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Vague Graph Structures: Some New Concepts and Applications

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ABSTRACT

A graph structure is a useful tool in solving the combinatorial problems in different areas of computer science and computational intelligence systems. In this paper, we apply the concept of vague sets to graph structures. We introduce certain notions, including vague graph structure (VGS), strong vague graph structure, vague B_i -cycle, and illustrate these notions by several examples. We study ϕ -complement, self-complement, strong self- complement, totally strong self-complement in vague graph structures, and investigate some of their properties. Finally, an application of vague influence graph structure is given.

1. Introduction

Graph theory has numerous applications to problem in computer science, electrical engineering, system analysis, operation research, economics, networking routing, and transportation. Also, the major role of graph theory in computer applications is the development of graph algorithms. A number of algorithms are used to solve problems, that are modelled in the form of graphs and the corresponding computer science application problems. A graph structure, introduced by Sampath kumar [18], is a generalization of undirected graph which is quite useful in studying some structure including graphs, signed graphs, and graphs in which every edge is labelled or colored. A graph structure helps to study the various relations and the corresponding edges simultaneously. A fuzzy set, introduced by Zadeh [20], gives the degree of membership of an object in a given set. Gau and Buehrer [7] proposed the concept of vague set in 1993, by replacing the value of an element in a set with a subinterval of [0, 1]. Namely, a true-membership function and a false membership function are used to describe the boundaries of the membership degree. Kaufmann defined in [8] a fuzzy graph. Rosenfeld [17] described the structure of fuzzy graphs obtaining analogs of several graph theoretical concepts. Bhattacharya [2] gave some remarks on fuzzy graphs. Several concepts on fuzzy graphs were introduced by Mordesom et al. [9].

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Dinesh [6] introduced the notion of a fuzzy graph structure and discussed some related properties. Ramakrishna [10] introduced the concept of vague graphs and studied some of their properties. Pal and Rashmanlou [16] studied irregular interval-valued fuzzy graphs. Also, they defined antipodal interval-valued fuzzy graphs [14], balanced interval-valued fuzzy graphs [15], some properties of highly irregular interval-valued fuzzy graphs [16], and new concepts on bipolar fuzzy graphs [12, 13]. Rashmanlou and Jun [11] investigated complete interval-valued fuzzy graphs. Akram [1] defined bipolar fuzzy graphs. Sunitha and Vijayakumar [19] studied some properties of complement on fuzzy graphs. Borzooei [3, 4, 5] studied on vague graphs. In this paper, we define the certain notions including vague graph structure (VGS), strong vague graph structure, vague cycle, and illustrate these notions by several examples. We present complement, self-complement, strong self-complement in vague graph structures, and introduce some of their interesting properties.

2. Preliminaries

In this section, we review some definitions that are necessary for this paper. A graph structure $G^* = (U, E_1, E_2, ..., E_k)$, consists of a non-empty set U together with relations $E_1, E_2, ..., E_k$ on U, which are mutually disjoint such that each E_i is reflexive and symmetric. If $(u,v) \in E_i$ for some $1 \le i \le k$, we call it an E_i-edge and write it as uv.

i. A graph structure $G^* = (U, E_1, E_2, ..., E_k)$, is complete, if each edge $E_i, 1 \le i \le k$ appears at least once in G^*;

ii. Between each pair of vertices u, v in U, uv is an E_i-edge for some $1 \le i \le k$.

A graph structure $G^* = (U, E_1, E_2, ..., E_k)$ is connected if the underlying graph is connected. In a graph structure, E_i-path between two vertices *u* and *v*, is the path which consists of only E_i-edges for some i, and similarly, E_i-cycle is the cycle, which consists of only E_i-edges for some *i*. Let $S \subseteq U$, then the subgraph structure $< S >$ induced by *S*, has vertex set *S*, where two vertices *u* and *v* in ⟨*S*⟩ are joined by an E_i-edge if and only if, they are joined by an E_i-edge in G^* for $1 \le i \le k$.

For some $i, 1 \le i \le k$ the E_i-sub graph induced by *S*, is denoted $E_i - < S >$ and it has only E_i-edges joining the vertices in *S*. If *T* is a subset of edge set in G^*, then sub graph structure ⟨*T*⟩ induced by *T* has the vertex set, the end vertices in *T*, and whose edges are those in *T*.

Let $G^* = (U, E_1, E_2, ..., E_k)$ and $H^* = (U_2, E'_1, E'_2, ..., E'_n)$, be graph structures then G^* and H^* are isomorphic, if $m = n$ and there exists a bijection $f : U_1 \to U_2$ and a permutation $\phi : \{E_1, E_2, \ldots, E_n\} \to \{E'_1, E'_2, \ldots, E'_n\}$, say $E_i \to E'_j, 1 \le i, j \le n$ such that for all $u, v \in U_1$, $uv \in E_i$ implies $f(u)f(v) \in E'_j$.

Two graph structures $G^* = (U, E_1, E_2, ..., E_k)$ and $H^* = (U_2, E'_1, E'_2, ..., E'_n)$, on the same vertex set *U*, are identical, if there exists a bijection $f : U \to U$, such that for all *u* and *v* in *U* and an E_i-edge *uv* in G^*, $f(u)f(v)$ is an E'-edge in H^*, where $1 \le i \le k$ and $E_i \simeq E'_i$ for all i.

Let ϕ be a permutation on $\{E_1, E_2, ..., E_k\}$ then the ϕ-cyclic complement of G^* denoted by $(G^*)^{\phi c}$ is obtained by replacing E_i with $\phi(E_i)$ for $1 \le i \le k$. Let $G^* = (U, E_1, E_2, ..., E_k)$, be a graph structure and ϕ be a permutation on $\{E_1, E_2, ..., E_k\}$, then

i. G^* is ϕ-self complementary, if G^* is isomorphic to $(G^*)^{\phi c}$, then the ϕ-complement of G^*, and G^* is self-complementary, if $\phi \neq$ *identity permutation*;

ii. G^* is strong ϕ-self complementary, if G^* is identical to $(G^*)^{\phi c}$, the ϕ-complement of G^* and G^* is strong self-complementary, if $\phi \neq$ identity permutation.

Definition 1. ([20]) A fuzzy subset μ on a set X is a map $\mu : X \to [0,1]$. A fuzzy binary relation on X is a fuzzy subset μ on $X \times X$. By a fuzzy relation we mean a fuzzy binary relation given by $\mu : X \to [0,1]$.

Definition 2. ([6]) Let $G^* = (U, E_1, E_2, \ldots, E_k)$, be a graph structure and let $v, \rho_1, \rho_2, \ldots, \rho_k$ be the fuzzy subset of $U, E_1, E_2, \ldots, E_k$, respectively, such that $0 \leq \rho_i(xy) \leq \mu(x) \wedge \mu(x) \wedge \mu(y), \forall x, y \in u, i = 1,2,\ldots,k$ Then $G = (v, \rho_1, \rho_2, \ldots, \rho_k)$ is a fuzzy graph structure of G^*.

Definition 3. ([6]) Let $G = (v, \rho_1, \rho_2, \ldots, \rho_k)$ be a fuzzy graph structure of a graph structure $G^* = (U, E_1, E_2, \ldots, E_k)$, then $F = (v, \tau_1, \tau_2, \ldots, \tau_k)$ is a partial fuzzy spanning subgraph structure of G if $\tau_i \subseteq \rho_i$ *for* $i = 1,2,\ldots,k$.

