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Malmquist Productivity Index Using Two-Stage DEA Model in Heart Hospitals

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Abstract

Heart patients displays several symptoms and it is hard to point them. Data envelopment analysis (DEA) provides a comparative efficiency degree for each decision-making units (DMUs) with several inputs and outputs. Evaluating of hospitals is one of the major applications in DEA. In this study, a comparison of additive model with standard input oriented and output oriented Malmquist productivity index (MPI) are used. The MPI is calculated to measure productivity growth relative to a reference technology. Two primary subjects are addressed in computation of MPI growth. What are generally referred to as a "catching-up" effect or technical efficiency change (TEC) and a "frontier shift" effect or technological change (TC). The data covers a six-year span from 2011 to 2016 for 15 local heart hospitals. Two inputs, one intermediate element and two outputs are chosen in two-stage model and these factors reflect the main function of hospitals. Conversion of two-stage to single-stage model is introduced. This model is proposed to fix the efficiency of a two-stage process, and avoid the dependence to various weights. Finally, the results indicated that geometry average of MPI in input oriented pure technical efficiency (PTE) in the tenth Hospital (2.1517) is introduced as the highest performance hospital with highest productivity growth.

Keywords:
DEA
MPI
Intermediate element
Two-stage model

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INTRODUCTION

Many people annually die from heart diseases. Health specialist bearing diverse investigations on heart diseases and create evidence of heart problems, many indications and disease symptoms development. Thus, there are valued materials unseen in their dataset to be mined (Mendis et al., 2011). The greatest public recycled method, Data Envelopment Analysis (DEA) created on linear programming, appeals a borderline of finest performs, displays which health administrations are efficient, gives the amount of inefficiency and directs the means of refining efficiency by giving goals estimates for each of the inputs or outputs exclusively (Lobo et al., 2010). To discover the unfamiliar inclinations in heart disease, all the accessible hospitals datasets are gathered to an exclusive model and their DMUs efficiency are matched. A dataset for 15 hospitals with two inputs, one intermediate element and two outputs in two-stage DEA model to test and justify the differences between hospitals are used. DEA authorities all unit in the data to have its private creation and then it calculates the efficiency of that solitary unit by associating it to the efficiency of the other units in the dataset.

LITERATURE REVIEW

Global rising number of patients, have inspired investigators to organize comprehensive study to expose concealed configurations in medical datasets. This unit offers an outline of preceding computational instructions on framework acknowledgment in hospitals. Not only are diverse models lectured, but also several hospitals inputs and outputs are enclosed to have a reasonable judgment. In conclusion, the gap in present literature, which was the key incentive of this study is also delivered. Some of the important studies are introduced. In specific study in hospitals, they utilized the data for the year 2005 and CCR was implemented. Inputs were "number of doctors" and "number of beds" and the outputs were "weighted admissions", "first consultations", "successive consultations", and "number of surgical interventions". In conclusion,6 efficient units were recognized (Caballer-Tarazona et al., 2010). Another researchers measured the functioning efficiency of Taiwan hospitals employing yearly data completed the period 1996-1997. "Number of patient beds", "number of physicians", "number of nurses", and "number of supporting medical members" were the inputs and "number of patient days", "number of clinic or outpatient visits", and "number of patients receiving surgery" were the outputs. Municipal hospitals be present fewer efficient (Chang et al., 2004). Giokas utilized the "total cost" in place of the solitary input and the "number of inpatient days in medical care", "number of inpatient days in surgical care", "number of outpatient visits", and "number of ancillary services" by means of outputs. According to the outcomes 37% of teaching hospitals and 15% of public hospitals were efficient. The regular efficiency ranking was 84.7% used for the teaching hospitals and 75.1% for the public hospitals. By dint of expending DEA specific beneficial material were delivered to researchers around refining a hospital's effective efficiency (Giokas, 2001). As a result, pattern recognition in hospitals can be addressed through different computational models. Regarding ranking hospitals efficiency, other respected works, focused on diverse aspects of hospitals on different models can be mentioned (Bilsel et al., 2011; Aboueljinane et al., 2013; Bhattacharjee et al., 2014; Bwana, 2015; Kawaguchi et al., 2014; Kose et al., 2014; Gul et al., 2015; Jehu et al., 2015). Also, many computational techniques for other health care issues have been reported in the three works of literature (Mirmozaffari et al., 2017a; Mirmozaffari et al., 2017b; Mirmozaffari et al., 2017c). It is observed that various DEA model are frequently utilized in different studies to compare and rank efficiency. Therefore, a comprehensive comparison of different hospitals efficiency practically provides an insight into hospital's performances. This comparison is of great importance to medical practitioners who desire to predict heart failure at a proper step of its progression. Finally, a comparison of standard input-oriented, output oriented and additive model are used. A unique utilizing MPI, a two-stage model with new approach is applied which eventually results in comparing various efficient and inefficient DMUs, covered in this study.

