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Ridge Regression With Intuitionistic Fuzzy Input and Output: A Parametric Approach

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A B S T R A C T

Ridge regression is a model that is frequently used and has numerous effective applications, particularly in the management of correlated factors in a multiple regression model. Additionally, multicollinearity poses a significant risk in fuzzy regression models when it comes to predictions. In order to solve this problem, we bring together the fuzzy regression model with the ridge regression technique. Regarding the evaluation of the coefficients of the ridge fuzzy regression model, the algorithm that we have suggested makes use of the parametric estimation approach. In this article, we examine the ridge regression in the intuitionistic fuzzy environment. We assume that the input and output data are intuitionistic fuzzy numbers. Since in the regression analysis we need to calculate the distance between the variables, we define a new fuzzy parametric distance. Also, the goodness of fit of the model with the indicators of the mean square of the prediction error has been investigated in simulation examples and real data.

1. Introduction

In statistics, linear regression is a linear model approach between the response variable and one or more explanatory variables. Regression is often used to discover the model of linear relationship between variables. In this case, it is assumed that one or more descriptive variables whose value is independent of the other variables or under the control of the researcher, can be effective in predicting the response variable whose value is not dependent on the descriptive variables and under the control of the researcher. The purpose of regression analysis is to identify the linear model of this relationship. Since in the real world we often face imprecise data, it is better to use fuzzy logic to model the inherent uncertainty in these data. In a general division, the types of fuzzy regression can be divided into the following three models:

- Fuzzy regression in the case where the relationship between the variables is assumed to be fuzzy. In other words, the regression equation coefficients are considered fuzzy.

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- Fuzzy regression in the case that the variables (either prediction or response) are imprecise and fuzzy.
- Fuzzy regression in the case that both variables and coefficients of the model are considered fuzzy.

It should be mentioned that the variation in fuzzy regression is not limited to the above modes. Rather, the methods that have been proposed for each mode have created a lot of diversity in fuzzy regression methods. Like classical regression, which is based on the principle or principles on which model parameters are estimated, fuzzy regression can be divided into possible regression methods, regression of the least squares of the fuzzy error, and regression of the least absolute magnitude of the fuzzy error.

Hassamian et al. [10] investigated fuzzy non-parametric regression model with fuzzy responses and exact predictors. Li et al. [14] investigated fuzzy multiple linear regression with LR numbers. In it, they presented a calculation formula for regression parameters and introduced two new distances between fuzzy numbers. Authors of Flores-Sosa et al. [7] applied ordinary least squares and ordered weighted average to solve multiple linear regression. In Durso and Chachi [5], the authors proposed Ordered Weighted Averaging to solve regression models with exact/fuzzy inputs and fuzzy output. Hasanpour et al. [9] estimated the fuzzy using diamond meter. They tried to make the estimates as close to the real value as possible with the ideal planning method and by comparing the error of their method with the error of Diamond [4] method, they showed the superiority of their method. Rabiei et al. [16] using the fuzzy regression method for a set of data with input and fuzzy output.

Intuitionistic fuzzy sets (IFSs) were first introduced by Atanassov [2] as an extension of Zadeh's fuzzy sets [23]. These IFSs serve as a mathematical framework for representing sets that are non-crisp and characterized by uncertainty. In the context of these sets, we establish functions that determine both membership and non-membership. In this particular scenario, it is possible to establish a hesitation function, which can be defined as the disparity between the "membership function" and the "one minus non-membership function." Through the use of Interval Type-2 Fuzzy Sets (IFSs), we are able to effectively represent and analyze incomplete information. Numerous scholars have made significant contributions to the field of Interval Type-2 Fuzzy Systems (IFSS) in terms of both theoretical advancements and practical implementations (see Szmidt [18] for more details). Akram et al. [1] introduced a unique decision-making approach using hypergraphs within the context of intuitionistic fuzzy environments. This technique was subsequently used in practical scenarios [17]. A hybrid technique that is based on recurrent neural networks was published in Karbasi et al., [11] for the purpose of approximating the coefficients (parameters) of a ridge fuzzy regression model that has LR-fuzzy output and crisp inputs. This problem was solved in Choi et al. [3] by the use of alpha-level estimation approach. The authors of reference Kim and Jung [13] developed a fuzzy ridge estimator that is independent of the distance between fuzzy numbers.

The use of distance and similarity measurements is crucial for identifying the dissimilarities between two entities. Decision making, pattern recognition, image processing, machine learning, market prediction, and so on are just some of the numerous possible uses for distance and similarity measures. Wang [21] first presented a computational formula for the similarity measure of fuzzy collections. Many scholars have taken an interest in this issue ever then and have gone into further depth. Different fuzzy set, intuitionistic fuzzy set, and fuzzy multiset distance and similarity metrics have been presented. There are a number of distance measures in common usage, but three of the most well-known are the Hamming distance, the Euclidean distance, and the Housdorff distance. The authors of Li et al. [15] examine the fuzziness of fuzzy sets, similarity measures, and connection measures. It was investigated by Ejegwa and Adamu [6] how far apart two intuitionistic fuzzy sets of second type may be. Weighted distance measure for intuitionistic fuzzy sets using the Choquet integral with regard to the non-monotonic fuzzy measure was presented by Torra and Narukawa [20]. Distance and similarity measures based on hesitant fuzzy sets were expanded by Xia and Xu [22].

Ridge regression is a method of estimating the coefficients of multiple-regression models in scenarios where the independent variables are highly correlated. It has been used in many fields including econometrics, chemistry, and engineering. Also known as Tikhonov regularization, named for Andrey Tikhonov, it is a method of regularization of ill-posed problems. It is particularly useful to mitigate the problem of multicollinearity in linear regression, which commonly occurs in models with large numbers of parameters Kennedy [12]. In general, the method provides improved efficiency in parameter estimation problems in exchange for a tolerable amount of bias [8].

When it comes to fuzzy multiple regression models, multicollinearity is a major concern, just as it is in conventional statistical theory when it comes to traditional multiple regression models. We offer the ridge fuzzy regression model, which is a combination of the ridge regression model and the fuzzy regression model. The purpose of this model is to lessen the impact of multicollinearity when it is present. We offer a parametric estimation approach that is based on the parametric distance measure in order to generate the ridge fuzzy regression model. Among the prominent features of the proposed meter is that decision makers can determine the distance between two fuzzy numbers based on their decision level by choosing the appropriate alpha values. And as a result, the regression model can be determined by changing the parameters of this distance according to the level of decision-making desired by the decision-makers. The remaining parts of this study are as follows:

The basics of IFSs, IFNs and related arithmetic operators are presented in Section 2. In this section, a new parametric approach for the distance of IFNs is proposed in general using the concepts of decision level and degree of uncertainty. The advantages of the proposed method are shown through a suitable example. In section 3, we will review the concepts of regression and estimation of regression parameters using the meter defined in the previous section. In the next section, we introduce ridge regression to estimate model parameters with an example.

