FUZZY OPTIMIZATION AND MODELLING 4(3) (2023) 1-15

E-ISNN: 2676-7007

Contents lists available at FOMJ

Fuzzy Optimization and Modelling

Journal homepage: http://fomj.qaemiau.ac.ir/

Paper Type: Research Paper

A Novel Approach for Solving Linear Programming Problems with Intuitionistic Fuzzy Numbers

Ali Mahmoodirad*

Department of Mathematics, Ayatollah Amoli Branch, Islamic Azad University, Amol, Iran

A R T I C L E I N F O

Article history:
Received 8 September 2023
Revised 3 October 2023
Accepted 8 October 2023
Available online 8 October 2023

Keywords:
Linear Programming
Trapezoidal Intuitionistic Fuzzy Number
Multi-objective Optimization
Fuzzy Programming Approach

A B S T R A C T

The literature on the linear programming problem with trapezoidal intuitionistic parameters is full of solution approaches which are mainly ranking function based. Use of the ranking function in the solution approaches could be a weakness as different ranking functions give different solutions. This paper, proposes a new solution approach without any ranking function for linear programming problem with trapezoidal intuitionistic parameters. For this aim, the trapezoidal intuitionistic fuzzy objective function is converted to a multi-objective function, and consequently, the problem is converted to a multi-objective crisp problem. As another contribution, in order to solve the obtained multi-objective problem for its efficient solutions, a new multi-objective optimization approach was developed and suited to the obtained multi-objective problem. The computational experiments of the study show the superiority of the proposed multi-objective optimization approach over the multi-objective optimization approaches of the literature.

1. Introduction

Linear programming is one of the most applied techniques in optimization problems, which consists of a set of linear constraints to be satisfied in order to optimize a linear objective function. In real world applications of this problem, the values of the parameters should be precisely defined and described [25, 35]. However, there are many diverse situations due to uncertainty in judgements, lack of evidence, etc, which make it impossible to get relevant precise data for the parameters. This type of imprecise data is not always well represented by the random variable selected from a probability distribution [14, 30]. Therefore, it may represent this data as fuzzy data. So, a fuzzy decision making method is needed in such cases. As a result, by considering fuzzy parameters in linear programming, fuzzy linear programming is defined.

According to Wan and Dong [40], fuzzy linear programming problems are of three types. The first class of these problems deals with parameters taking triangular fuzzy numbers (TFNs). Lai and Hwang [17] converted such problems to a multi-objective formulation to solve it. Lotfi et al. [19] applied lexicography and fuzzy approximate approaches to tackle linear programming with TFNs. Kumar et al. [16] also focused on linear

* Correspondig author
E-mail address: alimahmoodirad@gmail.com
DOI: 10.30495/fomj.2023.1995970.1113

programming with TFNs using a conversion based approach. The second class of these problems deals with parameters taking trapezoidal fuzzy numbers (TrFNs). Maleki et al. [23] focused on fully fuzzy linear programming with TrFNs. Meaning that, all parameters and variables are of TrFNs. Liu [18] introduced a method to measure the satisfaction of the constraints in linear programming with TrFNs. Maleki and Mashinchi [22] solved linear programming with TrFNs by a probabilistic approach. Allahviranloo et al. [1] proposed a ranking function based approach for linear programming with TrFNs. Ebrahimnejad [11] proposed a revised fuzzy simplex method for linear programming with TrFNs and obtained some new results. Das et al. [7] introduced a solution method for fully fuzzy linear programming with TrFNs. Dong and Wan [8] proposed a new approach for trapezoidal fuzzy linear programming problems considering the acceptance degree of fuzzy constraints violates.

The third class of these problems deal with parameters taking intuitionistic fuzzy numbers. Atanassov [4] introduced the theory of intuitionistic fuzzy set (IFS) that is an extension of fuzzy theory and is highly useful for real life problems to deal with vague information. The most important advantage of IFS comparing to fuzzy set is that it isolates the membership and non-membership degrees of a number of the set in a way that for an element, the sum of these degrees is less than or equal to one. Therefore, this theory seems to be very applicable when considering the vagueness in the estimation of parameters by a decision-maker. Angelove [3] introduced an approach based on rejection level of the constraints and values of the objective function of intuitionistic fuzzy linear programming to convert it to a crisp form. Dubey et al. [9] represented interval based linear programming with intuitionistic fuzzy linear programming. The studies of Kumar and Hussain [15] and Singh and Yadav [37] include ranking based approaches for intuitionistic transportation problem. In recent years, intuitionistic fuzzy numbers have been used in many optimization problems. As, multi-criteria decision making [28, 33], fuzzy multi-objective linear fractional optimization [24], selection problem [30, 6], data envelopment analysis [36], etc.

In this paper, we proposed a novel approach to solve intuitionistic fuzzy linear programming problems. The parameters are supposed to be trapezoidal intuitionistic fuzzy numbers which reflect the uncertain nature of these parameters. This type of fuzzy numbers is more suitable to reflect the fuzzy parameters. In trapezoidal intuitionistic fuzzy number, the most likely value is an interval of real numbers. This property makes this type of fuzzy values more suitable to reflect the uncertain parameters of linear programming problems. Linear programming with intuitionistic fuzzy parameters and its special cases like transportation problem have been previously focused [10, 27, 31, 32, 38, 15, 20, 37, 21, 29], etc.

In most of these studies, ranking functions are applied to crisp the intuitionistic fuzzy parameters and then solve the crisp formulation by classical methods. Using ranking function in the solution procedures is a weakness as using different ranking functions in a solution approach may result in obtaining different solutions. This weakness is a motivation for this study to introduce a solution approach which is not dependent to any ranking function. For this aim, the linear programming with trapezoidal intuitionistic fuzzy parameters is converted to a multi-objective crisp linear programming in which considering all the objective functions together gives an intuitionistic objective function value. As another contribution of this study, a new multi-objective optimization approach is proposed to solve the obtained multi-objective crisp linear programming. As an advantage of such approach, it gives more flexibility to decision-makers. The obtained results from the computational experiments of the study show the superiority of the proposed multi-objective optimization approach comparing to those of literature.

The rest of this paper is organized by the following sections. Section 2 presents some initial definitions of intuitionistic fuzzy numbers. Section 3 describes the mathematical formulation of linear programming with trapezoidal intuitionistic fuzzy parameters and the proposed solution approach. Computational experiments are performed in Section 4. Finally, conclusions are drawn by Section 5.

2. Initial Definitions

Some basic definitions from the intuitionistic fuzzy set (IFS) and interval programming which will be applied later in this paper are explained in this sub-section.