RESEARCH METHODOLOGY

The aim of current research is to efficiently compare hospitals productivity with 3 approaches (additive, input oriented and output oriented model). By using a reasonable method, a model was established to define the characteristics of heart hospitals in terms of some DMUs. DEA methods in this research are more trustworthy for MPI productivity assessing and classification strategies. The Banxia Frontier Analyst (Version 4.3) has been used for analyzing the information. In general, the entire course can be divided into three stages as follows:

DEA models

The CCR model

The CCR model reflects a stable or constant return to scale (CRS). In fact, a comparative rise in altogether inputs outcomes are set to the related growth in outputs. Where θ_p signifies the technical efficiency score of unit DMU, λ_j signifies the dual variables that pinpoint the benchmarks for inefficient units. If θ_p is set to one, then the surveyed DMU is considered technically efficient and lies on the efficiency frontier and is collected from the set of efficient units. DEA measures the efficiency of each reflection relative to the frontier that envelopes all the observations. Inefficient DMUs can be value-added (moved to the efficient frontier) with suggested advices for development which are the points along the frontier. The distance to the efficiency frontier delivers a measure of the efficiency. The efficiency of a specified DMU is considered using the CCR model as follows:

$$Min\ \theta_p$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_p x_{ip} \qquad , i=1 \ldots m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp} \qquad , r=1 \ldots s \tag{1}$$

$$\lambda_j \ge 0, \ j=1,\ldots,n$$

The BCC Model

On the other hand, the BCC model by Banker-Charnes-Cooper transformed the Constant Return to Scale (CRS) concept to Variable Return to Scale (VRS). The DMU controls under variable returns to scale and it is suspected that an increase in inputs does not result in a comparative change in the outputs. The BCC model splits the Technical Efficiency (TE) resulting from the CCR model into two parts:

1) Pure Technical Efficiency (PTE): PTE, which overlooks the influence of scale size by only comparing a DMU to a unit of similar scale and measures how a DMU utilizes its sources under exogenous environment.

2) Scale Efficiency (SE): SE, which measures how the scale size affects efficiency. If after applying both CRS, VRS model on the same data, there is a alteration in the two technical efficiencies, this designates that DMU has a scale efficiency and can be calculated by:

$$SE = TE/PTE \tag{2}$$

The BCC is represented as follows:

$$Min\ \theta_p$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_p x_{ip} \qquad , i=1 \ldots m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp} \qquad , r=1 \ldots s \tag{3}$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0, \ j=1,\ldots,n$$

The additive model

The additive models are often called slack based measure (SBM). In mathematical terms the AM is given as:

$$Min\ Z_. = -\sum_{r=1}^{s} s_r^+ - \sum_{i=1}^{m} s_i^-$$

$$s.t \ \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{i.} \ , i = 1, \ldots, m \tag{4}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r.} \ , r = 1, \ldots, s$$

$$\lambda_j, \ s_i^-, s_r^+ \ge 0, j=1,\ldots,n$$

The objective of the AM is to gain the maximal value of input and output slacks that are there in the given set of DMU's. This model reflects the input excess and the output shortfall simultaneously in arriving at a point on the efficient frontier (Chen et al., 2009; Chen et al., 2004).

A new approach in two-stage DEA model

In recent years, proposed various solutions regarding to the two-stage model (Chen et al., 2004; Kao et al., 2008; Chen et al., 2009a; Chen et al., 2009b; Wang et al., 2010 and Hosseinzadeh lotfi et al., 2012). A new two-stage model has been proposed in this study. In the proposed solution the two-stage model is considered as a single stage, where the intermediate elements are reflected as part of final inputs. Consider the input, intermediate element, and output are denoted by X, Z, and Y respectively, then the proposed model treats X+Z, and Y as input and output, respectively. This model is proposed to fix the efficiency of a two-stage process, and prevent the reliance to diverse weights. With respect to aforementioned methods or information the proposed model is represented in Fig. 1.