2. Fuzzy preliminaries

We summarize below the basic concepts of intuitionistic fuzzy set theory, such as intuitionistic fuzzy numbers, intuitionistic fuzzy arithmetic, and the ranking of intuitionistic fuzzy numbers.

Definition 1. Suppose $\mathcal{M}$ is the universal set, then $\tilde{A}$ is a fuzzy set by the following representation:

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | 0 \le \mu_{\tilde{A}}(x) \le 1, x \in \mathcal{M}\}, \tag{1}$$

where $\mu_{\tilde{A}}(x)$ correspond to membership value of each element of universal set respect to $\tilde{A}$ and is defined according to relation (2):

$$\mu_{\tilde{A}}: X \to [0,1], (x \in \mathcal{M}, \mu_{\tilde{A}}(x) \in [0,1]). \tag{2}$$

Definition 2. The intuitionistic fuzzy set $\tilde{A}$ has the following representation

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x)\rangle | 0 \le \mu_{\tilde{A}}(x) + \upsilon_{\tilde{A}}(x) \le 1, x \in \mathcal{M}\}, \tag{3}$$

where $\mu_{\tilde{A}}(x)$ and $\upsilon_{\tilde{A}}(x)$ respectively correspond to membership and non membership values and are defined according to equations (4) and (5):

$$\mu_{\tilde{A}}: X \to [0,1], (x \in \mathcal{M}, \mu_{\tilde{A}}(x) \in [0,1]). \tag{4}$$

$$\nu_{\tilde{A}}: X \to [0,1], (x \in \mathcal{M}, \nu_{\tilde{A}}(x) \in [0,1]). \tag{5}$$

A function $\pi_{\tilde{A}}(x)$ is called hesitancy function for each $x \in \mathcal{M}$ can be represented by relation:

$$\pi_{\tilde{A}}(x) = 1 - (\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x). \tag{6}$$

It is clear that the value of $\pi_{\tilde{A}}(x)$ is a number between zero and one.

Definition 3. For two intuitionistic fuzzy sets $\tilde{A}_1$ and $\tilde{A}_2$ in universal set $\mathcal{M}$, the following propositions are valid:

1. $\tilde{A}_1 \subseteq \tilde{A}_2 \Leftrightarrow \forall x \in \mathcal{M}\ \ \mu_{\tilde{A}_1}(x) \le \mu_{\tilde{A}_2}(x), \upsilon_{\tilde{A}_1}(x) \ge \upsilon_{\tilde{A}_2}(x)$
2. $\tilde{A}_1 = \tilde{A}_2 \Leftrightarrow \tilde{A}_1 \subseteq \tilde{A}_2, \tilde{A}_2 \subseteq \tilde{A}_1$
3. $\tilde{A}_1^c = \{\langle x, \upsilon_{\tilde{A}_1}(x), \mu_{\tilde{A}_1}(x)\rangle | x \in \mathcal{M}\}$

Definition 4. An arbitrary intuitionistic fuzzy number (IFN) such as $\tilde{A}$ defines an intuitionistic fuzzy set on the axis of real numbers that membership and non-membership functions are introduced corresponding to relations (7) and (8):

$$\mu_{\tilde{A}}(x)=\begin{cases}L(p^{-1}(a-x)), & x\le a\\ R(q^{-1}(x-a)), & x>a\end{cases} \quad (7)$$

$$\upsilon_{\tilde{A}}(x)==\begin{cases}1-L(p'^{-1}(a-x)), & x\le a\\ 1-R(q'^{-1}(x-a)), & x>a\end{cases} \quad (8)$$

where L and R are two continuous and strictly decreasing function from $\mathbb{R}^+$to $[0,1]$, and $L(0)=R(0)=1$. Also p, p' and q, q' are called respectively left and right spreads and $p'>p, q<q'$. A LR type IFN is denoted by $\tilde{A}=(a,p,q,p',q')$.

When $L(x)=R(x)=\max(0,1-x)$, we obtain a special type of IFNs called triangular intuitionistic fuzzy numbers (TriIFN), which are known as an important class of intuitionistic fuzzy numbers.

Definition 5. Let $\tilde{A}_1=(m_1,s_1,s_2,s'_1,s'_2)$. and $\tilde{A}_2=(m_2,l_1,l_2,l'_1,l'_2)$. are two TriIFNs and λ is an arbitrary positive number. Then,

1. $\tilde{A}_1\oplus\tilde{A}_2=(m_1+m_2,s_1+l_1,s_2+l_2,s'_1+l'_1,s'_2+l'_2)$.
2. $\lambda\tilde{A}_1=(\lambda m_1,\lambda s_2,\lambda s_2,\lambda s'_1,\lambda s'_2)$.

Definition 6. Let $\tilde{A}=(a,b,c,b',c')$ be an TriIFN. α cut, β cut and (α,β) cut for $\tilde{A}$ are respectively:

1. $\tilde{A}_\alpha=\{x|\mu_{\tilde{A}}(x)\ge\alpha\}=[a+\alpha(b-a),c(b-c)]$,
2. $\tilde{A}_\beta=\{x|\nu_{\tilde{A}}(x)\le\beta\}=[a'+(1-\beta)(b'-a'),c'+(1-\beta)(b'-c')]$,
3. $\tilde{A}_{\alpha,\beta}=\{x|\mu_{\tilde{A}}(x)\ge\alpha,\nu_{\tilde{A}}(x)\le\beta\}=[a+\alpha(b-a),c(b-c)]\cap[a'+(1-\beta)(b'-a'),c'+(1-\beta)(b'-c')]$,

In the following, we express the parametric form of a fuzzy number.

Definition 7. Parametric form of a fuzzy number including two functions $(\underline{u}_1,\underline{u}_2), 0\le r\le 1$ which apply in the following conditions:

1. $\underline{u}$ is a bounded left continuous non-decreasing function over $[0,1]$.
2. $\bar{u}$ is a bounded left continuous non-increasing function over $[0,1]$.
3. $\underline{u}\le\bar{u}$.

Similar to Definition 7, we can also define parametric IFNs. The parametric form of an IFN $\tilde{A}$ is as $(\underline{u},\bar{u},\underline{v},\bar{v})$ where $\underline{u},\bar{u},\underline{v}$ and $\bar{v}$ which apply in the conditions of Definition 7.