Definition 1 [4]**.** Let X be a universe of discourse. Then an IFS $\tilde{A}^I$ in X is defined by a set of ordered triples

$$\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)\rangle : x \in X\} \tag{1}$$

where, $\mu_{\tilde{A}^I}, \upsilon_{\tilde{A}^I} : X \to [0,1]$ and $0 \le \mu_{\tilde{A}^I}(x) + \upsilon_{\tilde{A}^I}(x) \le 1$ ($x \in X$) are held. In this definition $\mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)$ $(x \in X)$ are called degree of membership and degree of non-membership respectively. Also for $x \in X$ the following relation calculates degree of hesitation ($h(x)$).

$$h(x) = 1 - \mu_{\tilde{A}^I}(x) - \upsilon_{\tilde{A}^I}(x) \tag{2}$$

Definition 2 [4]**.** An IFS $\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)\rangle : x \in X\}$, must have the following two conditions,

i. There should be a real number r such that $\mu_{\tilde{A}^I}(r) = 1$ and, $\upsilon_{\tilde{A}^I}(r) = 0$,

ii. $\mu_{\tilde{A}^I}(x)$ and $\upsilon_{\tilde{A}^I}(x)$ are piecewise continuous mapping from the set of real numbers to the interval $[0,1]$ where $0 \le \mu_{\tilde{A}^I}(x) + \upsilon_{\tilde{A}^I}(x) \le 1$.

The membership and non-membership functions of the trapezoidal intuitionistic fuzzy number (TrIFN) $\tilde{A}^I = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$ are defined as follows,

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \frac{a_4 - x}{a_4 - a_3} & a_3 \le x \le a_4 \\ 0 & \text{Otherwise} \end{cases} \tag{3}$$

$$\upsilon_{\tilde{A}^I}(x) = \begin{cases} \frac{a_2 - x}{a_2 - a_1'} & a_1' \le x \le a_2 \\ 0 & a_2 \le x \le a_3 \\ \frac{x - a_3}{a_4' - a_3} & a_3 \le x < a_4' \\ 1 & \text{Otherwise} \end{cases} \tag{4}$$

where, $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$. These functions are schematically shown in Figure 1.

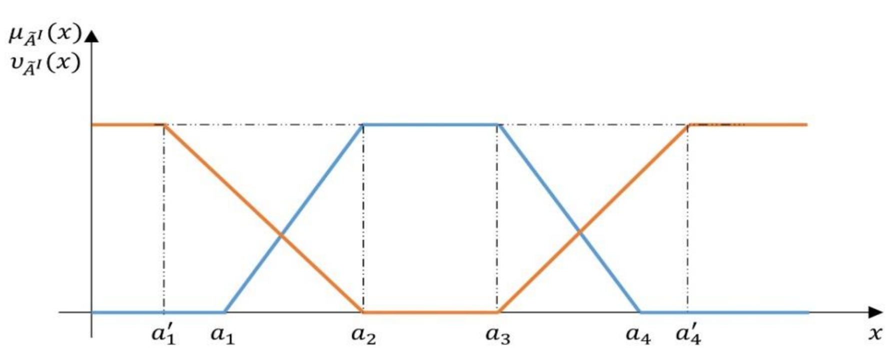

Figure 1. Membership and non-membership functions for a TrIFN.

Definition 3 [37]**.** The following operations can be done on TrIFNs $\tilde{A}^I=(a_1,a_2,a_3,a_4;a_1',a_2,a_3,a_4')$ and, $\tilde{B}^I=(b_1,b_2,b_3,b_4;b_1',b_2,b_3,b_4')$,

$$\tilde{A}^I\oplus\tilde{B}^I=(a_1+b_1,a_2+b_2,a_3+b_3;a_1'+b_1',a_2+b_2,a_3'+b_3') \quad (5)$$

$$\tilde{A}^I\Theta\tilde{B}^I=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1;a_1'-b_4',a_2-b_3,a_3-b_2,a_4'-b_1') \quad (6)$$

$$\tilde{A}^I\otimes\tilde{B}^I=(l_1,l_2,l_3,l_4;l_1',l_2,l_3,l_4') \quad (7)$$

$$k\tilde{A}^I=(ka_1,ka_2,ka_3,ka_4;ka_1',ka_2,ka_3',ka_4') \qquad k\geq 0 \quad (8)$$

$$k\tilde{A}^I=(ka_4,ka_3,ka_2,ka_1;ka_4',ka_3',ka_2,ka_1') \qquad k<0 \quad (9)$$

where $l_1=\min\{a_1b_1,a_1b_4,a_4b_1,a_4b_4\}$, $l_2=\min\{a_2b_2,a_2b_3,a_3b_2,a_3b_3\}$, $l_3=\max\{a_2b_2,a_2b_3,a_3b_2,a_3b_3\}$, $l_4=\max\{a_1b_1,a_1b_4,a_4b_1,a_4b_4\}$, $l_1'=\min\{a_1'b_1',a_1'b_4',a_4'b_1',a_4'b_4'\}$, $l_4'=\max\{a_1'b_1',a_1'b_4',a_4'b_1',a_4'b_4'\}$.

Theorem 1. The TrIFNs $\tilde{A}^I=(a_1,a_2,a_3,a_4;a_1',a_2,a_3,a_4')$ and, $\tilde{B}^I=(b_1,b_2,b_3,b_4;b_1',b_2,b_3,b_4')$ are equal if and only if $a_1=b_1$, $a_2=b_2$, $a_3=b_3$, $a_4=b_4$, $a_1'=b_1'$, and, $a_4'=b_4'$.

Proof. The proof is straightforward. ■

Definition 4 [29]. If $A=[a_l,a_u]$ be an interval number, the formulation,

$$\begin{aligned}&\min\{A\}\\&\text{subject to}\\&A\in\Omega\end{aligned} \quad (10)$$

is converted to the following bi-objective formulation where Ω is the set of constraints of the problem (10).

$$\begin{aligned}&\min\left\{\frac{1}{2}(a_l+a_u),a_u\right\}\\&\text{subject to}\\&A\in\Omega\end{aligned} \quad (11)$$

3. Intuitionistic Fuzzy Linear Programming

In extension of linear programming with trapezoidal fuzzy numbers, in this section of the paper a linear programming with TrIFNs with n variables and m constraints is defined in two different cases. The notations used in the formulations are defined by Table 1 in advance.