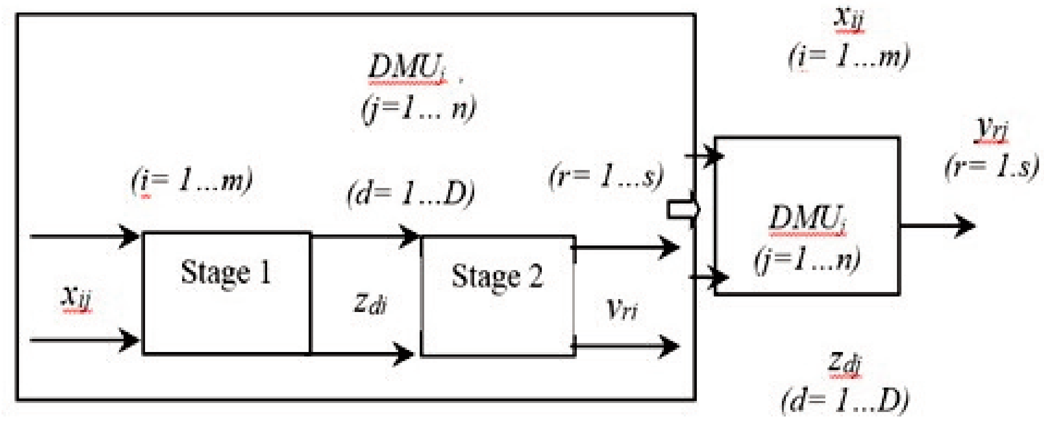

Fig.1. Conversion of two-stage model to one-stage model

Fig. 1 elaborates the proposed model. In fact, NN with ND units represent X or inputs, NB unit is Z or intermediate elements and OT with IT units are Y or outputs, which will introduce in Table 1. Finally, in a more detailed discussion $\mathrm{CCR_{IO}}$, $\mathrm{BCC_{IO}}$, $\mathrm{CCR_{OO}}$, $\mathrm{BCC_{OO}}$ and additive proposed model within all approaches, are thoroughly discussed below, at first linear $\mathrm{CCR_{IO}}$:

$$
\begin{aligned}
& Max \textstyle\sum_{r=1}^{s} u_r y_{rp} \\
& s.t. \textstyle\sum_{i=1}^{m} v_i x_{ij} + \sum_{d=1}^{D} w_d z_{dj} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, j = 1, \dots, n \\
& u_r, v_i, w_d \geq \varepsilon
\end{aligned}
\tag{5}
$$

Dual proposed model in $\mathrm{CCR_{IO}}$:

$$
\begin{aligned}
& Min\ \theta_p \\
& s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{ip} \qquad , i = 1, \dots, m \\
& \sum_{j=1}^{n} \lambda_j z_{dj} \leq \theta z_{dp} \qquad , d = 1, \dots, D \\
& \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp} \qquad , r = 1, \dots, s \\
& \lambda_j \geq 0,\ \theta_p\ free
\end{aligned}
\tag{6}
$$

Linear proposed model in $\mathrm{BCC_{IO}}$:

$$Max \sum_{r=1}^{s} u_r y_{rp} + w$$

$$s.t. \sum_{i=1}^{m} v_i x_{ij} + \sum_{d=1}^{D} w_d z_{dj} = 1 \quad (7)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{d=1}^{D} w_d z_{dj} + w \le 0, j = 1, \dots, n$$

$$u_r, v_i, w_d \ge \varepsilon, w\ free$$

$$\lambda_j,\ s_i^-, s_d^-, s_r^+ \ge 0, j=1,\dots,n$$

Dual proposed model in BCC_{IO}:

$$Min\ \varphi_p$$

$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \varphi x_{ip} \quad , i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp} \quad , r = 1, \dots, s \quad (8)$$

$$\sum_{j=1}^{n} \lambda_j z_{dj} \le \varphi z_{dp} \quad , d = 1, \dots, D$$

$$\sum_{i=1}^{n} \lambda_j = 1$$

$$\lambda_i \ge 0,\ \varphi_{..}\ free$$

Linear proposed model in CCR_{OO}

$$Min \sum_{i=1}^{m} v_i x_{ij} + \sum_{d=1}^{D} w_d z_{dj}$$

$$s.t. \sum_{r=1}^{s} u_r y_{rp} = 1 \quad (9)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{d=1}^{D} w_d z_{dj} \le 0, j = 1, \dots, n$$

$$u_r, v_i, w_d \ge \varepsilon$$

Dual proposed model in CCR_{OO}:

$$Max\ \theta_p$$

$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ip} \quad , i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge \theta y_{rp} \quad , r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j z_{dj} \le z_{dp} \quad , d = 1, \dots, D \quad (10)$$

$$\lambda_j \ge 0,\ \theta_p\ free$$

linear proposed model in BCC_{OO}:

$$Min \sum_{i=1}^{m} v_i x_{ij} + \sum_{d=1}^{D} w_d z_{dj} + w$$

$$s.t. \sum_{r=1}^{s} u_r y_{rp} = 1 \quad (11)$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp} \quad , r = 1, \dots, s$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{d=1}^{D} w_d z_{dj} + w \le 0, j = 1, \dots, n$$

$$u_r, v_i, w_d \ge \varepsilon$$

Dual proposed model in BCC_{OO}:

$$Max\ \varphi_p$$

$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ip} \quad , i = 1, \dots, m \quad (12)$$

$$\sum_{j=1}^{n} \lambda_j z_{dj} \le z_{dp} \quad , d = 1, \dots, D$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_p y_{rp} \quad , r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0,\ \varphi_p\ free$$