Now we can define a parametric distance measure between two numbers according to what was said about the representation of fuzzy numbers. Also, according to the mentioned materials, a TriIFN can be displayed as follows: (μ',υ') where $\mu'(r)=(\underline{p}(r),\bar{p}(r))=(a-p(1-r),a+q(1-r))$ and $\nu'(r)=(\underline{q}(r),\bar{q}(r))=(a-p'r,a+q'r)$.

Definition 8. Let $\tilde{A}=(\underline{u}_1,\bar{u}_1,\underline{v}_1,\bar{v}_1)$ and $\tilde{B}=(\underline{u}_2,\bar{u}_2,\underline{v}_2,\bar{v}_2)$ be two IFNs. The parametric distance between two numbers is defined as follows:

$$d_{\alpha,\beta}(\tilde{A},\tilde{B})=\left[\!\left[\int_\alpha^1(\underline{u}_1-\underline{u}_2)^2+(\bar{u}_1-\bar{u}_2)^2dr+\int_0^\beta(\underline{v}_1-\underline{v}_2)^2+(\bar{v}_1-\bar{v}_2)^2dr\right]\!\right]^{\frac{1}{2}} \quad (9)$$

Theorem 1. The defined function (9) is a meter, which means it applies to the following properties:

1. $d_{\alpha,\beta}(\tilde{A},\tilde{B})\ge 0, d_{\alpha,\beta}(\tilde{A},\tilde{A})=0$.
2. $d_{\alpha,\beta}(\tilde{A},\tilde{B})=d_{\alpha,\beta}(\tilde{B},\tilde{A})$.
3. $d_{\alpha,\beta}(\tilde{A},\tilde{C})\le d_{\alpha,\beta}(\tilde{A},\tilde{B})+d_{\alpha,\beta}(\tilde{B},\tilde{C})$.

Proof. Items 1 and 2 are clear. For item 3, we have:

$$d_{\alpha,\beta}(\tilde{A},\tilde{C}) = \left[\!\left[\int_\alpha^1 (\underline{u}_{\tilde{A}} - \underline{u}_{\tilde{C}})^2 + (\bar{u}_{\tilde{A}} - \bar{u}_{\tilde{C}})^2 dr + \int_0^\beta (\underline{\nu}_{\tilde{A}} - \underline{\nu}_{\tilde{C}})^2 + (\bar{\nu}_{\tilde{A}} - \bar{\nu}_{\tilde{C}})^2 dr\right]\!\right]^{\frac{1}{2}}$$

$$= \left[\!\left[\int_\alpha^1 (\underline{u}_{\tilde{A}} - \underline{u}_{\tilde{C}} + \underline{u}_{\tilde{B}} - \underline{u}_{\tilde{B}})^2 + (\bar{u}_{\tilde{A}} - \bar{u}_{\tilde{C}} + \bar{u}_{\tilde{B}} - \bar{u}_{B})^2 dr + \int_0^\beta (\underline{\nu}_{\tilde{A}} - \underline{\nu}_{\tilde{C}} + \underline{\nu}_{\tilde{B}} - \underline{\nu}_{\tilde{B}})^2 + (\bar{\nu}_{\tilde{A}} - \bar{\nu}_{\tilde{C}} + \bar{\nu}_{\tilde{B}} - \bar{\nu}_{\tilde{B}})^2 dr\right]\!\right]^{\frac{1}{2}}$$

$$\leq \left[\!\left[\int_\alpha^1 (\underline{u}_{\tilde{A}} - \underline{u}_{\tilde{B}})^2 + (\underline{u}_{\tilde{B}} - \underline{u}_{\tilde{C}})^2 + (\bar{u}_{\tilde{A}} - \bar{u}_{\tilde{B}})^2 + (\bar{u}_{\tilde{B}} - \bar{u}_{\tilde{C}})^2 dr + \int_0^\beta (\underline{\nu}_{\tilde{A}} - \underline{\nu}_{\tilde{B}})^2 + (\underline{\nu}_{\tilde{B}} - \underline{\nu}_{\tilde{C}})^2 + (\bar{\nu}_{\tilde{A}} - \bar{\nu}_{\tilde{B}})^2 + (\bar{\nu}_{\tilde{B}} - \bar{\nu}_{\tilde{C}})^2 dr\right]\!\right]^{\frac{1}{2}} = d_{\alpha,\beta}(\tilde{A},\tilde{B}) + d_{\alpha,\beta}(\tilde{B},\tilde{C})$$ □

Assume $\tilde{A} = (a,p,q,p',q')$ and $\tilde{B} = (b,t,s,t',s')$be two LR IFNs. Then

$$d^2{}_{\alpha,\beta}(\tilde{A},\tilde{B}) = 2(a-b)^2 + (t-p)^2 \int_\alpha^1 (R^{-1}(r))^2\, dr + (t'-p')^2 \int_0^\beta (R^{-1}(r))^2 dr + (q-s)^2 \int_\alpha^1 (L^{-1}(r))^2\, dr + (q'-s')^2 \int_0^\beta (L^{-1}(r))^2\, dr + (a-b)(t-p) \int_\alpha^1 (R^{-1}(r))\, dr + (a-b)(t'-p') \int_0^\beta (R^{-1}(r)) dr - (a-b)(s-q) \int_\alpha^1 (L^{-1}(r))\, dr + (a-b)(s'-q')^2 \int_0^\beta (L^{-1}(r)) dr$$

In a special case where two numbers are triangular, we have:

$$d^2{}_{\alpha,\beta}(\tilde{A},\tilde{B}) = 2(a-b)^2[1-\alpha] + (t-p)^2\left(\frac{(1-\alpha)^3}{3}\right) + (t'-p')^2\left(\frac{\beta^3}{3}\right) + (q-s)^2\left(\frac{(1-\alpha)^3}{3}\right) + 2(a-b)^2\beta + (q'-s')^2\left(\frac{\beta^3}{3}\right) + 2(a-b)(t-p)(\frac{1}{2} - \alpha + \frac{\alpha^2}{2}) - 2(a-b)(s'-q')\left(\frac{\beta^2}{2}\right)$$

With simplify the above equation we have:

$$d^2{}_{\alpha,\beta}(\tilde{A},\tilde{B}) = 2(a-b)^2(1-\alpha+\beta) + [(t-p)^2 + (q-s)^2]\left(\frac{(1-\alpha)^3}{3}\right) + [(t'-p')^2 + (q'-s')^2]\left(\frac{\beta^3}{3}\right) + (1-\alpha)^2(a-b)\ [(t-p)-(s-q)] + \beta^2(a-b)\ [(t'-p')-(s'-q')]$$