Table 1. Description of the notations used in the formulations

Notation	Description	Expanded form
$\mathbf{x}$	Vector of decision variables	$\mathbf{x}=(x_1,x_2,\ldots,x_n)^T$
$\tilde{Z}$	Triangular intuitionistic objective function value	
$\mathbf{c}$	Vector of objective function coefficients (crisp values)	$\mathbf{c}=(c_1,c_2,\ldots,c_n)^T$
$\mathbf{b}$	Vector of resource availabilities (crisp values)	$\mathbf{b}=(b_1,b_2,\ldots,b_n)^T$
$\mathbf{A}$	Matrix of technological coefficients (crisp values)	$\mathbf{A}=(a_{ij})_{m\times n}$
$\tilde{\mathbf{c}}$	Vector of TrIFNs for objective function coefficients	$\mathbf{c}_1=(c_{1,1},c_{2,1},\ldots,c_{n,1})^T$, $\mathbf{c}_2=(c_{1,2},c_{2,2},\ldots,c_{n,2})^T$, $\mathbf{c}_3=(c_{1,3},c_{2,3},\ldots,c_{n,3})^T$, $\mathbf{c}_4=(c_{1,4},c_{2,4},\ldots,c_{n,4})^T$, $\mathbf{c}_1'=(c_{1,1}',c_{2,1}',\ldots,c_{n,1}')^T$, $\mathbf{c}_4'=(c_{1,4}',c_{2,4}',\ldots,c_{n,4}')^T$

$\tilde{\mathbf{b}}$	Vector of TrIFNs for resource availabilities	$\mathbf{b}_1=(b_{1,1},b_{2,1},\dots,b_{m,1})^T$, $\mathbf{b}_2=(b_{1,2},b_{2,2},\dots,b_{m,2})^T$, $\mathbf{b}_3=(b_{1,3},b_{2,3},\dots,b_{m,3})^T$, $\mathbf{b}_4=(b_{1,4},b_{2,4},\dots,b_{m,4})^T$, $\mathbf{b}'_1=(b'_{1,1},b'_{2,1},\dots,b'_{m,1})^T$, $\mathbf{b}'_4=(b'_{1,4},b'_{2,4},\dots,b'_{m,4})^T$
$\tilde{\mathbf{A}}$	Matrix of TrIFNs for technological coefficients	$\mathbf{A}_1=(a_{ij,1})_{m\times n}$, $\mathbf{A}_2=(a_{ij,2})_{m\times n}$, $\mathbf{A}_3=(a_{ij,3})_{m\times n}$, $\mathbf{A}_4=(a_{ij,4})_{m\times n}$, $\mathbf{A}'_1=(a'_{ij,1})_{m\times n}$, $\mathbf{A}'_4=(a'_{ij,4})_{m\times n}$

3.1. Case 1: Linear programming with $\tilde{\mathrm{c}}$, $\mathbf{b}$, and $\mathbf{A}$

In this case of intuitionistic fuzzy linear programming, coefficients of the objective function are TrIFNs, while technological coefficients and resource availabilities are crisp values. The formulation is as follow,

$$\begin{aligned}&\min \tilde{Z}=\tilde{\mathbf{c}}^{\mathbf{T}}\mathbf{x}\\&\text{subject to}\\&\mathbf{Ax}\geq\mathbf{b}\\&\mathbf{x}\geq\mathbf{0}\end{aligned}\qquad(12)$$

Definition 5. The feasible solution of the problem (12) consists of a set of numbers $x\geq 0$ which satisfies constraint $\mathbf{Ax}\geq\mathbf{b}$. The feasible solution x^* is said to be an optimal solution if there exist any other feasible solution of the problem consisting the set of non-negative numbers x such that satisfies constraint $\mathbf{Ax}\geq\mathbf{b}$, then, $\tilde{\mathbf{c}}^{\mathbf{T}}\mathbf{x}^*\leq\tilde{\mathbf{c}}^{\mathbf{T}}\mathbf{x}$.

The objective function of model (12) is a TrIFN as $\tilde{z}=\left((\mathbf{c}_1)^T\mathbf{x},(\mathbf{c}_2)^T\mathbf{x},(\mathbf{c}_3)^T\mathbf{x},(\mathbf{c}_4)^T\mathbf{x};(\mathbf{c}'_1)^T\mathbf{x},(\mathbf{c}_2)^T\mathbf{x},(\mathbf{c}_3)^T\mathbf{x},(\mathbf{c}'_4)^T\mathbf{x}\right)$ (see Figure 2). This objective function should be minimized as a TrIFN. The TrIFN of Figure 2 has three parts as, the membership function, left side of the non-membership function, and right side of the non-membership function. To do such minimization respecting the nature of TrIFNs, the following issues should be considered.

- The trapezoidal membership function is minimized according to the method of Wan and Dong [40] by simultaneous optimization of $\min(\mathbf{c}_3)^T\mathbf{x}$, $\min\frac{1}{2}\left((\mathbf{c}_2)^T\mathbf{x}+(\mathbf{c}_3)^T\mathbf{x}\right)$, $\max(\mathbf{c}_2)^T\mathbf{x}-(\mathbf{c}_1)^T\mathbf{x}$, and, $\min(\mathbf{c}_4)^T\mathbf{x}-(\mathbf{c}_3)^T\mathbf{x}$.
- As the left side of non-membership function is related to the smaller values of $\mathbf{c}'^{\mathbf{T}}\mathbf{x}$, therefore the term $(\mathbf{c}_2)^T\mathbf{x}-(\mathbf{c}'_1)^T\mathbf{x}$ should be minimized.
- As the right side of non-membership function is related to the larger values of $\mathbf{c}'^{\mathbf{T}}\mathbf{x}$, therefore the term $(\mathbf{c}'_4)^T\mathbf{x}-(\mathbf{c}_3)^T\mathbf{x}$ should be maximized.