Linear proposed additive model:

$$Min\ Z_. = -\sum_{i=1}^{m} s_i^- - \sum_{d=1}^{D} s_d^- - \sum_{r=1}^{s} s_r^+$$

$$s.t \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{i.} \quad , i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j z_{dj} + s_d^- = z_{d.} \quad , d = 1, \dots, D \quad (13)$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r.} \quad , r = 1, \dots, s$$

$$\lambda_j,\ s_i^-, s_d^-, s_r^+ \ge 0, j=1,\dots,n$$

Dual proposed additive model:

$$Max\ Y. = \sum_{r=1}^{s} y_r.u_r - \sum_{i=1}^{m} x_i.v_i - \sum_{d=1}^{D} z_d.w_d + w$$
$$\sum_{r=1}^{s} y_r.u_r - \sum_{i=1}^{m} x_i.v_i - \sum_{d=1}^{D} z_d.w_d + w \le 0 \quad , j = 1, \dots, n \sum_{r=1}^{s} u_r \ge 1$$
$$\sum_{d=1}^{D} w_d \ge 1$$
$$\sum_{i=1}^{m} v_i \ge 1$$
$$u_r, v_i, k_d \ge \varepsilon, w\ free$$
(14)

EXPERIMENT AND REAULTS

Dataset description

The standard dataset, compiled in this study contains 6 periods (2011-2016), which is collected from 12 hospitals, under the supervision of National Health Ministry. The Number of Doctors (NN) and the Number of Beds (NB) in the hospitals are inputs in first stage. The Numbers of Nurses and secretaries (NN) is intermediate element. The Outpatient Treated (OT) and the Inpatient Treated (IT) in the hospitals are outputs in second stage. So, two inputs, one intermediate elements and two outputs for first hospital which is 2011 to 2016 are presented in Table 1.

Table 1: The datasets for 6 DMDs in the first hospital

Period	First Input (NN)	Second Input (ND)	Intermediate Element (NB)	First Output (OT)	Second Output (IT)
2011	16200	200	1450	25300	5693
2012	11046	158	6100	17200	4463
2013	15000	260	560	24100	5000
2014	13601	188	7100	16100	1800
2015	14300	201	2450	17693	4500
2016	12100	303	482	27500	9800

Evaluation in MPI for hospitals

The MPI is computed to evaluate productivity growth relative to a reference technology. Two main issues are addressed in computation of MPI growth. The first issue is the measurement of productivity change over the period, while the second is to decompose changes in productivity into what are generally denoted to as a ‘catching-up’ effect or technical efficiency change (TEC) and a ‘frontier shift’ effect or technological change (TC). MPI evaluates the total factor productivity change of a DMU between two periods. The concept of productivity usually referred to labor productivity, this concept is very much related to TFP, defined as the product of efficiency change (catch-up) and technological change (frontier-shift). If TFP value is greater than one this indicates a positive TFP growth from period (t) to period (t+1), whereas a value less than one shows a decrease in TFP growth or performance relative to the previous year. The frontier obtained in the current (t) and future (t+1) time periods are labeled accordingly. When inefficiency exists, the relative movement of any given DMU over time will therefore depend on both its position relative to the corresponding frontier (technical efficiency) and the position of the frontier itself (technical change), In fact:

$$\text{MPI} = \text{TEC} \times \text{TC} \tag{15}$$

The productivity change is explored by calculating the MPI: technical efficiency change (TEC) and technological change (TC). Table 2 and Fig. 2 present the results of input-oriented TEC, TC and MPI for TE and PTE in the First hospital during the period from 2011 to 2016.