Definition 4. [6] Let G^* be a graph structure and let G be a fuzzy graph structure of G^*. If $xy \in sup(\rho_i)$, then we say that "xy" is a ρ_i-edge of G.

Definition 5. [6] The strength of a ρ_i-path $x_\circ, x_1, \ldots, x_n$ of a fuzzy graph structure G is $\wedge_j^n \rho_i(x_j - 1x_j)$, $for\, i = 1,2,\ldots,k$.

Definition 6. [6] In a fuzzy graph structure G, for any $m \geq 2$.

$$\rho_i^2(xy) = (\rho_i \, o \, \rho_i)(xy) \vee_z \{\rho_i(xz) \wedge \rho_i(zy)\},$$

$$\rho_i^j(xy) = (\rho_i^{j-1} \, o \, \rho_i)(xy) = \vee_z \{\rho_i^{j-1}(xz) \wedge \rho_i(zy), j = 1,2,\ldots,m,$$

Also, $\rho_i^\infty(xy) = \vee\{\rho_i^j(xy), i = 1,2,..\}$.

Definition 7. [6] $G = (v, \rho_1, \rho_2, \ldots, \rho_k)$ is a ρ_i-cycle if and only if

$$(supp(v), supp(\rho_1), supp(\rho_2), \ldots, supp(\rho_k))$$

is a E_i-cycle.

Definition 8. [6] $G = (v, \rho_1, \rho_2, \ldots, \rho_k)$ *is a* ρ_i*-cycle* if and only if

$$(supp(v), supp(\rho_1), supp(\rho_2), \ldots, supp(\rho_k))$$

is a E_i*-cycle* and there exists no unique xy in $supp(\rho_i)$ such that: $\rho_i(xy) = \wedge\{\rho_i(uv \mid uv \in supp(\rho_i)\}$.

Definition 9. [7] A vague set A on an ordinary finite non-empty set X is a pair (t_A, f_A), where $t_A : X \to [0,1]$ *and* $f_A : X \to [0,1]$ are true and false membership functions, respectively such that $t_A(x) + f_A(x) \leq 1$, *for all* $x \in X$.

Definition 10. [7] Let X and Y be ordinary finite non-empty sets. Then we call a vague relation to be a vague subset of $X \times Y$, that is an expression R defined by:

$$R = \{\langle (x,y), t_R(x,y(, f_R(x,y)\rangle \mid x \in X, y \in Y\},$$

where $t_R : X \times Y \to [0,1], f_R : X \times Y \to [0,1]$, which satisfies condition $0 \leq t_R(x,y) + f_R(x,y) \leq 1$ for all $(x,y) \in X \times Y$.

Definition 11. [10] A vague graph is a pair $G=(A,B)$, where $A=(t_A, f_A$ is a vague set on V and $B=(t_B,f_B)$ is a vague set on $E\subseteq V\times V$ such that $t_B(xy)\le \min(t_A(x),t_A(y))$ and $f_B(xy)\ge \max(f_A(x),f_A(y))$ for $xy\in E$.

3. Vague graph structures

Definition 12. $\hat{G}_v=(A,B_1,B_2,\ldots,B_n)$ is called a vague graph structure (VGS) of a graph structure

$$(GS)\, G^*=(U,E_1,E_2,\ldots,E_k),$$

If $A=(t_A,f_A)$ is a vague set on U and for each $i=1,2,\ldots,n$; $B_i=(t_{B_i},f_{B_i})$ is a vague set on E_i such that:
$t_{B_i}(xy)\le t_A(x)\wedge t_A(y)$, $f_{B_i}(xy)\ge f_A(x)\vee f_A(y)$ $\forall xy\in E_i\subseteq U\times U$.

Note that $t_{B_i}(xy)=0=f_{B_i}(xy)$ for all $xy\in U\times U-E_i$ and $0\le t_{B_i}(xy)\le 1$, $0\le f_{B_i}(xy)\le 1, \forall xy\in E_i$ where U and $E_i (i=1,2,\ldots,n)$ are called underlying vertex set and underlyingedge set of $\hat{G}_v$, respectively.

Example 1. Let $G^*=(U,E_1,E_2)$, be a graph structure such that $U=\{a_1,a_2,a_3,a_4\}$, $E_1=\{a_1a_2,a_2a_3\}$, $E_2=\{a_3a_4,a_1a_4\}$. Let A, B_1 *and* B_2 be vague subsets of U, E_1, and E_2 respectively, such that:

$$A=\{(a_1,0.4,0.5),(a_2,0.4,0.6),(a_3,0.3,0.4),(a_4,0.4,0.4)\},$$

$$B_1=\{(a_1a_2,0.4,0.6),(a_2a_3,0.3,0.6)\},$$

$$B_2=\{(a_3a_4,0.3,0.4),(a_1a_4,0.3,0.5)\}.$$

Then $\hat{G}_v=(A,B_1,B_2)$ is a VGS of G^* as shown in Figure 1.

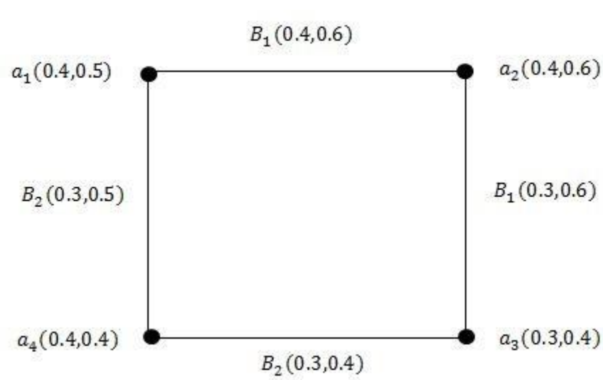

Figure 1. $VGS\ \hat{G}_v=(A,B_1,B_2)$

Definition 12. (*i*) $A\,VGS\ \hat{H}_v=(C,D_1,D_2,\ldots,D_n)$ is said to be a vague sub graph structure of a $VGS\ \hat{G}_v=(A,B_1,B_2,\ldots,B_n)$ with underlying vertex set U, *if* $C\subseteq A$ and $D_i\subseteq C_i$, for all i, that is:

$$t_C(x)\le t_A(x)\ , f_C(x)\ge f_A(x)\ ;\ for\ all\ x\in U,$$

and for $i = 1,2,\ldots,n$

$$t_{D_i}(xy) \le t_{B_i}(xy)\ , f_{D_i}(xy) \ge f_{B_i}(xy);\ \text{for all } xy \in U \times U.$$

(ii) $\breve{H}_v$ is called a vague spanning sub graph structure of a *VGA* $\hat{G}_v$*, if* $C = A$

(iii) $\breve{H}_v$ is called a vague partial spanning sub graph structure of a *VGS* $\hat{G}_v$, if it excludes some edges *of* $\hat{G}_v$.