Example 1. Let $\tilde{A} = (4{,}0.5{,}1{,}1.5{,}2.3)$ and $\tilde{B} = (8{,}3{,}1{,}2{,}1)$ be two TriIFNs. In this case, we have:

$$d^2{}_{\alpha,\beta}(\tilde{A},\tilde{B}) = 32(1-\alpha+\beta) + 6.25\left(\frac{(1-\alpha)^3}{3}\right) + 1.94\left(\frac{\beta^3}{3}\right) + 2.5(1-\alpha)^2 + 1.8\beta^2$$

Table 1 shows the value of distance $\tilde{A}$ and $\tilde{B}$ for different α and β . Also, in Figure 1, you can see the membership function of two IFNs $\tilde{A}$ and $\tilde{B}$. that alpha and beta represent different levels of decision making. As alpha increases, a higher decision level is selected, and conversely, if alpha is a lower value, it means that this problem is solved at a lower level of decision making. It is the opposite for beta, i.e. a low beta value indicates problem solving at a higher level of decision making. The data in the Table 1 shows that the distance between these two fuzzy numbers for members of the set that have a membership function of 0.8 and a non-membership function of 0.2 is equal to 0.023 (second row of the table). The members of the set whose membership is less than 0.4 and their non-membership is 0.8 is estimated at 0.21.

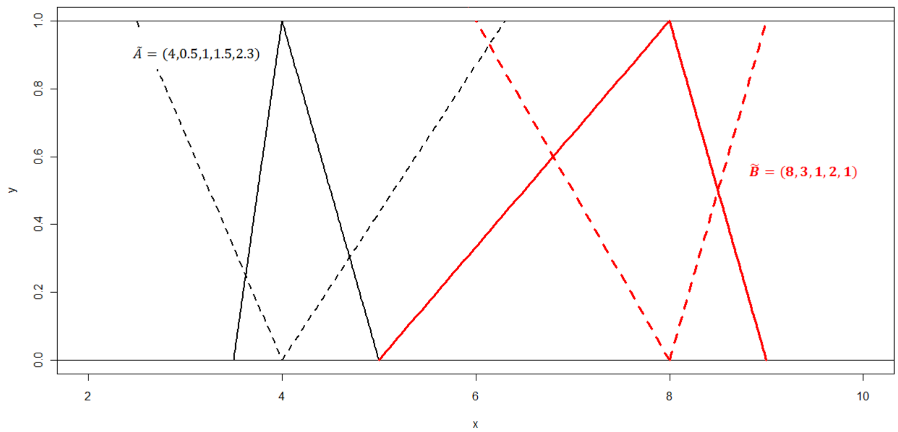

Figure 1. Membership functions of $\tilde{A}$ and $\tilde{B}$ in Example 1.

Table 1. $\tilde{A}$ and $\tilde{B}$ distance values for different α and β in Example1

Num.	α	β	$d^2_{\alpha,\beta}(\tilde{A},\tilde{B})$
1	0.2	0.8	0.22
2	0.8	0.2	0.023
3	0.5	0.5	0.11
4	0.4	0.8	0.21
5	0.6	0.6	0.13
6	0.3	0.4	0.12
7	0	1	0.31
8	1	0	0

3. Regression model

In classical regression, it is assumed that the variables and their related observations are accurate. Also the difference between the observed value for the dependent variable and the value obtained through the model, and the overall error of the model, is attributed to the random error related to observations and measurements, the absence of some variables, etc. About these random error statements and its possible distribution, assumptions such as normality, non-correlation, stability of variance... are considered. Based on these assumptions, statistical analysis such as estimation of parameters, prediction of the value of the dependent variable and hypothesis tests related to the model can be performed. But in many cases, one or more of the above assumptions may not be true, or, for example, due to the small sample size, it is not possible to ensure the correctness of some assumptions. For example, in a study, observations related to variables may be inaccurate or inaccurately reported; Or the variables under study may have an imprecise relationship. Also, assumptions such as normality and non-correlation of random error sentences may not hold. In such a situation, classical tools cannot provide suitable criteria for data modeling. One of the possible ways is to use the concept of fuzzy sets to model data in such conditions.

In linear regression with fuzzy input and output, it is assumed that coefficients are crisp and ambiguity in the input and output.

$$\tilde{Y}_i = c_0 + c_1\tilde{X}_{i1} + c_2\tilde{X}_{i2} + c_3\tilde{X}_{i3} + \cdots + c_m\tilde{X}_{im} + \epsilon_i, \qquad i = 1,2,\dots,n \tag{10}$$

Or

$$\tilde{Y}_i = c_0 + \sum_{j=1}^{m} c_j\tilde{X}_{ij} + \epsilon_i, \qquad i = 1,2,\dots,n \tag{11}$$

Based on observations

$$(\tilde{Y}_i, \tilde{X}_{i1}, \tilde{X}_{i2}, \dots, \tilde{X}_{im}), \qquad i = 1,2, \dots, n$$

where $c_0, c_1, \dots, c_m$ are model coefficients. To simplify the relationships, we assume that the regression and independent variables of both intuitive triangular symmetric fuzzy numbers are as follows.

$$\tilde{Y}_i = (y_i, l_i, r_i, l'_i, r'_i), \quad i = 1,2, \dots, n$$

$$\tilde{X}_{ij} = (x_{ij}, s_{ij}, w_{ij}, s'_{ij}, w'_{ij}), \quad i = 1,2, \dots, m.$$

Our goal here is to find the coefficients $c_i, i = 1, 2, \dots, m$ so that the distance between the $\hat{\tilde{Y}}_i = (\mu_{\hat{\tilde{Y}}_i}, \upsilon_{\hat{\tilde{Y}}_i})$ and $\tilde{Y}_i = (\mu_{\tilde{Y}_i}, \upsilon_{\tilde{Y}_i})$ is the smallest. For this purpose, the following minimization problem based on the distance defined in Definition 7 should be solved

$$minimize \sum_{i=1}^{n} d^2 (\tilde{Y}_i, \hat{\tilde{Y}}_i)$$

The least squares function is written as follows:

$$\begin{aligned} S(c_0, c_1, \dots, c_m) &= \sum_{i=1}^{n} d_{\alpha,\beta}{}^2 (\tilde{Y}_i, \hat{\tilde{Y}}_i) \\ &= \sum_{i=1}^{n} 2(y_i - c_0 - \sum_{j=1}^{m} c_j x_{ij})^2 (1 - \alpha + \beta) + [(l_i - c_0 - \sum_{j=1}^{m} c_j s_{ij})^2 \\ &+ (r_i - c_0 - \sum_{j=1}^{m} c_j w_{ij})^2] \left(\frac{(1-\alpha)^3}{3}\right) + [(l'_i - c_0 - \sum_{j=1}^{m} c_j s'_{ij})^2 \\ &+ (r'_i - c_0 - \sum_{j=1}^{m} c_j w'_{ij})^2] \left(\frac{\beta^3}{3}\right) + (1-\alpha)^2 (y_i - c_0 - \sum_{j=1}^{m} c_j x_{ij}) \quad [(l_i - c_0 - \sum_{j=1}^{m} c_j s_{ij}) - (r_i \\ &- c_0 - \sum_{j=1}^{m} c_j w_{ij})] + \beta^2 (y_i - c_0 - \sum_{j=1}^{m} c_j x_{ij}) \quad [(l'_i - c_0 - \sum_{j=1}^{m} c_j s'_{ij}) - (r'_i - c_0 - \sum_{j=1}^{m} c_j w'_{ij})] \end{aligned}$$

This function should be minimized with respect to $c_0, c_1, \dots, c_m$. If we write these equations in the form of a matrix, the data and results will be displayed more compactly. For this purpose, we write the least squares equation as follows:

$$\begin{aligned} S(c_0, c_1, \dots, c_m) &= 2(1 - \alpha + \beta)((\boldsymbol{Y} - \boldsymbol{XC})'(\boldsymbol{Y} - \boldsymbol{XC})) + A((\boldsymbol{L} - \boldsymbol{SC})'(\boldsymbol{L} - \boldsymbol{SC}) + (\boldsymbol{R} - \boldsymbol{WC})'(\boldsymbol{R} - \boldsymbol{Wc})) \\ &+ B((\boldsymbol{L}^* - \boldsymbol{S}^*\boldsymbol{C})'(\boldsymbol{L}^* - \boldsymbol{S}^*\boldsymbol{C}) + (\boldsymbol{R}^* - \boldsymbol{W}^*\boldsymbol{C})'(\boldsymbol{R}^* - \boldsymbol{W}^*\boldsymbol{C}) \\ &+ D[(\boldsymbol{Y} - \boldsymbol{XC})'((\boldsymbol{L} - \boldsymbol{SC}) - (\boldsymbol{R} - \boldsymbol{WC}))] + E[(\boldsymbol{Y} - \boldsymbol{XC})'((\boldsymbol{L}^* - \boldsymbol{S}^*\boldsymbol{C}) - (\boldsymbol{R}^* - \boldsymbol{W}^*\boldsymbol{C}] \end{aligned}$$

Where in $A = \frac{(1-\alpha)^3}{3}, B = \frac{\beta^3}{3}, D = (1-\alpha)^2$ and $E = \beta^2$

$$\boldsymbol{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix}, \quad \boldsymbol{R} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}, \quad \boldsymbol{L}^* = \begin{bmatrix} l'_1 \\ l'_2 \\ \vdots \\ l'_n \end{bmatrix}, \quad \boldsymbol{R}^* = \begin{bmatrix} r'_1 \\ r'_2 \\ \vdots \\ r'_n \end{bmatrix}$$

$$\boldsymbol{X}=\begin{bmatrix}1 & x_{11} & x_{12} & \cdots & x_{1m}\\ 1 & x_{21} & x_{22} & \cdots & x_{2m}\\ & \vdots & & \ddots & \vdots\\ 1 & x_{n1} & x_{n2} & \cdots & x_{nm}\end{bmatrix},\quad \boldsymbol{S}=\begin{bmatrix}1 & s_{11} & s_{12} & \cdots & s_{1m}\\ 1 & s_{21} & s_{22} & \cdots & s_{2m}\\ & \vdots & & \ddots & \vdots\\ 1 & s_{n1} & s_{n2} & \cdots & s_{nm}\end{bmatrix},\quad \boldsymbol{W}=\begin{bmatrix}1 & w_{11} & w_{12} & \cdots & w_{1m}\\ 1 & w_{21} & w_{22} & \cdots & w_{2m}\\ & \vdots & & \ddots & \vdots\\ 1 & w_{n1} & w_{n2} & \cdots & w_{nm}\end{bmatrix},\boldsymbol{S}^*=$$

$$\begin{bmatrix}1 & s'_{11} & s'_{12} & \cdots & s'_{1m}\\ 1 & s'_{21} & s'_{22} & \cdots & s'_{2m}\\ & \vdots & & \ddots & \vdots\\ 1 & s'_{n1} & s'_{n2} & \cdots & s'_{nm}\end{bmatrix},\quad \boldsymbol{W}^*=\begin{bmatrix}1 & w'_{11} & w'_{12} & \cdots & w'_{1m}\\ 1 & w'_{21} & w'_{22} & \cdots & w'_{2m}\\ & \vdots & & \ddots & \vdots\\ 1 & w'_{n1} & w'_{n2} & \cdots & w'_{nm}\end{bmatrix},\quad \boldsymbol{C}=\begin{bmatrix}c_0\\ c_1\\ \vdots\\ c_m\end{bmatrix}$$

The least squares estimators should apply in the following relationship:

$$\frac{\partial S(c_0, c_1, \dots, c_m)}{\partial \boldsymbol{C}}|_{\hat{C}} = 0$$

Therefore, the least squares estimator of $\boldsymbol{C}$, under the condition of having the inverse of $\boldsymbol{D}$, becomes as follows.

$$\widehat{\boldsymbol{C}} = (4(1-\alpha+\beta)\boldsymbol{X}'\boldsymbol{X} + 2A(\boldsymbol{S}'\boldsymbol{S} + \boldsymbol{W}'\boldsymbol{W}) + 2B(\boldsymbol{S}^{*\prime}\boldsymbol{S}^* + \boldsymbol{W}^{*\prime}\boldsymbol{W}^*) + (\boldsymbol{X}'\boldsymbol{S} - \boldsymbol{X}'\boldsymbol{W})D + E(\boldsymbol{X}'\boldsymbol{S}^* - \boldsymbol{X}'\boldsymbol{W}^*))^{-1}(4(1-\alpha+\beta)\boldsymbol{X}'\boldsymbol{Y} + 2A(\boldsymbol{S}'\boldsymbol{L} + \boldsymbol{W}'\boldsymbol{R}) + 2B(\boldsymbol{S}^{*\prime}\boldsymbol{L}^* + \boldsymbol{W}^{*\prime}\boldsymbol{R}^*) + D((\boldsymbol{S}-\boldsymbol{W})'\boldsymbol{Y} + \boldsymbol{X}'(\boldsymbol{L} - \boldsymbol{R}))(E((\boldsymbol{S}^*-\boldsymbol{W}^*)'\boldsymbol{Y} + \boldsymbol{X}'(\boldsymbol{L}^*-\boldsymbol{R}^*)))) \quad (12)$$