Table 2: Input oriented TEC, TC and MPI for TE and PTE in the first hospital

Period	TEC (Catch-up) For TE	TC (Frontier-Shift) For TE	MPI For TE	TEC (Catch-up) For PTE	TC (Frontier Shift) For PTE	MPI
011-2012	0.6707	1.2257	0.8221	0.6732	1.1333	0.7629
2012-2013	0.4421	2.3493	1.0386	0.8805	1.3153	1.1581
2013-2014	2.3698	0.2397	0.568	1.1831	0.4343	0.5139
2014-2015	0.6858	1.5652	1.0734	0.7817	1.2803	1.0009
2015-2016	1.9859	1.0032	1.9922	1.824	1.0097	1.8416
Geo. Avg.	**1.2308**	**1.2766**	**1.0988**	**1.0685**	**1.03458**	**1.0554**
Std.	0.8803	0.7724	0.5931	0.5976	0.35703	0.5025
CV	0.7152	0.60504	0.53977	0.55930	0.34509	0.4761
Min	0.4421	0.2397	0.5680	0.6732	0.4343	0.5139
Max	2.3698	2.3493	1.9922	1.824	1.3153	1.8416

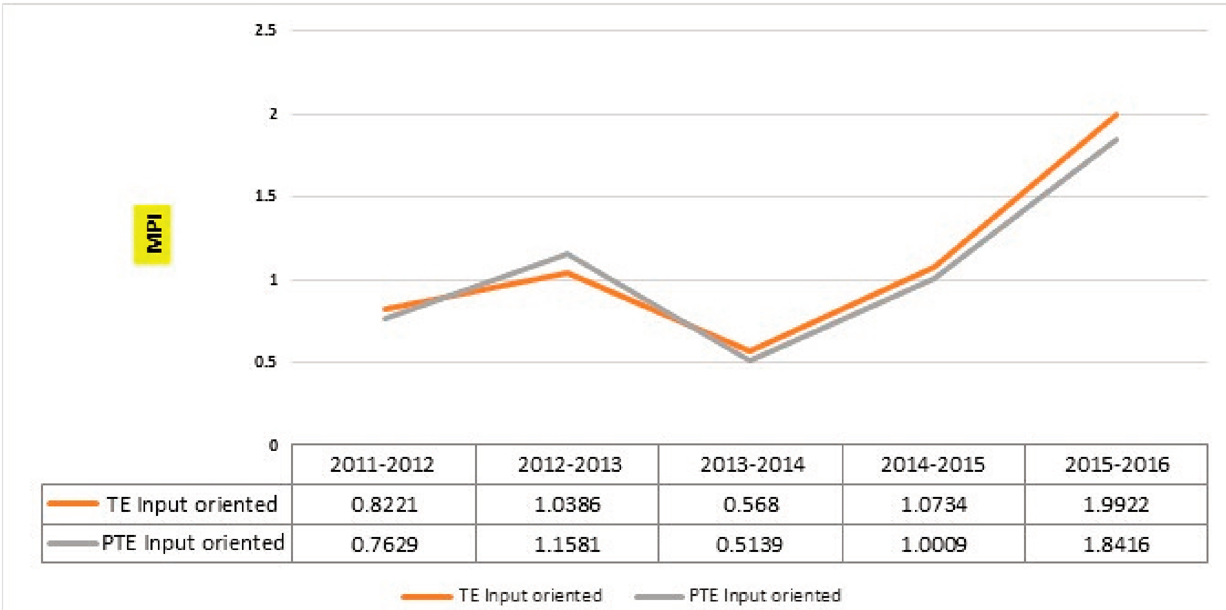

Fig. 2 . A comparison of input oriented MPI for TE and PTE in first hospital

Table 3 and Fig.3 present the results of output-oriented TEC, TC and MPI for TE and PTE in the first hospital during the period from 2011 to 2016.

Table 3 : Output oriented TEC, TC and MPI for TE and PTE in the first hospital

Period	TEC (Catch-up) For TE	TC (Frontier-Shift) For TE	MPI For TE	TEC (Catch-up) For PTE	TC (Frontier Shift) For PTE	MPI
011-2012	0.6707	1.2257	0.8221	0.8116	0.9418	0.7643
2012-2013	0.4421	2.3493	1.0386	0.5176	2.3962	1.2403
2013-2014	2.3698	0.2397	0.568	1.7717	0.2865	0.5076
2014-2015	0.6858	1.5652	1.0734	1.0336	1.0584	1.0940
2015-2016	1.9859	1.0032	1.9922	1.2999	1.1780	1.5314
Geo. Avg.	**1.2308**	**1.2766**	**1.0988**	**1.0868**	**1.1721**	**1.0275**
Std.	0.880381	0.772411	0.593132	0.597615	0.35703	0.502595
CV	0.7152	0.60504	0.5397	0.5593	0.34509	0.4761
Min	0.4421	0.2397	0.5680	0.5176	0.2865	0.5076
Max	2.3698	2.3493	1.9922	1.7717	2.3962	1.5314