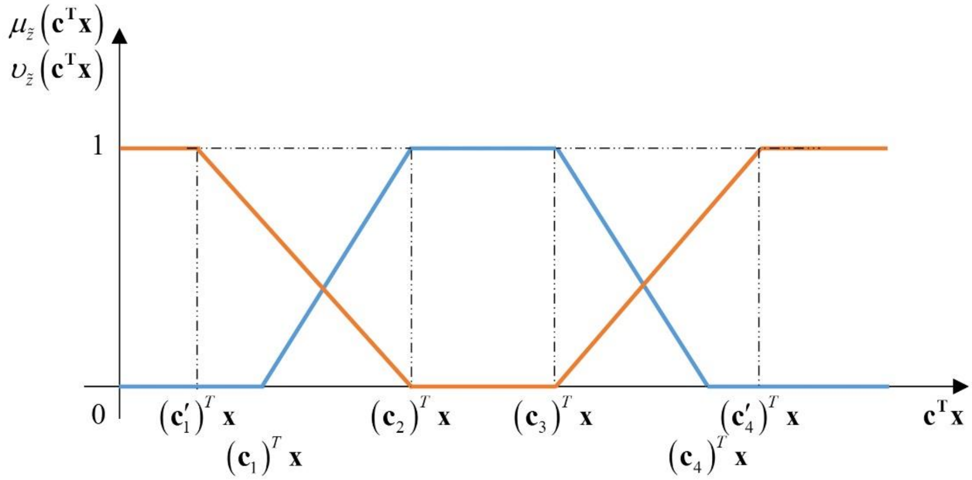

Figure 2. The TrIFN of the objective function of models (12)

Considering the above-mentioned issues, the objective function of the model (12) is replaced by six objective functions and the following multi-objective model is introduced,

$$
\begin{aligned}
&\max Z_1 = (\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}_1)^T \mathbf{x} \\
&\min Z_2 = \frac{1}{2}\left((\mathbf{c}_2)^T \mathbf{x} + (\mathbf{c}_3)^T \mathbf{x}\right) \\
&\min Z_3 = (\mathbf{c}_3)^T \mathbf{x} \\
&\min Z_4 = (\mathbf{c}_4)^T \mathbf{x} - (\mathbf{c}_3)^T \mathbf{x} \\
&\min Z_5 = (\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}_1')^T \mathbf{x} \\
&\max Z_6 = (\mathbf{c}_4')^T \mathbf{x} - (\mathbf{c}_3)^T \mathbf{x} \\
&\text{subject to} \\
&\mathbf{Ax} \geq \mathbf{b} \\
&\mathbf{x} \geq \mathbf{0}
\end{aligned}
\tag{13}
$$

Definition 6 [12, 13]. A vector $\mathbf{x}^*$ is called an efficient (Pareto optimal) solution to the multi-objective problem (13), if there is no other feasible solution $\mathbf{x}$ such that $Z_i \geq Z_i^*,\ i = 1,6,\ Z_k \leq Z_k^*,\ k = 2,3,4,5$ and $Z_i > Z_i^*,\ Z_k < Z_k^*$ for at least one $i = 1,6,\ k = 2,3,4,5$.

Theorem 1. The Pareto-optimal solution of the multi-objective formulation (13) is an optimal solution of the problem (12).

The above-mentioned multi-objective formulation should be solved for efficient (Pareto optimal) solutions. In the literature of multi-objective optimization, various approaches such as goal programming, ε-constraint approach, fuzzy programming approach, have been proposed and applied to multi-objective optimization problems. Zimmermann [41] for the first time applied a fuzzy programming approach (max-min operator [5]) to solve a multi-objective model. Unfortunately, his solution approach may not give an efficient (Pareto-optimal) solution in some cases [2]. This weakness of fuzzy programming approach later was focused by the studies that introduced the hybrid versions of fuzzy programming method. LH [17], TH [39], ABS [1], SMN [34], MMNV [26], are some of these proposed methods. Here, a new hybrid version of fuzzy programming approach is proposed to solve the model (13). The method is explained in the following steps.

Step 1. Solve the following sub-models to obtain the positive ideal solution (PIS) and the negative ideal solution (NIS) of each objective function of model (13).

$$Z_1^{PIS} = \max(\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}_1)^T \mathbf{x}$$
subject to
Constraints of model (13) (14)

$$Z_1^{NIS} = \min(\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}_1)^T \mathbf{x}$$
subject to
Constraints of model (13) (15)

$$Z_2^{PIS} = \min\frac{1}{2}\left((\mathbf{c}_2)^T \mathbf{x} + (\mathbf{c}_3)^T \mathbf{x}\right)$$
subject to
Constraints of model (13) (16)

$$Z_2^{NIS} = \max\frac{1}{2}\left((\mathbf{c}_2)^T \mathbf{x} + (\mathbf{c}_3)^T \mathbf{x}\right)$$
subject to
Constraints of model (13) (17)

$$Z_3^{PIS} = \min(\mathbf{c}_3)^T \mathbf{x}$$
subject to
Constraints of model (13) (18)

$$Z_3^{NIS} = \max(\mathbf{c}_3)^T \mathbf{x}$$
subject to
Constraints of model (13) (19)

$$Z_4^{PIS} = \min(\mathbf{c}_4)^T \mathbf{x} - (\mathbf{c}_3)^T \mathbf{x}$$
subject to
Constraints of model (13) (20)

$$Z_4^{NIS} = \max(\mathbf{c}_4)^T \mathbf{x} - (\mathbf{c}_3)^T \mathbf{x}$$
subject to
Constraints of model (13) (21)

$$Z_5^{PIS} = \min(\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}'_1)^T \mathbf{x}$$
subject to
Constraints of model (13) (22)

$$Z_5^{NIS} = \max(\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}'_1)^T \mathbf{x}$$
subject to
Constraints of model (13) (23)

$$Z_6^{PIS} = \max(\mathbf{c}'_4)^T \mathbf{x} - (\mathbf{c}_3)^T \mathbf{x}$$
subject to (24)

Constraints of model (13)

$$Z_6^{NIS}=\min\left(\mathbf{c}_4'\right)^T\mathbf{x}-\left(\mathbf{c}_3\right)^T\mathbf{x}$$
subject to
Constraints of model (13). (25)

Step 2. As each objective function, can be related to a fuzzy membership function (MF), the MFs of the objective functions are calculated through the following relationships,

$$\mu_r\left(Z_r\right)=\begin{cases}1 & \text{if} \quad Z_r\le Z_r^{PIS}\\ \dfrac{Z_r^{NIS}-Z_r}{Z_r^{NIS}-Z_r^{PIS}} & \text{if} \quad Z_r^{PIS}\le Z_r\le Z_r^{NIS}\\ 0 & \text{if} \quad Z_r\ge Z_r^{NIS}\end{cases} \quad \text{for minimum type objective functions} \tag{26}$$

$$\mu_r\left(Z_r\right)=\begin{cases}0 & \text{if} \quad Z_r\le Z_r^{NIS}\\ \dfrac{Z_r-Z_r^{NIS}}{Z_r^{PIS}-Z_r^{NIS}} & \text{if} \quad Z_r^{NIS}\le Z_r\le Z_r^{PIS}\\ 1 & \text{if} \quad Z_r\ge Z_r^{PIS}\end{cases} \quad \text{for maximum type objective functions} \tag{27}$$

where $\mu_r\left(Z_r\right)$ for $r\in\{1,2,\dots,R\}$ (in this case $R=6$) is the linear MF of the objective function Z_r.

