM to the three-compartment fractional order model of $CD4^{+}T$ cells. The most important reason for using DQ-RBFM is that the method can achieve high order of accuracy by using small number of nodal points. The next reason is that DQ-RBFM can solve the ill-conditioning of interpolation matrix and the other one is that the method converts the differential equation to a algebraic equations system that solves the problem more convenient than traditional The results showed that when the centralization of susceptible $CD4^{+}T$ cells and infected $CD4^{+}T$ cells increase, the free HIV particles decrease. Since the algorithm of the method did not apply in whole domain, the DQ-RBF method was distinct from other methods that mentioned. Since the exact solutions were not available, the residual functions had been plotted. The numerical results which were obtained in the section showed that the method was effective to approximate solutions of the problem. Because of the interpolation matrix of Gaussian RBFs was conditionally positive definite, the non-singularity of interpolant matrix was guaranteed and it was invertible. There were different ways to find out shape parameter such as Rippa algorithm and choice of radial basis functions and finding shape parameter are open objects in solving problems with RBFs. Computations in this work were done by using the package Maple 18 and the cpu time of the method was 124.00ms. Research for finding more applications of this method and other techniques based on RBFs is one of the goals in our next studies.

Acknowledgements

The authors are very grateful to the reviewers for carefully reading of the paper and for their comments and suggestions which have improved the paper.

REFERENCES

Ahmadi, N., Vahidi, A. R., & Allahviranloo, T. (2017). An efficient approach based on radial basis functions for solving stochastic fractional differential equations. Math. Sci, 11, 113–118.

Arafa, A. A. M., Hanafy. I. M., & Gouda. M. I. (2016). Stability analysis of fractional order-HIV Infection of CD4+T cells with numerical solutions, *Journal of Fractional Calculus and Applications,* 7 (1) 36-45.

Biazar, J., Sadri, Kh., & Ebrahimi, H. (2011). Variational iteration method for a model of HIV infection of cells. *Journal of Nature Science and Sustainable Technology*, 2, 83-90.

Biazar, J., Hosseini, K. (2016). A modified Adomian decomposition method for singular initial value Emden-Fowler type equations. *International Journal of Applied Mathematical Research*, 5, 69-72.

Biazar, J., Ebrahimi, H. (2009). Variational Iteration Method for Fredholm integral equations of the second kind. *Iranian Journal of Optimization*, 1,11-17.

Biazar, J., Hosseini, K. (2017). An Effective modification of Adomian Decomposition Method for Solving Emden-Fowler Type Systems. *The National Academy of Sciences*, 40, 258-209.

Biazar, J., Hosseini, K. (2017). Analytic approximation of Volterra's population model, *Journal of Applied Mathematics Statistics and Informatics*, 13, 5-17.

Chandhini, G., Prashanthi, K. S., & Antony, V. (2017), A radial basis function method for fractional Darboux problems. Engineering Analysis with Boundary Elements, 000, 1-18.

Dehghan, M., &Abbaszadeh, M. (2017). Solution of multi-dimensional Klein–Gordon–Zakharov and Schrödinger /Gross–Pitaevskii equations via local Radial Basis Functions–Differential Quadrature (RBF–DQ) technique on non-rectangular computational domain. Engineering Analysis with Boundary Elements,000, 1-15.

Dehghan, M., & Abbaszadeh, M. (2018). A combination of proper orthogonal decomposition–discrete empirical interpolation method (POD–DEIM) and meshless local RBF-DQ approach for prevention of groundwater contamination. Computers and Mathematics with Applications, 75, 1390-1412.

Dehghan,M., & Abbaszadeh,M. (2018) . An upwind local radial basis functions-differential quadrature (RBF-DQ) method with proper orthogonal decomposition (POD) approach for solving compressible Euler equation, Engineering Analysis with Boundary Elements, 000, 1-13.

Gökdogan, A., Yildirim, A., & Merdan, M. (2011). Solving a fractional order model of HIV infection of CD4+ T cell, Mathematical and Computer Modelling, 54, 2132–2138.

Hardy, R, L. (1971). Multiquadric equations of topography and other irregular surface, Journal of geophysical research, 76, 1905-1915.

Hosseini, K., Biazar, J., Ansari, R., & Gholamin, P. (2012). A new algorithm for solving differential equations. Math. Meth. Appl. Sci, 35, 993–999.

Hosseini, K., Daneshian, B., Amanifard, N., & Ansari, R. (2012). Homotopy Analysis Method for a Fin with Temperature Dependent Internal Heat Generation and Thermal Conductivity. International Journal of Nonlinear Science, 14, 201-210.

Ilie, M., Biazar, J., Ayati, Z. (2017). Application of the Lie Symmetry Analysis for Second-order Fractional Differential Equations. *Iranian Journal of Optimization*, 2, 79-83.

Kansa, E. (1990). Multiquadrics a scattered data approximation scheme with Applications and to computational fluid dynamics–I surface approximations and partial derivative estimates. *Computers & Mathematics with Applications*, 19, 147–161.

Karamali., G., Dehghan, M., & Abbaszadeh, M. (2018). Numerical solution of a time fractional PDE in the electroanalytical chemistry by a local meshless method. *Engineering with Computers published online*.

Kazem, S., & Hatam, A. (2017). A modification on strictly positive definite RBF-DQ method based on matrix decomposition. *Engineering Analysis with Boundary Elements*, 76, 90-98.

Liu, J., Li, X., & Hu, X. (2019). A RBF-based differential quadrature method for solving two-dimensional variable-order time fractional advection-diffusion equation. *Journal of Computational Physics*. 384, 222-238.

Moradweysi, P., Ansari, R., Hosseini, K., & Sadeghi, F. (2018). Application of modified Adomian decomposition method to pull-in instability of nano-switches using nonlocal Timoshenko beam theory. Applied Mathematical Modelling, 54, 594-604.

Ongun,M. Y. (2011). The Laplace Adomian Decomposition Method for solving a model for HIV infection of CD4+T cells. Mathematical and Computer Modelling, 53, 597–603.

Parand, K., & Hashemi, S. (2018). RBF-DQ Method for Solving Equations of Lane-Emden type. *Ain Shams Engineering Journal*, 4, 615-629.

Senthilkumaran, M., & Arul Joseph, G. (2017). Stability Analysis for Fractional Order Model of HIV infection of CD4+T-Cells with Independent Delays. International Journal of Computational and Applied Mathematics, 12, 635-645.