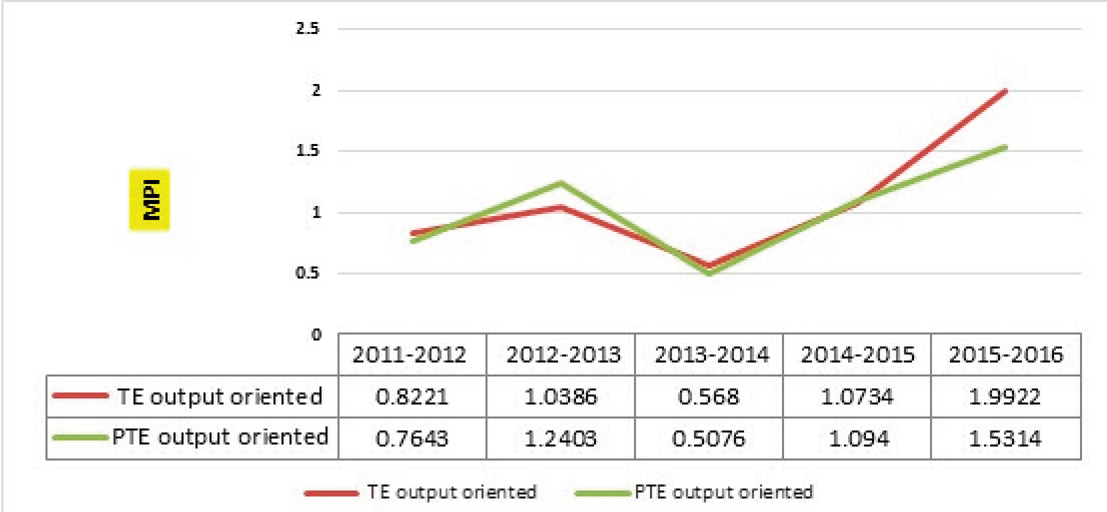

Fig. 3. A comparison of output oriented MPI for TE and PTE in first hospital

Table 4 and Fig. 4 show the results of additive model TEC, TC and MPI for TE and PTE in the first hospital during the period from 2011 to 2016.

Table 4: Additive model TEC, TC and MPI for TE and PTE in the first hospital

Period	TEC (Catch-up) For TE	TC (Frontier-Shift) For TE	MPI For TE	TEC (Catch-up) For PTE	TC (Frontier Shift) For PTE	MPI
011-2012	0.5951	1.6426	0.9774	0.6733	0.9892	0.6660
2012-2013	0.6158	1.4097	0.8681	0.4331	3.8642	1.6737
2013-2014	0.9367	1.4229	1.3328	0.8474	0.5042	0.4273
2014-2015	1.2152	0.8301	1.0088	1.5852	0.5680	0.9004
2015-2016	1.6366	0.6543	1.0709	2.5527	0.7276	1.8575
Geo. Avg.	**0.9998**	**1.1919**	**1.0516**	**1.2183**	**1.3304**	**1.1049**
Std.	0.880381	0.772411	0.593132	0.597615	0.35703	0.502595
CV	0.71525	0.60504	0.53971	0.5593	0.34509	0.4761
Min	0.5951	0.6543	0.8681	0.4331	0.5042	0.4273
Max	1.6366	1.6426	1.3328	2.5527	3.8642	3.8642