Step 3. (Proposed single-objective model) Convert model (13) to the following single objective formulation,

$$\max\ \gamma\lambda_0+\left(1-\gamma\right)\sum_{r=1}^{R}\lambda_r$$
subject to
$$\lambda_0+\lambda_r\le\mu_r\left(Z_r\right) \qquad \forall r\in\{1,2,\dots,R\}$$
$$\lambda_0,\lambda_r\in[0,1] \qquad \forall r\in\{1,2,\dots,R\}$$
Constraints of model (13). (28)

In the formulation (28), the continuous and non-negative variables λ_0 and λ_r are used to control the minimum satisfaction level of the objective functions as well as their compromise degrees.

The value γ ($0\le\gamma\le1$) is also used to indicate the importance of λ_0 and λ_r. In the literature of multi-objective optimization γ is set to 0.4 experimentally.

Theorem 2. The optimal solution of the proposed single-objective model (28) is an efficient (Pareto-optimal) solution to the multi-objective model (13).

Step 4. Solve the single-objective model (28) with a given value for γ. If the obtained solution is satisfactory for the decision maker, stop. Otherwise, do one of the following changes and repeat the steps 1 to 4 until a satisfactory solution is obtained;

- Increase the NIS value for maximization type objective functions.
- Decrease the PIS value for minimization type objective functions.
- Change the given value for γ.

In order to obtain the intuitionistic fuzzy objective function value of model (12), the obtained optimal solution from the steps 1-4, is supplied to the objective function formula of model (12), and its intuitionistic fuzzy value is calculated.
The overall solution approach for linear programming with TrIFNs is shown by the flowchart of Figure 3.

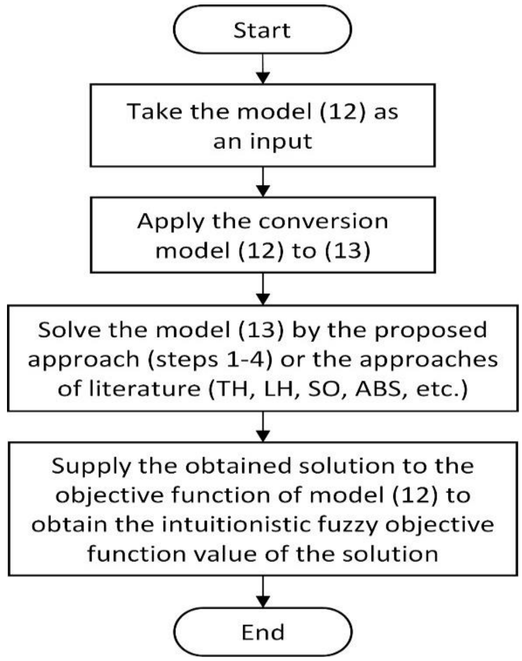

Figure 3. The flowchart of the proposed solution approach for intuitionistic fuzzy linear programming

3.2. Case 2: Linear programming with $\tilde{\mathbf{c}}$, $\tilde{\mathbf{b}}$, and $\tilde{\mathbf{A}}$

In this case of intuitionistic fuzzy linear programming, all coefficients and resource availabilities are TrIFNs. The formulation of this case is as follows,

$$\begin{aligned} &\min \tilde{Z} = \tilde{\mathbf{c}}^{\mathbf{T}}\mathbf{x} \\ &\text{subject to} \\ &\tilde{\mathbf{A}}\mathbf{x} \geq \tilde{\mathbf{b}} \\ &\mathbf{x} \geq \mathbf{0} \end{aligned} \tag{29}$$

To convert model (29) to its crisp version, the objective function and the constraints should be changed in the conversion process. The same as Case 1, the objective function is converted to six crisp objective functions. On the other hand, considering the concept of Theorem 1, the constraints set of model (29) is converted to six sets of constraints and the equivalent multi-objective crisp formulation is written as follow,

$$\begin{aligned} &\max Z_1 = (\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}_1)^T \mathbf{x} \\ &\min Z_2 = \frac{1}{2}\left((\mathbf{c}_2)^T \mathbf{x} + (\mathbf{c}_3)^T \mathbf{x}\right) \\ &\min Z_3 = (\mathbf{c}_3)^T \mathbf{x} \\ &\min Z_4 = (\mathbf{c}_4)^T \mathbf{x} - (\mathbf{c}_3)^T \mathbf{x} \\ &\min Z_5 = (\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}'_1)^T \mathbf{x} \\ &\max Z_6 = (\mathbf{c}'_4)^T \mathbf{x} - (\mathbf{c}_3)^T \mathbf{x} \\ &\text{subject to} \\ &\mathbf{A}_1\mathbf{x} \geq \mathbf{b}_1 \\ &\mathbf{A}_2\mathbf{x} \geq \mathbf{b}_2 \end{aligned} \tag{30}$$

$$\mathbf{A}_3\mathbf{x} \geq \mathbf{b}_3$$
$$\mathbf{A}_4\mathbf{x} \geq \mathbf{b}_4$$
$$\mathbf{A}_1'\mathbf{x} \geq \mathbf{b}_1'$$
$$\mathbf{A}_4'\mathbf{x} \geq \mathbf{b}_4'$$
$$\mathbf{x} \geq \mathbf{0}$$

The multi-objective formulation (30) can be solved by the hybrid fuzzy programming approach introduced in the previous sub-section for obtaining efficient (Pareto optimal) solutions.

It is notable to mention that, for a maximization type intuitionistic fuzzy problem, the same concept that resulted in converting the objective function and constraints of model (29) to the model (30) is used. Therefore, the model,

$$\begin{aligned} &\max \tilde{Z} = \tilde{\mathbf{c}}^{\mathbf{T}}\mathbf{x} \\ &\text{subject to} \\ &\tilde{\mathbf{A}}\mathbf{x} \leq \tilde{\mathbf{b}} \\ &\mathbf{x} \geq \mathbf{0} \end{aligned} \tag{31}$$

is converted to the following model.