Example 2. Consider a $VGS\ \hat{G}_v = (A, B_1, B_2)$ as shown in Figure 1. Let:

$$C = \{(a_1, 0.3, 0.6), (a_2, 0.0, 0.7), (a_3, 0.3, 0.4), (a_4, 0.3, 0.5)\},$$

$$D_1 = \{(a_1a_2, 0, 0.7), (a_2a_3, 0, 0.7)\},$$

$$D_2 = \{(a_3a_4, 0.2, 0.5), (a_1a_4, 0.3, 0.6)\}.$$

$$C_1 = \{(a_1a_2, 0.2, 0.6), (a_2a_3, 0.3, 0.6)\},$$

$$C_2 = \{(a_3a_4, 0.2, 0.4), (a_1a_4, 0.3, 0.7)\}.$$

By routine calculation, it is easy to see that $\breve{H}_v = (C, D_1D_2), J_v = (A, C_1, C_2), K_v = (A, F_1, F_2)$ are respectively the vague sub graph structure, vague spanning sub graph structure, and vague partial spanning sub graph structure of $\breve{G}_v$.

Their respective drawings are shown in Figure 2, Figure 3 and Figure 4.

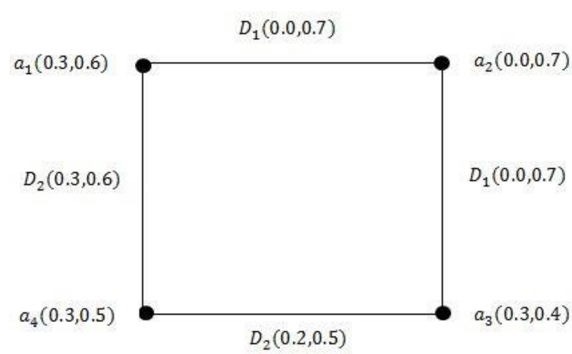

Figure 2. Vague subgraph structure $\breve{H}_v = (C, D_1D_2)$

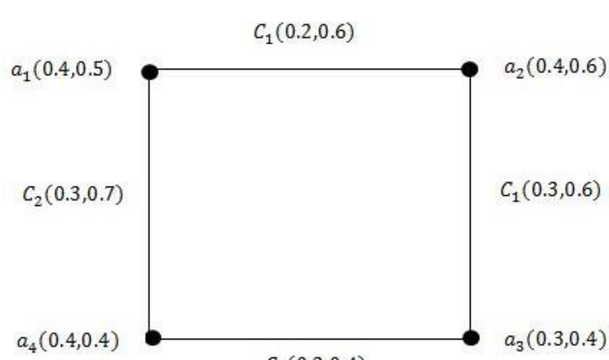

Figure 3. Vague spanning subgraph structure $J_v = (A, C_1, C_2)$

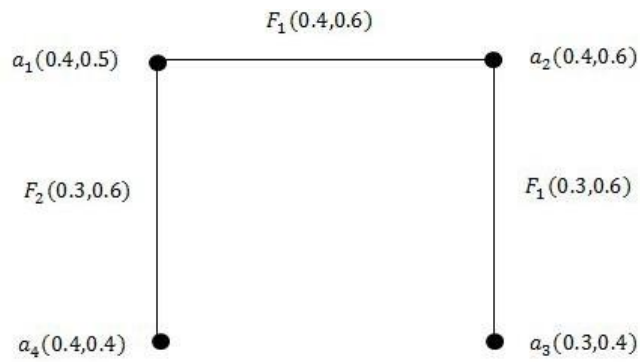

Figure 4. Vague partial spanning sub graph structure $K_v = (A, F_1, F_2)$.

Definition 13. Let $\breve{G}_v = (A, B_1, B_2, \ldots, B_n)$ be a VGS with underlying vertex set U. Then there is a B_i-edge between two vertices x and y of U, if one of the following is true:

(i) $t_{B_i}(xy) > 0$ and $f_{B_i}(xy) > 0$

(ii) $t_{B_i}(xy) > 0$ and $f_{B_i}(xy) = 0$

(ii) $t_{B_i}(xy) = 0$ and $f_{B_i}(xy) > 0$, for some i.

Definition 14. For a vague graph structure $\breve{G}_v = (A, B_1, B_2, \ldots, B_n)$ with vertex set U, support of B_i is given by:

$$supp(B_i) = \{xy \in U \times U : t_{B_i}(xy) \neq 0 \text{ or } f_{B_i}(xy) \neq 0\}, \quad i = 1, 2, \ldots, n.$$

Definition 15. B_i-path of a VGS $\breve{G}_v = (A, B_1, B_2, \ldots, B_n)$ with underlying vertex set U, is a sequence of distinct vertices $v_1, v_2, \ldots, v_m \in U$ (except the choice $v_m = v_1$), such that $v_{j-1}v_j$ is a B_i-edge for all $j = 2, 3, \ldots, m$.

Definition 16. In a VGS $\breve{G}_v = (A, B_1, B_2, \ldots, B_n)$ with underlying vertex set U, two vertices x and y of U are said to be B_i-connected, if they are joined by a B_i-path, for $i \in \{1, 2, 3, \ldots, n\}$

Definition 17. A VGS $\breve{G}_v = (A, B_1, B_2, \ldots, B_n)$ with underlying vertex set U, is said to be B_i-strong, if for all B_i-edges xy, $t_{B_i}(xy) = t_A(x) \wedge t_A(y)$, $f_{B_i}(xy) = f_A(x) \vee f_A(y)$, for some $i \in \{1, 2, 3, \ldots, n\}$.

Example 3. Consider the VGS $\breve{G}_v = (A, B_1, B_2, \ldots, B_n)$, as shown in Figure 1. Then

(i) a_1a_2, a_2a_3 are B_1-edges and a_3a_4, a_1a_4 are B_2-edges;

(ii) $a_1a_2a_3$ and $a_3a_4a_1$ are B_1 and B_1 paths, respectively;

(iii) a_1 and a_3 are B_1-connected vertices of U;

(iv) $\breve{G}_v$ is a B_1-strong since $supp(B_1) = \{a_1a_2, a_2a_3\}$ and we have:

(i) $t_{B_1}(a_1a_2) = 0.4 = (t_A(a_1) \wedge t_A(a_2))$,

(ii) $f_{B_1}(a_1a_2) = 0.6 = (f_A(a_1) \vee f_A(a_2))$,

(iii) $t_{B_1}(a_2a_3) = 0.3 = (t_A(a_2) \wedge t_A(a_3))$,

(iv) $f_{B_1}(a_2a_3) = 0.6 = (f_A(a_2) \vee f_A(a_3))$,

Definition 18. A VGS $\breve{G}_v = (A, B_1, B_2, \dots, B_n)$ is said to be strong, if it is B_i -strong for all $i \in \{1, 2, 3, \dots, n\}$.

Definition 19. A VGS $\breve{G}_v = (A, B_1, B_2, \dots, B_n)$ with underlying vertex U, is called complete or $B_1, B_2, \dots, B_n$ -complete if

(i) $\breve{G}_v$ is a strong VGS.

(ii) $supp(B_i) \neq \phi$, for all $i \in \{1, 2, 3, \dots, n\}$

(iii) For each pair of vertices $xy \in u$, *xy* is a B_i-edge for some *i*.

Example 4. Let $\breve{G}_v = (A, B_1, B_2)$ shown in Figure 5, be VGS of the graph structure $G^* = (U, E_1, E_2)$ where $U = \{a_1, a_2, a_3, a_4\}$, $E_1 = \{a_1a_2, a_2a_3, a_1a_3\}$ and $E_2 = \{a_1a_4, a_3a_4, a_2a_4\}$. Then $\breve{G}_v$ is a strong VGS since it is both B_1 -strong and B_2-strong. Moreover $supp(B_1) \neq \phi, supp(B_2) \neq \phi$, every pair of vertices belonging to u, is either a B_1-edge or a B_2-edge, so $\breve{G}_v$ is a complete or B_1B_2 -complete VGS as well.