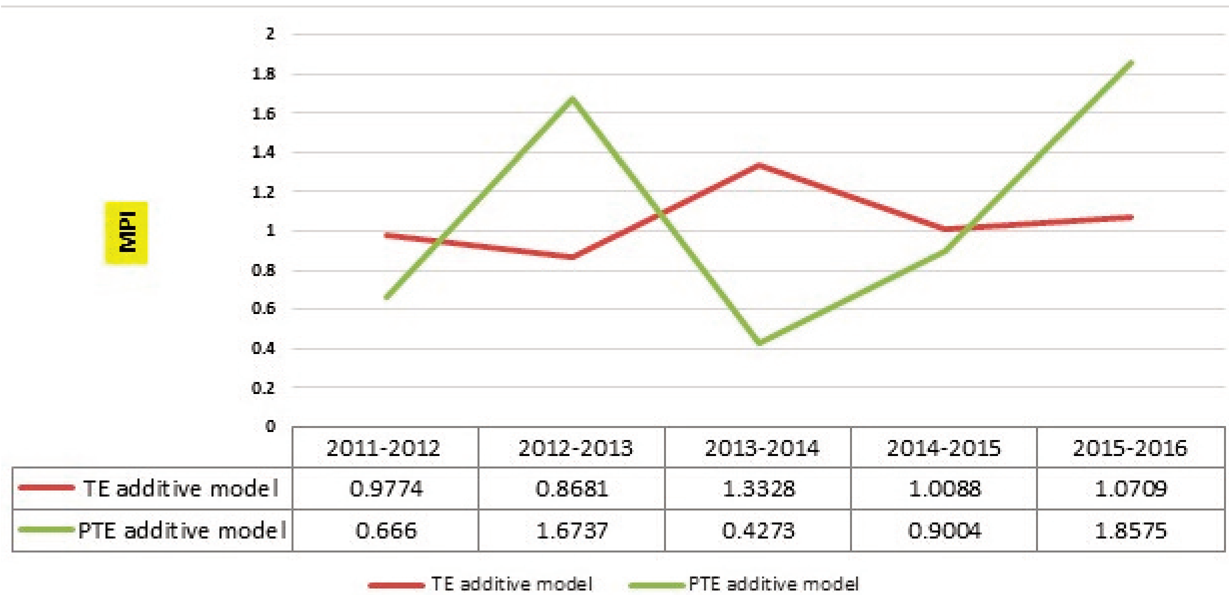

Fig. 4. A comparison of additive model MPI for TE and PTE in first hospital

In input oriented, the minimum value of MPI in TE is 0.568, while the maximum value is 1.9922. Further, it is noted that TE in the first hospital had the larger geometric average of MPI compare with PTE, with a growth of 9.886%. This productivity increase was attributed to technological change growth of 27.662%, since the technical efficiency change is 23.086%. While PTE had a geometric average MPI increase of 5.055%. This productivity increase was attributed to technological change growth of 3.458% and the technical efficiency change is 6.85%. All aforementioned analyze can be inferred from output oriented and additive model. In a more detail discussion for both TE and PTE in input oriented, output oriented and additive model three-step are considered from 2011 to 2016:

- Except for TE in additive mode, an increase in productivity during 2011 to 2013. For example in input oriented, increasing MPI from 0.8221 to 1.0386 for TE and from 0.7629 to 1.1581 for PTE are represented.
- Except for TE in additive mode, a decrease during 2013 to 2014. For example in input oriented, decreasing MPI from 1.0386 to 0.568 for TE and decreasing MPI from 1.1581 to 0.5139 for PTE are introduced.
- Except for TE in additive mode, an increase during 2014 to 2016. For example, in input oriented, increasing MPI from 0.568 to 1.0734 and finally increasing from 1.0734to 1.9922 for TE and increasing from 0.5139 to 1.0009 and finally increasing from 1.0009 to 1.8416.

Moreover, MPI geometric average growth among all input oriented, output oriented and additive model are considered and finally:

- MPI geometric average growth for PTE in additive model with growth of 10.49% is introduced as a model which has highest productivity growth.
- MPI geometric average growth for PTE in output oriented model with growth of 2.75% is introduced as a model which has lowest productivity growth.

In a more detail discussion of Table 2, 3 and 4, Table 5, 6 and 7 compares geometric average of MPI, TEC and TC in all hospitals for TE and PTE input oriented, output oriented and additive model.

Table 5:Input oriented geometric average of MPI, TEC and TC in all of the hospitals for TE and PTE

Hospital	TEC (Catch-up) For TE	TC (Frontier-Shift) For TE	MPI For TE	TEC (Catch-up) For PTE	TC (Frontier Shift) For PTE	MPI
1	1.2308	1.2766	1.0988	1.0685	1.0345	1.0554
2	1.0566	1.1853	1.1383	1.0894	1.1514	1.1804
3	1.1824	1.3587	1.4599	1.1400	1.1319	1.2088
4	0.9836	1.3572	1.3902	1.0008	1.2232	1.3035
5	1.3008	0.9955	1.3071	1.0384	1.2570	1.4312
6	1.0087	1.2739	1.3939	1	1.3979	1.3979
7	1.1588	1.1717	1.3694	1.1279	1.1535	1.3216
8	1.2703	1.1374	1.3744	1.1625	1.1021	1.3255
9	0.9479	1.2369	1.0476	0.9334	1.2561	1.1811
10	1.1319	1.5961	1.4456	1.0914	2.0892	2.1517
11	0.9514	0.9380	0.8897	1	1.1764	1.1764
12	1.0140	1.7493	1.7852	1	1.1115	1.1115
13	1.2308	1.2766	1.0988	1.0685	1.0345	1.0554
14	1.0087	1.2739	1.3939	1	1.3979	1.3979
15	0.9836	1.3572	1.3902	1.0008	1.2232	1.3035
Avg.	**1.0973**	**1.2789**	**1.3055**	**1.0481**	**1.2493**	**1.3067**

Table 6: Output oriented geometric average of MPI, TEC and TC in all of the hospitals for TE and PTE