If the triangular fuzzy numbers are intuitively symmetric, then $\boldsymbol{L}=\boldsymbol{L}^*, \boldsymbol{R}=\boldsymbol{R}^*, \boldsymbol{S}=\boldsymbol{S}^*$ and $\boldsymbol{W}=\boldsymbol{W}^*$. In this case, the least squares estimator is equal to:

$$\hat{C} = (4(1-\alpha+\beta)\boldsymbol{X}'\boldsymbol{X} + 2(\boldsymbol{S}'\boldsymbol{S} + \boldsymbol{W}'\boldsymbol{W})(A+B) + (\boldsymbol{X}'\boldsymbol{S} - \boldsymbol{X}'\boldsymbol{W})(D+E))^{-1}(4(1-\alpha+\beta)\boldsymbol{X}'\boldsymbol{Y} + \boldsymbol{2}(\boldsymbol{S}'\boldsymbol{L} + \boldsymbol{W}'\boldsymbol{R})(A+B) + (\boldsymbol{S}-\boldsymbol{W})'\boldsymbol{Y} + \boldsymbol{X}'(\boldsymbol{L}-\boldsymbol{R})(D+E)) \quad (13)$$

4. Ridge regression

Ridge regression is a statistical technique used to estimate the coefficients of multiple-regression models in situations when the independent variables exhibit significant levels of correlation. The phrase "ridge regression" is often used to describe a linear regression model in which the coefficients are estimated using a ridge estimator instead of ordinary least squares (OLS). The ridge estimator, is known to have a lower variance compared to the OLS estimator. In some instances, the ridge estimator exhibits a mean squared error that is comparatively less than that of the ordinary least squares (OLS) estimator. This is due to the ridge estimator's ability to effectively balance its variance and the square of its bias. When the independent variables are related to each other so that the correlation coefficient between them is statistically significant, the problem of collinearity has arisen. As a result, other variables in the regression model may be able to estimate the effect of each variable. In this way, the estimators become very sensitive, and their variance will also be large. As mentioned earlier, the occurrence of multiple collinearity between the predictor variables in linear regression analysis may cause severe instability in the estimates of the least squares of the regression parameters, which means that the magnitude and sign of the parameters in different samples are significantly different. It will be stable, as a result of which, the estimation of the lowest second power obtained will not be reliable.

The occurrence of multiple collinearity between the predictor variables in the linear regression analysis may cause severe instability in the estimates of the least squares of the regression parameters, which means that the magnitude and sign of the parameters in different samples are significantly unstable. It will be that as a result, the estimation of the lowest second power obtained will not be reliable. In order to solve this problem, in this section, we express ridge regression for TriIFNs based on the proposed distance measure in the Definition 8.

$$minimize \sum_{i=1}^{n} d^2\,(\tilde{Y}_i, \tilde{\hat{Y}}_i) + \lambda \sum_{j=1}^{m} c_j^2$$

The Lagrangian parameter, sometimes called the ridge or biasing parameter, is the penalty parameter that affects the magnitude of the coefficients and the degree of regularization. Other names for this parameter are biasing parameter and ridge parameter. The symbol λ compels the coefficients to approach zero while preventing them from really reaching it. (The coefficient will be decreased in proportion to the severity of the imposed punishment.) As approaches 0, we will have arrived at the solution with the fewest squares. The estimate converges to zero as approaches infinity in the intercept-only model. There is an answer to each and every problem. As a result, the λ lay out a strategy for finding a solution.

Therefore, the estimator of $\boldsymbol{C}$, becomes as follows.

$$\widehat{\boldsymbol{C}} = (4(1-\alpha+\beta)\boldsymbol{X}'\boldsymbol{X} + 2A(\boldsymbol{S}'\boldsymbol{S} + \boldsymbol{W}'\boldsymbol{W}) + 2B(\boldsymbol{S}^{*\prime}\boldsymbol{S}^{*} + \boldsymbol{W}^{*\prime}\boldsymbol{W}^{*}) + (\boldsymbol{X}'\boldsymbol{S} - \boldsymbol{X}'\boldsymbol{W})D + E(\boldsymbol{X}'\boldsymbol{S}^{*} - \boldsymbol{X}'\boldsymbol{W}^{*}) + 2\lambda\boldsymbol{I})^{-1}(4(1-\alpha+\beta)\boldsymbol{X}'\boldsymbol{Y} + 2A(\boldsymbol{S}'\boldsymbol{L} + \boldsymbol{W}'\boldsymbol{R}) + 2B(\boldsymbol{S}^{*\prime}\boldsymbol{L}^{*} + \boldsymbol{W}^{*\prime}\boldsymbol{R}^{*}) + D((\boldsymbol{S}-\boldsymbol{W})'\boldsymbol{Y} + \boldsymbol{X}'(\boldsymbol{L}-\boldsymbol{R}))(E((\boldsymbol{S}^{*} - \boldsymbol{W}^{*})'\boldsymbol{Y} + \boldsymbol{X}'(\boldsymbol{L}^{*} - \boldsymbol{R}^{*})))) \quad (14)$$

5. Numerical example

In the following example, we will utilize a basic ridge linear regression model with a triangular intuitionistic fuzzy output in order to demonstrate how to apply the parametric distance measure as well as the least square approach.

Example 2. We apply our proposed fuzzy multiple linear regression model and ridge regression model to the house price data taken from Tanaka [19]. The house price data are shown in Table 2. We have converted these data into intuitive fuzzy. Table 2 shows these data.

able 2. Input-output data concerning house prices taken from Tanaka [19]..