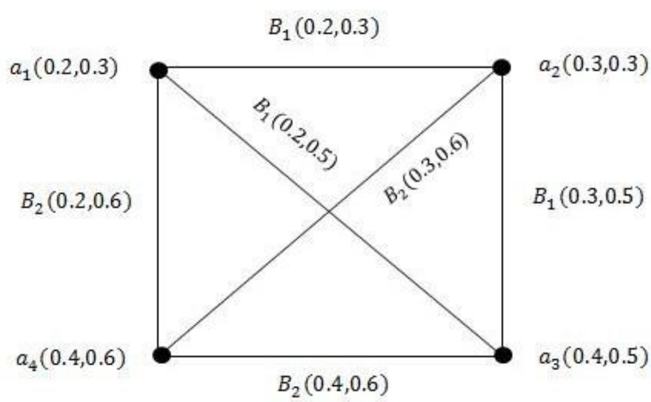

Figure 5. VGS $\breve{G}_v = (A, B_1, B_2)$

Definition 20. In a VGS $\breve{G}_v = (A, B_1, B_2, \dots, B_n)$ with underlying vertex set U, t_{B_i} and f_{B_i} -strength of a B_i -path" $P_{B_i} = v_1 v_{2m}$ " are denoted by $\delta \cdot P_{B_i}$ and $\Delta \cdot P_{Bi}$, respectively, such that $\delta . P_{B_i} = \wedge_{j=2}^{m}[t_{B_i}(v_{j-1}v_j)]$ and $\Delta . P_{B_i} = \vee_{j=2}^{m}[f_{B_i}(v_{j-1}v_j)]$. Then we write, strength of the path $P_{B_i} = (\delta, P_{B_i}, \Delta.P_{B_i})$.

Example 5. In $\breve{G}_v = (A, B_1, B_2)$ shown in Figure 5, $P_1 = a_1a_2a_3a_1$ is a B_1 -path and $a_1a_4a_3$ is a B_2 -path and we have:

$$\delta.P_1 = t_{B_1}(a_1a_2) \wedge t_{B_1}(a_2a_3) \wedge t_{B_1}(a_3a_1) = 0.2 \wedge 0.3 \wedge 0.2 = 0.2$$

$$\Delta.P_1 = f_{B_1}(a_1a_2) \vee f_{B_1}(a_2a_3) \vee f_{B_1}(a_3a_1) = 0.3 \vee 0.5 \vee 0.5 = 0.5$$

$$\delta.P_2 = t_{B_2}(a_1a_4) \wedge t_{B_2}(a_4a_3) = 0.2 \wedge 0.4 = 0.2$$

$$\Delta.P_2 = f_{B_2}(a_1a_4) \vee f_{B_2}(a_4a_3) = 0.6 \vee 0.6 = 0.6.$$

Thus strength of B_1 -path is $P_1 = (0.2, 0.5)$, strength of P_2 -path is $P_2 = (0.2, 0.6)$.

Definition 21. *In a VGS* $\breve{G}_v = (A, B_1, B_2, \ldots, B_n)$ with underlying vertex set *U :*

(i) t_{B_i} -strength of connectedness between *x* and *y*, is defined by $t^{\infty}_{B_i}(xy) = \vee_{j\geq 1}\{t^{j}_{B_i}(xy)\}$, *where* $t^{i}_{B_i}(xy) = (t^{j-1}_{B_i} \, o \, t_{B_i})(xy)$, for $j \geq 2$ and $t^{2}_{B_i}(xy) = (t_{B_i} \, o \, t_{B_i})(xy) = \vee_z\{t_{B_i}(xz \wedge t_{B_i}(zy)\}$;

(ii) f_{B_i} -strength of connectedness between *x* and *y*, is defined by $f^{\infty}_{B_i}(xy) = \wedge_{j\geq 1}\{t^{j}_{B_i}(xy)\}$, *where* $f^{i}_{B_i}(xy) = (f^{j-1}_{B_i} \, o \, f_{B_i})(xy)$, for $j \geq 2$ and $f^{2}_{B_i}(xy) = (f_{B_i} \, o \, f_{B_i})(xy) = \wedge_z\{f_{B_i}(xz \wedge f_{B_i}(zy)\}$.

Example 6. Let $\breve{G}_v = (A, B_1, B_2)$ *be a VGS* of graph structure $G^* = (U, E_1, E_2)$ as shown in Figure 6, such that $U = \{a_1, a_2, a_3\}$, $E_1 = \{a_1a_2, a_1a_3\}$ *and* $E_2 = \{a_2a_3\}$.

Since $t_{B_1}(a_1a_2) = 0.2$, $t_{B_1}(a_1a_3) = 0.2$, $t_{B_1}(a_2a_3) = 0$.

Therefore;

$$t^2_{B_1}(a_1a_2) = (t_{B_1} \ o \ t_{B_1})(a_1a_2) = t_{B_1}(a_1a_3) \wedge t_{B_1}(a_3a_2) = 0.2 \wedge 0.3 = 0.2$$
$$t^2_{B_1}(a_2a_3) = (t_{B_1} \ o \ t_{B_1})(a_2a_3) = t_{B_1}(a_2a_1) \wedge t_{B_1}(a_1a_3) = 0.2 \wedge 0.2 = 0.2$$
$$t^2_{B_1}(a_1a_3) = (t_{B_1} \ o \ t_{B_1})(a_1a_3) = t_{B_1}(a_1a_2) \wedge t_{B_1}(a_2a_3) = 0.2 \wedge 0.3 = 0.2$$
$$t^3_{B_1}(a_1a_2) = (t^2_{B_1} \ o \ t_{B_1})(a_1a_2) = t^2_{B_1}(a_1a_3) \wedge t_{B_1}(a_3a_2) = 0.2 \wedge 0.3 = 0.2$$
$$t^3_{B_1}(a_2a_3) = (t^2_{B_1} \ o \ t_{B_1})(a_2a_3) = t^2_{B_1}(a_2a_1) \wedge t_{B_1}(a_1a_3) = 0.2 \wedge 0.2 = 0.2$$
$$t^3_{B_1}(a_1a_3) = (t^2_{B_1} \ o \ t_{B_1})(a_1a_3) = t^2_{B_1}(a_1a_2) \wedge t_{B_1}(a_2a_3) = 0.2 \wedge 0.3 = 0.2$$

Thus, we have:

$$t^{\infty}_{B_1}(a_1a_2) = \vee\{0.2, 0.2, 0.2\} = 0.2,$$
$$t^{\infty}_{B_1}(a_2a_3) = \vee\{0.2, 0.2, 0.2\} = 0.2,$$
$$t^{\infty}_{B_1}(a_1a_3) = \vee\{0.2, 0.2, 0.2\} = 0.2.$$