$$\begin{aligned} &\min Z_1 = (\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}_1)^T \mathbf{x} \\ &\max Z_2 = (\mathbf{c}_2)^T \mathbf{x} \\ &\max Z_3 = \frac{1}{2}\left((\mathbf{c}_2)^T \mathbf{x} + (\mathbf{c}_3)^T \mathbf{x}\right) \\ &\max Z_4 = (\mathbf{c}_4)^T \mathbf{x} - (\mathbf{c}_3)^T \mathbf{x} \\ &\max Z_5 = (\mathbf{c}_2)^T \mathbf{x} - (\mathbf{c}_1')^T \mathbf{x} \\ &\min Z_6 = (\mathbf{c}_4')^T \mathbf{x} - (\mathbf{c}_3)^T \mathbf{x} \\ &\text{subject to} \\ &\mathbf{A}_1\mathbf{x} \leq \mathbf{b}_1 \\ &\mathbf{A}_2\mathbf{x} \leq \mathbf{b}_2 \\ &\mathbf{A}_3\mathbf{x} \leq \mathbf{b}_3 \\ &\mathbf{A}_4\mathbf{x} \leq \mathbf{b}_4 \\ &\mathbf{A}_1'\mathbf{x} \leq \mathbf{b}_1' \\ &\mathbf{A}_4'\mathbf{x} \leq \mathbf{b}_4' \\ &\mathbf{x} \geq \mathbf{0} \end{aligned} \tag{32}$$

4. Computational Experiments

In this section, to study the performances of the proposed solution methodology of Section 3, we present a numerical example and a case study in transportation problem and compare the performance of the proposed approach with the LH [17], TH [39], ABS [1], SMN [34], MMNV [26], MMNV [26], approaches of the literature. To do these computations, the required mathematical formulations are coded in GAMS 23.5 optimization software. All codes are run on a machine with Intel Core 2 GHZ CPU, 2 GB RAM.

4.1. Numerical example: Linear programming

As the numerical example, we consider the following linear programming formulation with TrIFN parameters.

$$
\begin{aligned}
&\min \tilde{Z}^I = (1,3,4,7;0,3,4,10)\,x_1 + (4,6,8,10;2,6,8,13)\,x_3 + (0,5,14,15;-2,5,14,17)\,x_4 \\
&\quad -(5,15,24,30;2,15,24,35)\,x_6 + (1,3,4,7;0,3,4,9)\,x_7 - (2,5,18,22;-1,5,18,25)\,x_9 \\
&\quad -(3,5,7,9;1,5,7,12)\,x_{11} \\
&\text{subject to} \\
&(2,4,6,8;1,4,6,13)\,x_1 + (3,5,9,12;2,5,9,16)\,x_2 + (3,7,14,17;2,7,14,19)\,x_3 \\
&+(5,8,10,14;2,8,10,15)\,x_4 \le (10,15,20,25;7,15,20,35) \\
&(4,7,10,13;2,7,10,15)\,x_4 + (3,6,9,14;1,6,9,18)\,x_5 + (5,8,10,13;2,8,10,15)\,x_6 \\
&+(1,5,8,14;0,5,8,17)\,x_7 \le (10,20,25,30;5,20,25,40) \\
&(4,7,10,13;2,7,10,15)\,x_7 + (5,6,7,9;2,6,7,12)\,x_8 + (3,6,9,12;1,6,9,15)\,x_9 \\
&+(5,8,10,15;2,8,10,20)\,x_{10} \le (12,15,25,35;8,15,25,50) \\
&(4,7,10,17;2,7,10,20)\,x_1 + (3,6,9,13;1,6,9,16)\,x_2 + (4,8,12,18;3,8,12,20)\,x_3 \\
&+(5,8,10,20;2,8,10,30)\,x_{11} \le (5,8,10,20;3,8,10,30) \\
&(4,7,10,12;1,7,10,15)\,x_4 + (4,7,10,11;2,7,10,12)\,x_5 + (3,6,9,14;1,6,9,16)\,x_6 \\
&+(5,8,10,12;2,8,10,15)\,x_7 \le (7,15,25,30;3,15,25,35) \\
&x_i \ge 0
\end{aligned}
\tag{33}
$$

According to the solution methodology proposed in Section 3, formulation (33) is converted to the following multi-objective formulation.

As a multi-objective formulation, the model (34) was solved by the proposed solution approach (steps 1-4 of sub-section 3.1) as well as the approaches LH, TH, ABS, SMN and MMNV. The obtained results are shown in Table 2. It is notable to be mentioned that the value of γ is set to 0.4 in the proposed approach and the approaches LH, TH, ABS, SMN and MMNV (if needed). And also in some of the approaches LH, TH, ABS, SMN and MMNV the objective functions are weighted equally (if needed).

Table 2. The obtained solution for Example 1 by different approaches

Approach	Optimal solution	$\tilde{Z}$
LH	$x_1^* = 0.83$, $x_6^* = 0.77$, $x_9^* = 1.53$	(-55.89, -43.50, -15.85, -1.1; -65.15, -43.50, -15.85, 8.29)
TH	$x_6^* = 1.09$, $x_9^* = 1.5$	(-65.78, -53.22, -23.9, -8.46; -75.74, -53.22, -23.9, -0.691)
ABS	$x_6^* = 0.96$, $x_9^* = 1.31$	(-57.47, -46.5, -20.89, -7.4; -66.18, -46.5, -20.89,-0.6)
SMN	$x_1^* = 0.78$, $x_6^* = 0.8$, $x_9^* = 1.50$	(-56.27, -43.89, -16.36, -1.51; -65.56, -43.89, -16.36, 7.76)
MMNV	$x_6^* = 0.96$, $x_9^* = 1.31$	(-57.47, -46.5, -20.89, -7.4; -66.18, -46.5, -20.89,-0.6)
The proposed	$x_6^* = 2$, $x_9^* = 2.5$, $x_{11}^* = 1$	(-124, -100, -47, -18; -144.5, -100, -47, -2.5)

The obtained intuitionistic fuzzy objective function values of Table 2 are such obvious that no ranking function is needed for their comparison. According to these results, the lowest intuitionistic fuzzy objective function value is obtained by the proposed approach. After these approaches, the TH approach performs better than the other approaches.

4.1. Case study: transportation problem

In order to further study the performance of the proposed approach, a case study of honey transportation is considered here. The data is gathered from a honey producer in north of Iran. This producer, has three honey production sites in Babolkenar (A), Firoozkooh (B), and Hamedan (C). Because of the high quality of the produced honey, there are a variety of demands from the cities Ghaemshahr (1), Babolsar (2), Damavand (3), and Babol (4). According to variety in the flowers, weather, and unstable market, the transportation costs between the production sites and the customers are uncertain with TrIFNs. But the production rates and demand values are to be of crisp type (see the data of Table 3). The producer needs to have a plan for sending the honey from the production sites to the customers in order to minimize the TrIFN value of the transportation cost.