No.	$\tilde{Y}_i = (y_i; l_i, r_i)_T$	$\tilde{X}_{i1} = (x_{i1}; s_{i1}, w_{i1})_T$	$\tilde{X}_{i2} = (x_{i2}; s_{i2}, w_{i2})_T$	$\tilde{X}_{i3} = (x_{i3}; s_{i3}, w_{i3})_T$
1	$(606; 100,165)_T$	$(38.09; 3.5,5.25)_T$	$(36.43; 2.3,4.6)_T$	$(5; 0.09,0.13)_T$
2	$(710; 50,86)_T$	$(62.1; 3.7,5.55)_T$	$(26.5; 1.8,2.4)_T$	$(6; 0.11,0.17)_T$
3	$(808; 97,193)_T$	$(63.76; 2.7,4.05)_T$	$(44.71; 3.1,2.1)_T$	$(7; 0.13,0.2)_T$
4	$(826; 102,203)_T$	$(74.52; 2.9,4.35)_T$	$(38.09; 2.7,1.6)_T$	$(8; 0.2,0.3)_T$
5	$(865; 87,173)_T$	$(75.38; 3.8,5.7)_T$	$(41.4; 3.1,2.6)_T$	$(7; 0.1,0.15)_T$
6	$(852; 197,93)_T$	$(52.99; 2.9,4.35)_T$	$(26.49; 0.98,1.3)_T$	$(4; 0.15,0.23)_T$
7	$(917; 200,204)_T$	$(62.93; 3.2,4.8)_T$	$(26.49; 1.9,2.3)_T$	$(5; 0.13,0.19)_T$
8	$(1031; 109,219)_T$	$(72.04; 1.8,2.7)_T$	$(33.12; 2.4,3.2)_T$	$(6; 0.13,0.2)_T$
9	$(1092; 380,259)_T$	$(76.12; 4.0,6.0)_T$	$(43.06; 3.7,1.9)_T$	$(7; 0.17,0.26)_T$
10	$(1203; 92,183)_T$	$(90.26; 2.8,4.2)_T$	$(42.64; 2.8,2.4)_T$	$(7; 0.2,0.3)_T$
11	$(1394; 350,226)_T$	$(85.70; 1.3,1.95)_T$	$(31.33; 2.6,1.5)_T$	$(6; 0.17,0.25)_T$
12	$(1420; 219,237)_T$	$(95.27; 1,9,2.85)_T$	$(27.64; 0.99,1.23)_T$	$(6; 0.22,0.32)_T$
13	$(1601; 222,244)_T$	$(105.98; 7.1,4.65)_T$	$(27.64; 2.4,3.6)_T$	$(6; 0.35,0.53)_T$
14	$(1632; 209,218)_T$	$(79.25; 3.1,4.65)_T$	$(66.81; 3.6,3.2)_T$	$(6; 0.23,0.35)_T$
15	$(1699; 519,238)_T$	$(120.50; 11.7,7.55)_T$	$(32.25; 1.7,4.2)_T$	$(6; 0.17,0.26)_T$

Models (13) are applied to obtain the set of the parameters of model $\tilde{Y}_i = c_0 + c_1\tilde{X}_{i1} + c_2\tilde{X}_{i2} + c_3\tilde{X}_{i3} + \epsilon_i, i = 1,2,\ldots,15$ .The parameters values obtained are described in Table 3. In this table, the estimated values of the response variable and error are also recorded for different values of alpha and beta.

Table 3. Estimation of parameters and $\tilde{\tilde{Y}}$ for independent variables $\tilde{X}_1 = (35.5; 2.75,4.2)_T$, $\tilde{X}_2 = (40.2; 3.14,2.87)_T$ and $\tilde{X}_3 = (7; 0.15,0.98)_T$ in Example 2

Num.	α	β	$\hat{C} = (C_0, C_1, C_2, C_3)_{org}$	$\hat{Y} = \widetilde{(\hat{y}, \hat{l}, \hat{r})}$	MSE
1	0.2	0.8	(147.8,18.87,14.54,-167.08)	(232.9662,72,51,-42.72)	32504.59
2	0.8	0.2	(248.99,17.98,13.46,-165.19)	(272.11,66.94,-47.74)	507.86
3	0.5	0.5	(174.88,18.48,14.08,-163.56)	(252.1946,70.51,-42.24)	7935.62
4	0.4	0.8	(152.58,18.77,14.42,-165.75)	(238.42,72.04,-42.21)	23108.69
5	0.6	0.6	(170.60,18.51,14.12,-163.51)	(250.91,70.72,-41.95)	8887.91
6	0.3	0.4	(161.56,18.61,14.23,-164.04)	(246.33,71.28,-41.73)	12919.25
7	0	1	(138.94,19.11,14.81,-170.34)	(220.47,73.52,-44.16)	63485.7
8	0.99	0.01	(285.89,17.68,13.09,-164.94)	(284.96,64.97,-49.84)	0.064
9	0.4	0.3	(171.41,18.49,14.09,-163.13)	(252.22,70.62,-41.78)	7713.43

For example, if in a new case, we observe $\tilde{X}_1 = (35.5; 2.75,4.2)_T$, $\tilde{X}_2 = (40.2; 3.14,2.87)_T$ and $\tilde{X}_3 = (7; 0.15,0.98)_T$ then for different values for α and β we predict $\hat{Y}$ in Table 3. To calculate ridge estimators, we use Equation (14) for different values of λ. Tables 4 show the values of the estimators and the MSE for different values of λ and $\alpha = 0.99$ and $\beta = 0.01$. As seen in Figure 2, at the confidence level of $\alpha = 0.99$, the lowest error value for λ is 0.4, where the ridge estimators are $\hat{C}_{0ridge} = 0.72$, $\hat{C}_{1ridge} = 16.48$, $\hat{C}_{2ridge} = 10.21$ and $\hat{C}_{3ridge} = -86.47$ with MSE $= 0.018346$.

Table 4. The error values and estimation of ridge parameters for the data of Example 2 ($= 0.99$ and $\beta = 0.01$).

Num.	λ	$\hat{C} = (C_0, C_1, C_2, C_3)_{org}$	MSE
1	0	(285.89,17.67,13.09,-164.95)	0.01835
2	0.001	(263.65,17.72,13.14,-162.21)	0.018352
3	0.003	(227.79,17.78,13.21,-157.73)	0.018355
4	0.007	(178.15,17.86,13.29,-151.28)	0.01836
5	0.009	(160.27,17.89,13.31,-148.85)	0.018361
6	0.015	(122.25,17.93,13.33,-143.36)	0.018365
7	0.035	(65.11,17.91,13.2,-133.12)	0.018369
8	0.07	(32.20,17.74,12.81,-123.34)	0.018366
9	0.127	(14.55,17.43,12.164,-112.64)	0.018359
10	0.225	(4.81,16,96,11.19,-99.20)	0.01835
11	0.35	(0.72,16.48,10.21,-86.47)	0.018346
12	0.65	(-1.74,15.69,8.61,-66.28)	0.018353
13	0.85	(-2.06,15.33,7.90,-57.36)	0.018362
14	0.94	(-2.11,15.20,7.63,-54.08)	0.018367
15	1	(-2.12,15.12,7.47,-52.09)	0.018369