Since $f_{B_1}(a_1a_2) = 0.6$, , $f_{B_1}(a_1a_3) = 0.7$, $f_{B_1}(a_2a_3) = 0$ Therefore;

$$f^2_{B_1}(a_1a_2) = (f_{B_1} \ o \ f_{B_1})(a_1a_2) = f_{B_1}(a_1a_3) \vee f_{B_1}(a_3a_2) = 0.7 \vee 0.7 = 0.7$$
$$f^2_{B_1}(a_2a_3) = (f_{B_1} \ o \ f_{B_1})(a_2a_3) = f_{B_1}(a_2a_1) \vee f_{B_1}(a_1a_3) = 0.6 \vee 0.7 = 0.6$$
$$f^2_{B_1}(a_1a_3) = (f_{B_1} \ o \ f_{B_1})(a_1a_3) = f_{B_1}(a_1a_2) \vee f_{B_1}(a_2a_3) = 0.6 \vee 0 = 0.6$$
$$f^3_{B_1}(a_1a_2) = (f^2_{B_1} \ o \ f_{B_1})(a_1a_2) = f^2_{B_1}(a_1a_3) \vee f_{B_1}(a_3a_2) = 0.6 \vee 0.7 = 0.7$$
$$f^3_{B_1}(a_2a_3) = (f^2_{B_1} \ o \ f_{B_1})(a_2a_3) = f^2_{B_1}(a_2a_1) \vee f_{B_1}(a_1a_3) = 0.7 \vee 0.7 = 0.7$$
$$f^3_{B_1}(a_1a_3) = (f^2_{B_1} \ o \ f_{B_1})(a_1a_3) = f^2_{B_1}(a_1a_2) \vee f_{B_1}(a_2a_3) = 0.7 \vee 0.7 = 0.7,$$

and

$$f^4_{B_1}(a_1a_2) = (f^3_{B_1} \ o \ f_{B_1})(a_1a_2) = f^3_{B_1}(a_1a_3) \vee f_{B_1}(a_3a_2) = 0.7 \vee 0.7 = 0.7$$

$$f_{B_1}^4(a_2a_3) = (f_{B_1}^3 \ o \, f_{B_1})(a_2a_3) = f_{B_1}^3(a_2a_1) \vee f_{B_1}(a_1a_3) = 0.7 \vee 0.7 = 0.7$$
$$f_{B_1}^4(a_1a_3) = (f_{B_1}^3 \ o \, f_{B_1})(a_1a_3) = f_{B_1}^2(a_1a_2) \vee f_{B_1}(a_2a_3) = 0.7 \vee 0.7 = 0.7.$$

Thus, we have:

$$f_{B_1}^{\infty}(a_1a_2) = \vee\{0.6, 0.7, 0.7, 0.7\} = 0.7,$$
$$f_{B_1}^{\infty}(a_2a_3) = \vee\{0.0, 0.7, 0.7, 0.7\} = 0.7,$$
$$f_{B_1}^{\infty}(a_1a_3) = \vee\{0.7, 0.6, 0.7, 0.7\} = 0.2.$$

By similarity way, we can calculate $t_{B_2}^{\infty}(a_1a_2), t_{B_2}^{\infty}(a_2a_3)$, and $t_{B_2}^{\infty}(a_1a_3)$.

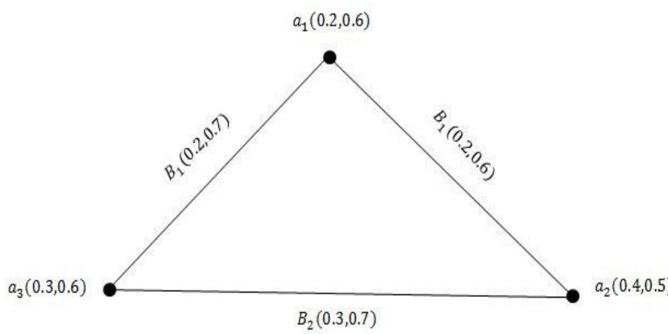

Figure 6. VGS $\breve{G}_v = (A, B_1, B_2)$

Definition 22. A VGS $\hat{G}_v = (A, B_1, B_2, ..., B_n)$ of a graph structure $\hat{G}_v = (A, B_1, B_2, ..., B_n)$ is a B_i-cycle, if G^* is an E_i-cycle.

Definition 23. A VGS $\hat{G}_v = (A, B_1, B_2, ..., B_n)$ of a graph structure $G^* = (U, E_1, E_2, ..., E_k)$, is a vague B_i-cycle, for some i, if following conditions hold:

i) G is a B_i-cycle;

ii) There is no unique B_i-edge uv in $\hat{G}_v$; such that $t_{B_i}(uv) = \min\{t_{B_i}(xy) : xy \in E_i = supp(B_i)\}$ or $f_{B_i}(uv) = \max\{f_{B_i}(xy) : xy \in E_i = supp(B_i)\}$

Example 7. VGS $\breve{G}_v = (A, B_1, B_2)$ in Figure 7, is a B_1 -cycle as well as vague B_1 -cycle, since $(supp(A), supp(A_1), supp(B_2))$ is an E_1 -cycle and there are two B_1 -edges with minimum degree of membership and two B_1 -edges with maximum degree of membership of all B_1 -edge.

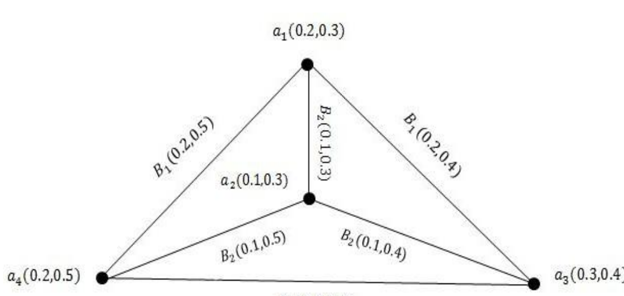

Figure 7. VGS $\breve{G}_v = (A, B_1, B_2)$

Definition 24. A VGS $\breve{G}_{v1}=(A_1,B_{11},B_{12},\ldots,B_{1n})$ of GS $G_1^*=(U_1,E_{11},E_{12},\ldots,E_{1n})$ is isomorphic to a $\breve{G}_{v2}=(A_2,B_{21},B_{22},\ldots,B_{2n})$ of $G_2^*=(U_2,E_{21},E_{22},\ldots,E_{2n})$ if there exist a bijective $f:U_1\to U_2$ and a permutation ϕ on the set $\{1,2,\ldots,n\}$, such that:

$$t_{A_1}(u_1)=t_{A_2}(f(u_1)),f_{A_1}(u_1)=f_{A_2}(f(u_1))$$

for all $u_1\in U_1$, and for $\phi(i)=j$,

$$t_{B_{1i}}(u_1u_2)=t_{B_{2i}}(f(u_1)f(u_2)),f_{B_{1i}}(u_1u_2)=f_{B_{2i}}(f(u_1)f(u_2))$$

for all $u_1u_2\in U_{1i}$, $i=1,2,\ldots,n$.

Definition 25. A VGS $\breve{G}_{v1}=(A_1,B_{11},B_{12},\ldots,B_{1n})$ of GS $G_1^*=(U_1,E_{11},E_{12},\ldots,E_{1n})$ is identical to a $VGS\ \breve{G}_{v2}=(A_2,B_{21},B_{22},\ldots,B_{2n})$ of $G_2^*=(U_2,E_{21},E_{22},\ldots,E_{2n})$ if there exist a bijective $f:U_1\to U_2$, such that:

$$t_{A_1}(u)=t_{A_2}(f(u)),f_{A_1}(u)=f_{A_2}(f(u))$$

for all $u\in U$,

$$t_{B_{1i}}(u_1u_2)=t_{B_{2i}}(f(u_1)f(u_2)),f_{B_{1i}}(u_1u_2)=f_{B_{2i}}(f(u_1)f(u_2))$$

for all $u_1u_2\in E_{1i}$, $i=1,2,\ldots,n$.