Table 3. The transportation costs, demand, and supply values for the case study

		Destinations				Supply
		1	2	3	4	
Sources	A	(18,22,23, 25; 15,22, 23, 29)	(22,24,26,30; 20,24,26,35)	(26,30,34,38; 23,30,34,40)	(19,23,25,27; 17,23,25,30)	1200
	B	(30,35,38,40; 25,35,38,45)	(32,35,39,42; 30,35,39,47)	(23,24,28,30; 21,24,28,35)	(31,32,33,37; 27,32,33,39)	950
	C	(39,43,44,45; 36,43,44,50)	(40,45,46,48; 38,45,46,50)	(35,37,40,43; 33,37,40,45)	(40,45,46,48; 38,45,46,50)	750
Demand		600	600	800	900	

Applying the methodology of Sub-section 3.1 to this case study, the following multi-objective formulation is obtained.

$$
\begin{aligned}
&\max Z_1 = 4x_{A1} + 2x_{A2} + 4x_{A3} + 4x_{A4} + 5x_{B1} + 3x_{B2} + x_{B3} + x_{B4} + 4x_{C1} + 5x_{C2} + 2x_{C3} + 5x_{C4} \\
&\min Z_2 = 22.5x_{A1} + 25x_{A2} + 32x_{A3} + 24x_{A4} + 36.5x_{B1} + 37x_{B2} + 26x_{B3} + 32.5x_{B4} + 43.5x_{C1} \\
&\qquad + 45.5x_{C2} + 38.5x_{C3} + 45.5x_{C4} \\
&\min Z_3 = 23x_{A1} + 26x_{A2} + 34x_{A3} + 25x_{A4} + 38x_{B1} + 39x_{B2} + 28x_{B3} + 33x_{B4} + 44x_{C1} + 46x_{C2} + 40x_{C3} + 46x_{C4} \\
&\min Z_4 = 2x_{A1} + 4x_{A2} + 4x_{A3} + 2x_{A4} + 2x_{B1} + 3x_{B2} + 2x_{B3} + 4x_{B4} + x_{C1} + 2x_{C2} + 3x_{C3} + 2x_{C4} \\
&\min Z_5 = 7x_{A1} + 4x_{A2} + 7x_{A3} + 6x_{A4} + 10x_{B1} + 5x_{B2} + 3x_{B3} + 5x_{B4} + 7x_{C1} + 7x_{C2} + 4x_{C3} + 7x_{C4} \\
&\max Z_6 = 6x_{A1} + 9x_{A2} + 6x_{A3} + 5x_{A4} + 7x_{B1} + 8x_{B2} + 7x_{B3} + 6x_{B4} + 6x_{C1} + 4x_{C2} + 5x_{C3} + 4x_{C4} \\
&\text{subject to} \\
&x_{A1} + x_{A2} + x_{A3} + x_{A4} = 1200 \\
&x_{B1} + x_{B2} + x_{B3} + x_{B4} = 950 \\
&x_{C1} + x_{C2} + x_{C3} + x_{C4} = 750 \\
&x_{A1} + x_{B1} + x_{C1} = 600 \\
&x_{A2} + x_{B2} + x_{C2} = 600 \\
&x_{A3} + x_{B3} + x_{C3} = 800 \\
&x_{A4} + x_{B4} + x_{C4} = 900 \\
&x_{ij} \geq 0, \quad i = A, B, C, \; j = 1, 2, 3, 4
\end{aligned}
\tag{35}
$$

The multi-objective model (35) was solved by the proposed solution approach (steps 1-4 of sub-section 3.1) as well as the approaches LH, TH, ABS, SMN and MMNV. The obtained results are shown in Table 4. It is notable to be mentioned that the value of γ is set to 0.4 in the proposed approach and the approaches TH and LH (if needed). And also in some of the approaches LH, TH, ABS, SMN and MMNV the objective functions are weighted equally (if needed).

Table 4. The obtained solution for the variables of case study by different approaches

Variable	Obtained value					
	LH	**TH**	**ABS**	**SMN**	**MMNV**	**The proposed**
x_{11}^*	449.67	0	81.05	287.79	81.05	0
x_{12}^*	291.1	367.68	218.96	437.78	218.96	600
x_{13}^*	281.98	185	0	162.22	0	0
x_{14}^*	177.26	647.32	900	312.22	900	600
x_{21}^*	123.07	335	0	312.22	0	0
x_{22}^*	308.91	0	150	0	150	0
x_{23}^*	518.03	615	800	637.78	800	800
x_{24}^*	0	0	0	0	0	150
x_{31}^*	518.02	265	518.96	0	518.96	600
x_{32}^*	0	232.32	231.05	162.22	231.05	0
x_{33}^*	0	0	0	0	0	0
x_{34}^*	722.74	252.68	0	587.78	0	150

Table 5. The obtained intuitionistic fuzzy objective function value of case study by different approaches

Approach	Intuitionistic fuzzy objective function
LH	(80661.78, 90662.41, 97603.61, 104831.4; 73733.67, 90662.41, 97603.61, 115513.06)
TH	(79128.04, 88967.68, 95952.68, 102593.04; 71873.04, 88967.68, 95952.68, 113788.4)
ABS	(76056.86, 84900, 91768.95, 97637.90; 69656.86, 84900, 91768.95, 109563.72)
SMN	(78996.61, 88869.91, 95544.34, 102544.34; 71809.95, 88869.91, 95544.34, 113395.47)
MMNV	(76056.86, 84900, 91768.95, 97637.90; 69656.86, 84900, 91768.95, 109563.72)
The proposed method	(77050, 84750, 91250, 97950; 70350, 84750, 91250, 110350)

The obtained intuitionistic fuzzy objective function values of Table 5 can be compared with no ranking function. According to these results, the lowest intuitionistic fuzzy objective function value is obtained by the ABS and MMNV approaches. These two approaches have similar performance. After these approaches, the proposed method performs better. Finally, the LH, TH and SMN approaches perform the same, as similar objective function values are obtained by them.

For an exact comparison of them, applying ranking functions of the literature of fuzzy theory is advised. In addition to these approaches, the method of Mahmoodirad et al. [21] is used to solve this example. According to this ranking function, the results are presented in Table 6. The results of this table, confirm the pervious results.