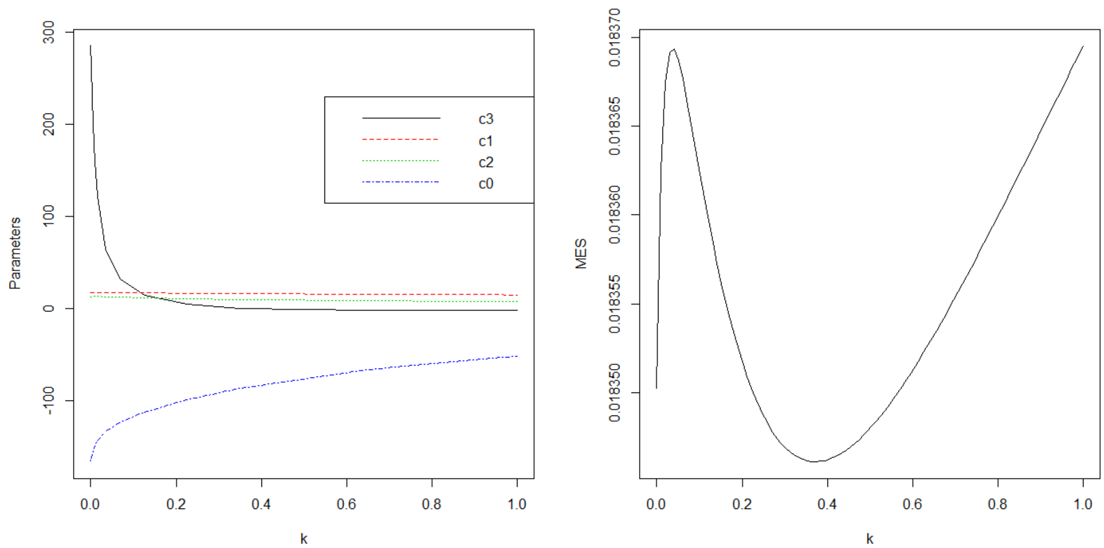

Figure 2. The coefficients and MSE for the ridge fuzzy regression model, $\alpha = 0.99, \beta = 0.01$

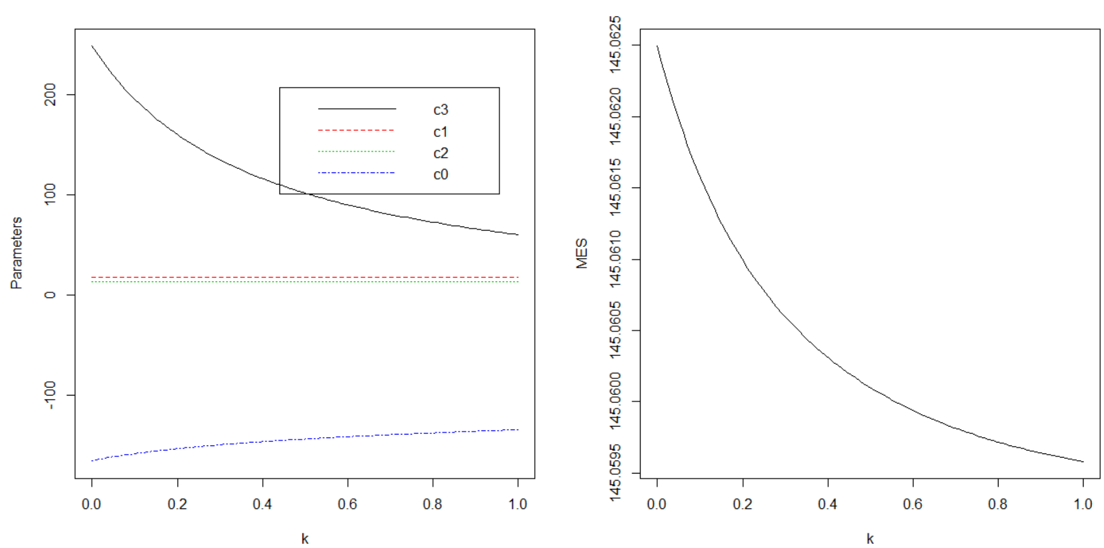

Figure 3. The coefficients and MSE for the ridge fuzzy regression model, $\alpha = 0.8, \beta = 0.2$

6. Conclusion

Addressing multicollinearity in multiple linear regression models is a crucial subject in the field of statistics. The phenomena might result in imprecise estimations of the regression coefficients, increase their standard errors, provide non-significant p-values, and diminish the predictive capacity of the fitted model. Ridge regression was first proposed as a method to mitigate the impact of multicollinearity when it is present. When two or more of the variables in a multiple linear regression model exhibit a strong correlation, the columns of the design matrix become linearly dependent. Therefore, it is not possible to estimate the standard regression parameter. Ridge regression may enhance model performance by reducing model complexity. As models increase in complexity, they have the ability to capture inconsequential local structures, which is referred to as overfitting. When more factors are added to the model, coefficient estimates in these circumstances become more susceptible to excessive variation. Ridge regression reduces the number of parameters, which in turn allows for a trade-off between bias and variance, resulting in improved model performance.

The fuzzy regression model is a commonly used statistical model in fuzzy statistical investigations. Similar to conventional regression models, fuzzy regression models also encounter the issue of multicollinearity. This study presents a parametric estimation approach that utilises a parametric distance measure to estimate the parameters of the ridge fuzzy regression model. The fuzzy ridge regression model creates a fuzzy linear function that minimises the combined total of squared error term and penalty term. On the other hand, a fuzzy multiple linear regression model generates a fuzzy linear function that minimises the squared error. In order to demonstrate the effectiveness of the suggested method, an examples is provided.

In this article, fuzzy ridge regression for exact input and fuzzy output variables, as well as ridge regression for fuzzy input and output and exact coefficients, were studied. For future research, the fuzzy ridge regression model can be used when both the input and output variables are fuzzy and the coefficients of the model. The novelty of this approach is its parametric nature, which allows decision makers to solve the desired regression model according to their decision-making level.

It is important to note that ridge regression is often favoured over lasso regression when the study purpose is to address multicollinearity without eliminating variables that have poor contributions. However, in cases when the data has a high number of dimensions and it is important to exclude collinear variables, lasso regression may be preferred over ridge regression. In future works, we want to expand the suggested parametric estimation technique for ridge fuzzy models to include lasso fuzzy regression models, in order to effectively handle such scenarios. The use of lasso fuzzy regression will be particularly advantageous in the modelling of associated genetic data sets.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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