Example 8. Let $\breve{G}_{v1}$ and $\breve{G}_{v2}$ be two VGSs of graph structures $G_1^*=(U_1,E_1,E_2)$ and $G_2^*=(U_2,E_1',E_2')$, respectively, as shown in Fig. 8. Here, $\breve{G}_{v1}$ iss isomorphic (not identical) to $\breve{G}_{v2}$ under the mapping $f:U_1\to U_2$, defined by $f(a_1)=b_1$, $f(a_2)=b_2$, and $f(a_3)=b_3$ and a permutation ϕ given by $\phi(1)=2,\phi(2)=1$, such that

$$t_{A_1}(a_i)=t_{A_2}(f(a_i)),f_{A_1}(a_i)=f_{A_2}(f(a_i))\ ,\forall a_i\in U \text{ for all } u\in U,$$

$$t_{B_k}(a_ia_j)=t_{B_{\phi(k)}}(f(a_i)f(a_j)),f_{B_k}(a_ia_j)=f_{B_{\phi(k)}}(f(a_i)f(a_j)),\forall a_ia_j\in E_k,k=1,2.$$

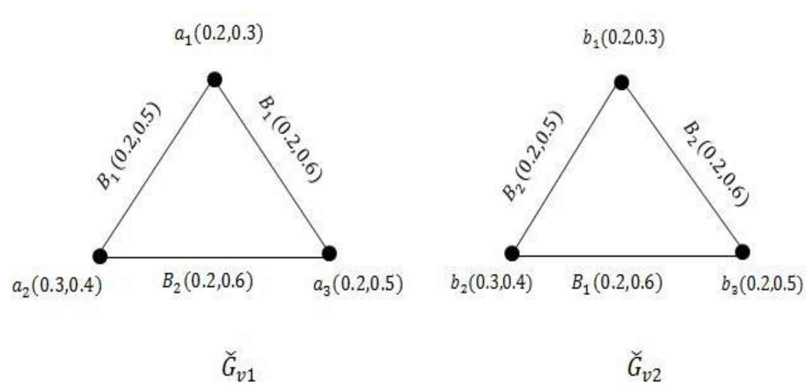

Figure 8. Isomorphic vague graph structures

Example 9. Let $\breve{G}_{v1}=(A,B_1,B_2)$ and $\breve{G}_{v2}=(A',B_1',B_2')$ be two VGSs of graph structures $G_1^*=(U_1,E_1,E_2)$ and $G_2^*=(U_2,E_1',E_2')$, respectively, as shown in Figure 9. Here, $\breve{G}_{v1}$ is isomorphic with $\breve{G}_{v2}$ under the mapping $f:U\to U'$, defined by $f(a_1)=b_6$, $f(a_2)=b_2$, $f(a_3)=b_4,f(a_5)=b_1$

and $f(a_6) = b_2$, a permutation ϕ given by , such that:

$t_A(a_i) = t_{A'}(f(a_i)), f_A(a_i) = f_{A'}(f(a_i))$, $\forall a_i \in U$ for all $u \in U$,

$t_{B_k}(a_i a_j) = t_{B'_{(k)}}(f(a_i)f(a_j)), f_{B_k}(a_i a_j) = f_{B'_{(k)}}(f(a_i)f(a_j)), \forall a_i a_j \in E_k, k = 1,2.$

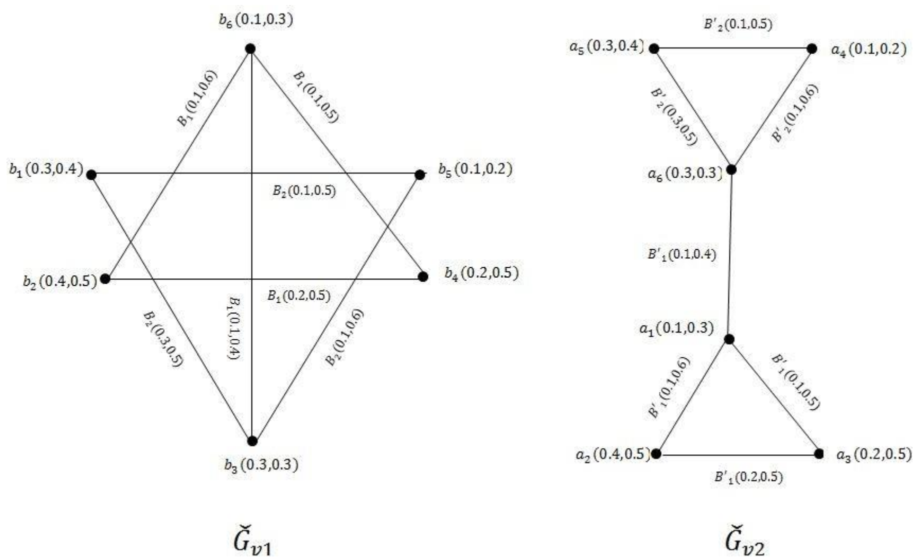

Figure 9. Identical vague graph structures

Definition 26. Let $\hat{G}_v = (A, B_1, B_2, ..., B_n)$ be a vague graph structure of a graph structure $G^* = (U, E_1, E_2, ..., E_k)$. Let ϕ denote a permutation on the set $\{E_1, E_2, ..., E_n\}$ and the corresponding permutation on $\{B_1, B_2, ..., B_n\}$, *i.e.*, $\phi(B_i) = B_j$ if and only if $\phi(E_i) = E_j$, for all i. If $xy \in B_r$ for some r and $t_{B_i^\phi}(xy) = t_A(x) \wedge t_A(y) - \vee_{j \neq i} t_{\phi(B_j)}(xy)$, $f_{B_i^\phi}(xy) = f_A(x) \vee f_A(y) - \vee_{j \neq i} f_{\phi(B_j)}(xy)$, $i = 1, 2, ..., n$, then $xy \in B_m^\phi$ while m is chosen such that $t_{B_m^\phi}(xy) \geq t_{B_i^\phi}(xy)$ and $f_{B_m^\phi}(xy) \geq f_{B_i^\phi}(xy)$, for all i. Then VGS $(A, B_1^\phi, B_2^\phi, ..., B_n^\phi)$ denoted by $G_v^{\phi c}$,is called the ϕ -complement of VGS.