Hospital	TEC (Catch-up) For TE	TC (Frontier-Shift) For TE	MPI For TE	TEC (Catch-up) For PTE	TC (Frontier Shift) For PTE	MPI
1	1.2308	1.2766	1.0988	1.0868	1.1721	1.0275
2	1.0566	1.1853	1.1383	1.1151	1.1010	1.0415
3	1.1824	1.3587	1.4599	1.0220	1.0985	1.1317
4	0.9836	1.3572	1.3902	0.9902	1.0958	1.0959
5	1.3008	0.9955	1.3071	1.2636	1.0271	1.2951
6	1.0087	1.2739	1.3939	1	1.0205	1.0205
7	1.1588	1.1717	1.3694	1.1004	1.1287	1.1985
8	1.2703	1.1374	1.3744	1.1020	1.0357	1.0572
9	0.9479	1.2369	1.0476	0.9537	1.1275	0.9443
10	1.1319	1.5961	1.4456	1.0166	0.9313	0.9459
11	0.9514	0.9380	0.8897	1	1.0368	1.0368
12	1.0140	1.7493	1.7852	1.0034	1.0422	1.0408
13	1.2308	1.2766	1.0988	1.0868	1.1721	1.0275
14	1.0087	1.2739	1.3939	1	1.0205	1.0205
15	0.9836	1.3572	1.3902	0.9902	1.0958	1.0959
Avg.	**1.0973**	**1.2789**	**1.3055**	**1.0487**	**1.0737**	**1.0653**

Table 7: Additive model geometric average of MPI, TEC and TC in all of the hospitals for TE and PTE

Hospital	TEC (Catch-up) For TE	TC (Frontier-Shift) For TE	MPI For TE	TEC (Catch-up) For PTE	TC (Frontier Shift) For PTE	MPI
1	0.9998	1.1919	1.0516	1.2183	1.3306	1.1049
2	1.1664	1.4818	1.0508	1.5116	2.1173	0.9876
3	1.3641	1.3609	1.1997	1.5275	1.2856	1.2468
4	2.0983	1.4667	1.4779	1.9385	1.1752	1.4007
5	1.2309	1.0833	1.3423	1.3144	1.4284	1.4284
6	1.0272	1.0085	1.0234	1	1.0141	1.0141
7	1.8570	1.8835	1.1698	1.7324	1.2686	1.2686
8	1.5387	1.2428	1.4425	1.4649	1.2905	1.2905
9	1.1124	1.1089	0.9774	1.6056	1.4216	1.4216
10	1.3450	1.2557	1.1301	2.4040	1.0876	1.0876
11	0.9358	1.0427	0.9648	1	1	1
12	1.0884	1.0846	1.1457	1.0242	0.9445	0.9445
13	0.9998	1.1919	1.0516	1.2183	1.3306	1.1049
14	1.0272	1.0085	1.0234	1	1.0141	1.0141
15	2.0983	1.4667	1.4779	1.9385	1.1752	1.4007
Avg.	**1.3259**	**1.2586**	**1.1685**	**1.4598**	**1.31508**	**1.181**

The average geometry of TEC, TC and MPI for TE, PTE in all hospitals, in input oriented, output oriented and additive model corresponds to each geometric average are considered and finally:

- Geometry average of MPI in input oriented PTE in the tenth hospital (2.1517) is considered as the only MPI which is higher than 2 and has highest MPI among all PTE and TE in input oriented, output oriented and additive model.
- Geometry average of MPI in input oriented and output oriented TE in the twelfth hos-

pital (1.7852) is considered as the two MPI which have second place for highest MPI among all PTE and TE in input oriented, output oriented and additive model.

- Geometry average of MPI in input oriented and output oriented TE in the eleventh hospital (0.8897) is considered as the only MPI which has lowest MPI among all of PTE and TE in input oriented, output oriented and additive model.

- MPI geometric average growth from all hospitals for PTE in input oriented model with growth of 30.67% is introduced as a model which has highest productivity growth among three suggested models.
- MPI geometric average growth from all hospitals for PTE in output oriented model with growth of 6.53% is introduced as a model which has lowest productivity growth among three suggested models.

CONCLUSION

Various heart hospitals efficiency in DEA were compared. In a more detail discussion, First hospital during the period from 2011 to 2016, are evaluated. A unique model consisting of converting two-stage model to a novel one stage model and evaluation methods are evolved. Input-oriented, output oriented and additive model with TE, PTE and SE as well as MPI with TEC and TC evaluation methods, are applied to find the superior hospitals. MPI in Tenth hospital (2.1517) in PTE input oriented is presented as the highest performance between hospitals. MPI in input oriented and output oriented TE in the eleventh hospital (0.8897) is introduced as the lowest performance between hospitals.

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