Example 10. Consider VGS $\hat{G}_v = (A, B_1, B_2)$ shown in Figure 10 and let ϕ be a permutation on the set $\{B_1, B_2\}$ such that $\phi(B_1) = B_2$ and $\phi(B_2) = B_1$. Now for $a_1 a_2 \in B_1$.

$$t_{B_1}^\phi(a_1a_2) = t_A(a_1) \wedge t_A(a_2) - \bigvee_{j \neq 1}[t_{\phi B_j}(a_1a_2)] = 0.2 - 0.2 = 0,$$

$$f_{B_1}^\phi(a_1a_2) = f_A(a_1) \vee f_A(a_2) - \bigvee_{j \neq 1}[f_{\phi B_j}(a_1a_2)] = 0.5 - 0.5 = 0,$$

$$f_{B_2}^\phi(a_1a_2) = f_A(a_1) \vee f_A(a_2) - \bigvee_{j \neq 2}[f_{\phi B_j}(a_1a_2)] = 0.5 - 0 = 0.5.$$

Clearly, $t_{B_2^\phi}(a_1a_2) = 0.2 > 0 = t_{B_1^\phi}(a_1a_2)$ *and* $f_{B_1^\phi}(a_1a_2) = 0.5 > 0 = f_{B_1^\phi}(a_1a_2)$ so $a_1a_2 \in B_2^\phi$.

$$t_{B_1^\phi}(a_1a_3) = t_A(a_1) \wedge t_A(a_3) - \vee[t_{\phi B_2}(a_1a_3)] = 0.2 - 0.2 = 0,$$

$$f_{B_1^\phi}(a_1a_3) = f_A(a_1) \vee f_A(a_3) - \vee[f_{\phi B_2}(a_1a_3)] = 0.4 - 0.4 = 0,$$

$$t_{B_2^\phi}(a_1a_3) = t_A(a_1) \wedge t_A(a_3) - \vee[t_{\phi B_1}(a_1a_3)] = 0.2 - 0 = 0.2,$$
$$f_{B_2^\phi}(a_1a_3) = f_A(a_1) \vee f_A(a_3) - \vee[f_{\phi B_1}(a_1a_3)] = 0.4 - 0 = 0.4.$$

So, $a_1a_3 \in B_2^\phi$.

And for a_2a_3 we have

$$t_{B_1^\phi}(a_2a_3) = t_A(a_2) \wedge t_A(a_3) - \vee[t_{\phi B_2}(a_2a_3)] = 0.2 - 0 = 0.2,$$
$$f_{B_1^\phi}(a_2a_3) = f_A(a_2) \vee f_A(a_3) - \vee[f_{\phi B_2}(a_2a_3)] = 0.5 - 0 = 0.5,$$
$$t_{B_2^\phi}(a_2a_3) = t_A(a_2) \wedge t_A(a_3) - \vee[t_{\phi B_1}(a_2a_3)] = 0.2 - 0.2 = 0,$$
$$f_{B_2^\phi}(a_2a_3) = f_A(a_2) \vee f_A(a_3) - \vee[f_{\phi B_1}(a_2a_3)] = 0.4 - 0 = 0.4,$$

So , $a_2a_3 \in B_1^\phi$.

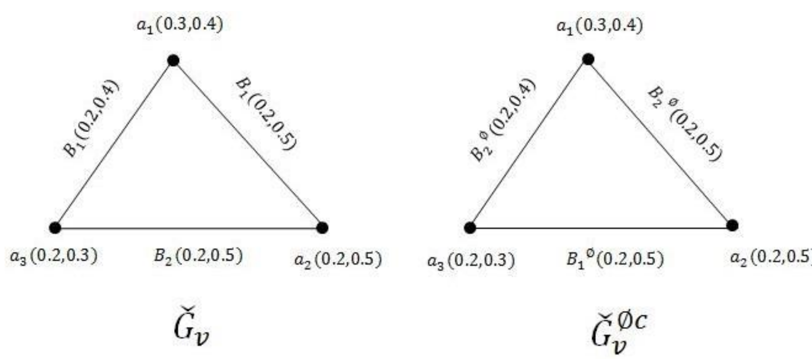

Figure 10. Vague graph structures $\breve{G}_v$ and $\breve{G}_v^{\phi c}$

Theorem 1. A ϕ -complement of a vague graph structure is always a strong VGS. Moreover, if $\phi(i) = r$, for $i \in \{1, 2, \ldots, n\}$, then all B_r -edges in VGS $\breve{G}_v = (A, B_1, B_2, \ldots, B_n)$ become B_i^ϕ -edges in $\breve{G}_v^{\phi c} = (A, B_1^\phi, B_2^\phi, \ldots, B_n^\phi)$.

Proof. First part is obvious from the definition of ϕ -complement $\breve{G}_v^{\phi c}$ of VGS $\breve{G}_v$ since for any B_i^ϕ -edge *xy*, $t_{B_i}^\phi(xy)$ and $f_{B_i}^\phi(xy)$ respectively have the maximum values of

$$[t_A(x) \wedge t_A(y)] - \bigvee_{j \neq i} t_{\phi B_j}(xy) \text{ and}$$
$$[t_A(x) \vee f_A(y)] - \bigvee_{j \neq i} f_{\phi B_j}(xy).$$

That is

$$t_{B_i}^\phi(xy) = t_A(x) \wedge t_A(y),$$
$$f_{B_i}^\phi(xy) = f_A(x) \vee f_A(y),$$

for all edges *xy* in $\breve{G}_v^{\phi c}$, hence $\breve{G}_v^{\phi c}$ is always a strong VGS. Now suppose on contrary that $\phi(i) = r$ but *xy* is B_s -edge in $\breve{G}_v$ with $s \neq r$ which implies that $\phi B_i \neq B_s$. Comparing expressions we get:

$$\underset{j\neq i}{\vee} t_{\phi B_j}(xy)=0 \quad , \quad \underset{j\neq i}{\vee} f_{\phi B_j}(xy)=0,$$

which is not possible because $B_s=\phi B_j$ for some $j\in\{1,2,\ldots,i-1,i+1,\ldots,n\}$.

So, Our supposition is wrong and xy must be a B_r-edge. Hence we can calculate that if $\phi(i)=r$, then all B_r-edge in VGS $\breve{G}_v=(A,B_1,B_2,\ldots,B_n)$ become B_i^{ϕ}-edges in $\breve{G}_v^{\phi c}=(A,B_1^{\phi},B_2^{\phi},\ldots,B_n^{\phi})$, for $r,i\in\{1,2,\ldots,n\}$.

Definition 27. Let $\breve{G}_v=(A,B_1,B_2,\ldots,B_n)$ be a VGS and ϕ be a permutation on the set $\{1,2,\ldots,n\}$. Then

(i) $\breve{G}_v$ is self-complementary, if it is isomorphic to $\breve{G}_v^{\phi c}$, the ϕ-complement of $\breve{G}_v$.

(ii) $\breve{G}_v$ is self-complementary, if it is identical to $\breve{G}_v^{\phi c}$.

(iii) $\breve{G}_v$ is totally self-complementary if it is isomorphic to $\breve{G}_v^{\phi c}$,the ϕ-complement of $\breve{G}_v$ for all permutations ϕ on the set $\{1,2,\ldots,n\}$.

(iiii) $\breve{G}_v$ is totally strong self-complementary, if it is identical to $\breve{G}_v^{\phi c}$, the ϕ-complement of $\breve{G}_v$, for all permutations ϕ on the set $\{1,2,\ldots,n\}$.

Theorem 2. A VGS $\breve{G}_v$ is strong if and only if $\breve{G}_v$ is totally self-complementary.

Proof. Let $\breve{G}_v$ be a strong VGS and ϕ be a permutation on the set $\{1,2,\ldots,n\}$ .If $\phi^{-1}(i)=j$, hen by Theorem 27, all B_i-edges in $\breve{G}_v=(A,B_1,B_2,\ldots,B_n)$ become B_i^{ϕ}-edges in $\breve{G}_v^{\phi c}=(A,B_1^{\phi},B_2^{\phi},\ldots,B_n^{\phi})$. Also $\breve{G}_v^{\phi c}$ is strong, so

$$t_{B_i}(a_1a_2)=t_A(a_1)\wedge t_A(a_2)=t_{B_j^{\phi}}(a_1a_2)$$
$$f_{B_i}(a_1a_2)=f_A(a_1)\vee f_A(a_2)=f_{B_j^{\phi}}(a_1a_2).$$

Then $\breve{G}_v$ is isomorphic to $\breve{G}_v^{\phi c}$, under the identity mapping $f:U\to U$ and a permutation ϕ, such that:

$$t_A(a)=t_A(f(a)) \quad , \quad f_A(a)=f_A(f(a)) \; , \; \text{for all } a\in U$$

and

$$t_{B_i}(a_1a_2)=t_{B_j^{\phi}}(a_1a_2)=t_{B_j^{\phi}}(f(a_1)f(a_2)),$$
$$f_{B_i}(a_1a_2)=f_{B_j^{\phi}}(a_1a_2)=f_{B_j^{\phi}}(f(a_1)f(a_2)),$$

for all $a_1a_2\in E_i$ .This hold for all the permutation on the set $\{1,2,\ldots,n\}$ .Hence $\breve{G}_v$ is totally self-complementary. Conversely, Let ϕ be any permutation on the set $\{1,2,\ldots,n\}$ and $\breve{G}_v$ and $\breve{G}_v^{\phi c}$ be isomorphic. From the definition of ϕ-complement and isomorphism of VGSs, we have:

$$t_{B_i}(a_1a_2)=t_{B_j^{\phi}}(f(a_1)f(a_2))=t_A(f(a_1))\wedge t_A(f(a_2))=t_{B_i}(a_1a_2)=t_A(a_1)\wedge t_A(a_2)$$
$$f_{B_i}(a_1a_2)=f_{B_j^{\phi}}(f(a_1)f(a_2))=f_A(f(a_1))\vee f_A(f(a_2))=f_{B_i}(a_1a_2)=f_A(a_1)\vee f_A(a_2)$$

for all $a_1a_2\in E_i$, $i=1,2,\ldots,n$. Hence, $\breve{G}_v$ is a strong VGS.

Theorem 3. If graph structure $G^* = (U, E_1, E_2, \dots, E_n)$ is totally strong self-complementary and A is a vague set of U with constant fuzzy mapping t_A and f_A, then a strong VGS $\breve{G}_v = (A, B_1, B_2, \dots, B_n)$ of G^* is totally strong self-complementary.

Proof. Consider a strong VGS $\breve{G}_v = (A, B_1, B_2, \dots, B_n)$ of a graph structure $G^* = (U, E_1, E_2, \dots, E_n)$ .Suppose that G^* is totally strong self-complementary and that for some constants $s, t \in [10,1]$, $A = (t_A, f_A)$ is a vague set of u such that $t_A(u) = s$, $f_A(u) = t$, for all $u \in U$. Then we have to prove that $\breve{G}_v$ is totally strong self-complementary.

Let ϕ be an arbitrary permutation on the set $\{1, 2, \dots, n\}$ and $\phi^{-1}(j) = i$ .Since G^* s totally strong self-complementary, so there exists a bijection $f : U \to U$, such that for every E_i-edge a_1a_2 in G^* , $f(a_1)f(a_2)$ is an E_i-edge in $(G^*)^{\phi^{-1}c}$.Consequently, for every B_i-edge a_1a_2 in $\breve{G}_v$, $f(a_1)f(a_2)$ is a B_i^{ϕ}-edge in $\breve{G}_v^{\phi c}$. From the definition of *A* and the definition of strong VGS $\breve{G}_v$

$$t_A(a) = s = t_A(f(a)), \quad f_A(a) = t = f_A(f(a)), \textit{ for all } a, f(a) \in U,$$

and

$$t_{B_i}(a_1a_2) = t_A(a_1) \wedge t_A(a_2) = t_A(f(a_1)) \wedge t_A(f(a_2)) = t_{B_j^{\phi}}(f(a_1)f(a_2))$$
$$f_{B_i}(a_1a_2) = f_A(a_1) \vee f_A(a_2) = f_A(f(a_1)) \vee f_A(f(a_2)) = f_{B_j^{\phi}}(f(a_1)f(a_2)),$$

for all $a_1a_2 \in E_i$, $i = 1, 2, \dots, n$, which shows $\breve{G}_v$ is strong self-complementary.

Hence, $\breve{G}_v$ is totally strong self-complementary, since ϕ is arbitrary.

Example 11. A VGS $\breve{G}_v = (A, B_1, B_2, \dots, B_n)$ of graph structure $G^* = (U, E_1, E_2, \dots, E_n)$ as shown in Fig. 11 is totally strong self-complementary.

4. Application example of vague influence graph structure

Graph models find wide application in many areas of mathematics, computer science, and the natural and social science. Often these models need to incorporate more structure than simply the adjacencies between vertices. In studies of group behavior, it is observed that certain people can influence thinking of others. A directed graph, called an influence graph, can be used to model this behavior. Each person of a group is represented by a vertex. There is a directed edge from vertex x to vertex y, when the person represented by vertex x influence the person represented by vertex y. This graph does not contain loops and it does not contain multiple directed edges. We now explore vague influence graph structure model to find out the influence person within a social group. In influence graph, the vertex (node) represents a power (authority) of a person and the edge represents the influence of a person on another person in the social group. Consider a vague influence graph structure of a social group.

In Figure 12, vague influence graph structure, the degree of power of a person is defined in terms of its trueness and falseness. The node of the vague influence graph structure shows the authority a person possesses in the group; for example C has 40% authority in the group, but he does not have 20% power, and 40% power is not decided, whereas the edges show the influence of a person on another in a group, for example C can influence B, 30% but he cannot convince him 60%, and remaining 10% is hesitation part. The degree of a vertex and edge in a vague influence graph structure is also characterized by an interval $[t_A(x), 1 - f_A(x)]$.

It is worth mentioning here that interval-valued fuzzy sets are not vague sets. In vague sets both are independently proposed by the decision maker. The node of the vague influence graph structure shows that likelihood of power a person possesses in the group.

For example, C possesses $t_C = 40\%$ to $1 - f_C = 80\%$ power, whereas the edges show the interval of influence a person has on another person in a social group C has $t_C = 30\%$ to $1 - f_C = 40\%$ influence on B and B has $t_B = 20\%$ to $1 - f_B = 40\%$ influence on E.

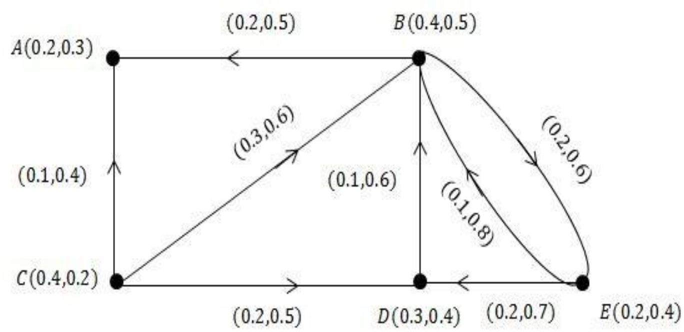

Figure 12. Vague influence graph structure

5. Conclusions

It is well known that graphs are among the most ubiquitous models of both natural and human- made structures. They can be used to model many types of relations and process dynamics in computer science, physical, biological, and social systems. In this paper, we introduced certain notions, including vague graph structure (VGS), strong vague graph structure, vague B_i -cycle, and illustrated these notions by several examples. We studied ϕ -complement, self-complement, strong self-complement, and totally strong self-complement in vague graph structures and investigated some of their properties. Finally an application of vague influence graph structure in social group is given.

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