Table 6. The obtained crisp objective function value

Approach	Crisp objective function	Rank
LH	93908.94	5
TH	92152.91	4
ABS	88281.65	1
SMN	91946.86	3
MMNV	88281.65	1
The proposed	88462.5	2

5. Concluding Remarks

A linear programming with trapezoidal intuitionistic fuzzy parameters was solved in this paper. As all of the studies of the literature of this problem are constructed based on ranking functions, for the first time an approach with no ranking function was employed for the problem. For this aim, the trapezoidal intuitionistic fuzzy objective function was decomposed to a multi-objective function to be solved instead of the initial problem. In order to solve the obtained multi-objective problem for its efficient solutions, a new multi-objective optimization approach was developed and suited to the obtained multi-objective problem. The computational experiments of the study, show the superiority of the proposed multi-objective optimization approach over the multi-objective optimization approaches of the literature of multi-objective optimization. As future study on this topic, the effect of ranking function based approaches can be studied and compared with the proposed approach and other approaches that apply no ranking function. Furthermore, fuzzy linear programming with uncertain parameters based on belief degree can be of interest for the researchers.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. Allahviranloo, T., Lotfi, F.H., Kiasary, M.Kh., Kiani, N.A., & Alizadeh, L. (2008). Solving fully fuzzy linear programming problem by the ranking function. *Applied Mathematical Sciences*, 2(1), 19–32.
2. Alavidoost, M.H., Babazadeh, H., & Sayyari, S.T. (2016). An interactive fuzzy programming approach for bi-objective straight and U-shaped assembly line balancing problem. *Applied Soft Computing*, 40, 221–235.
3. Angelove, P.P. (1997). Optimization in an intuitionistic fuzzy environment. *Fuzzy Sets and Systems*, 86(3), 299–306.
4. Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 87–96.
5. Bellman, R., & Zadeh, L.A. (1970). Decision making in fuzzy environment. *Management Science*, 17(B), 141–164.
6. Daami Remadi, F., & Moalla Frikha, H. (2023). The triangular intuitionistic fuzzy numbers CODAS method for solving green material selection problem. *International Journal of Operational Research*, 46 (3), 398-415.
7. Das, S. K., Mandal, T., & Edalatpanah, S. A. (2017). A mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers. *Applied Intelligence*, 46 (3), 509-517.
8. Dong, J. Y., & Wan, S. P. (2018). A new trapezoidal fuzzy linear programming method considering the acceptance degree of fuzzy constraints violated. *Knowledge-Based Systems*, 148, 100-114.
9. Dubey, D., Chandra, S., & Mehra, A. (2012). Fuzzy linear programming under interval uncertainty based on IFS representation. *Fuzzy Sets and Systems*, 188(1), 68–87.
10. Dubey, D., & Mehra, A. (2011). Linear programming with triangular intuitionistic fuzzy numbers. *Advances in Intelligent Systems Research*, 1, 563–569.
11. Ebrahimnejad, A. (2011). Some new results in linear programs with trapezoidal fuzzy numbers: finite convergence of the Ganesan and Veeramani's method and a fuzzy revised simplex method. *Applied Mathematical Modelling*, 35, 4526–4540.
12. He, Y., He, Z., & Huang, H. (2017). Decision making with the generalized intuitionistic fuzzy power interaction averaging operators. *Soft Computing*, 21(5), 1129–1144.
13. Huo, Y., Qiu, P., Zhai, J., Fan, D., & Peng, H. (2017). Multi-objective service composition model based on cost-effective optimization. *Applied Intelligence*, 48, 651-669.
14. Kaur, A., & Kumar, A. (2012). A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. *Applied Soft Computing*, 12, 1201–1213.
15. Kumar, P.S., & Hussain, R.J. (2014). Computationally simple approach for solving fully intuitionistic fuzzy real life transportation problems. *International Journal of System Assurance Engineering and Management*, 7, 90-101.
16. Kumar, A., Kaur, J., & Singh, P. (2011). A new method for solving fully fuzzy linear programming problems. *Applied Mathematical Modelling*, 35, 817–823.
17. Lai, Y.J., & Hwang, C.L. (1992). A new approach to some possibilistic linear programming problems. *Fuzzy Sets and Systems*, 49, 121–133.
18. Liu, X.W. (2001). Measuring the satisfaction of constraints in fuzzy linear programming, *Fuzzy Sets and Systems*, 122, 263–275.
19. Lotfi, F.H., Allahviranloo, T., Jondabeha, M.A., & Alizadeh, L. (2009). Solving a fully fuzzy linear programming using lexicography method and fuzzy approximate solution. *Applied Mathematical Modelling*, 33, 3151–3156.
20. Mahmoodirad, A., & Sanei, M. (2016). Solving a multi-stage multi-product solid supply chain network design problem by meta-heuristics. *Scientia Iranica E*, 23(3), 1428-1440.
21. Mahmoodirad A., Allahviranloo T., & Niroomand S. (2019). A new effective solution method for fully intuitionistic fuzzy transportation problem. *Soft Computing*, 23 (12), 4521-4530.
22. Maleki, H.R., & Mashinchi, M. (2004). Fuzzy number linear programming: a probabilistic approach (3). *Journal of Applied Mathematics and Computing*. 15(2), 333–341.
23. Maleki, H.R., Tata, M., & Mashinchi, M. (2000). Linear programming with fuzzy variables. *Fuzzy Sets and Systems*, 109, 21–33.
24. Mollalign Moges, D., Rangia Mushi, A., & Guta Wordofa, B. (2023).A new method for intuitionistic fuzzy multi-objective linear fractional optimization problem and its application in agricultural land allocation problem. *Information Sciences*, 625, 457-475.

25. Mirzaei, N., Mahmoodirad, A. & Niroomand, S. (2019). An Uncertain Multi-objective Assembly Line Balancing Problem: A Credibility-Based Fuzzy Modeling Approach. *International Journal of Fuzzy Systems*, 21, 2392–2404.
26. Mosallaeipour, S., Mahmoodirad, A., Niroomand, S., & Vizvari, B. (2017). Simultaneous selection of material and supplier under uncertainty in carton box industries: a fuzzy possibilistic multicriteria approach. *Soft Computing*, 22, 2891-2905.
27. Nagoorgani, A., & Ponnalagu, K. (2012). A new approach on solving intuitionistic fuzzy linear programming problem. *Applied Mathematical Sciences*, 6, 3467–3474.
28. Nayagam, V.L.G., Jeevaraj, S., & Dhanasekaran, P. (2016). An intuitionistic fuzzy multi-criteria decision-making method based on non-hesitance score for interval-valued intuitionistic fuzzy sets. *Soft Computing*, 21, 7077-7082.
29. Niroomand, S., Garg, H., & Mahmoodirad, A. (2020). An intuitionistic fuzzy two stage supply chain network design problem with multi-mode demand and multi-mode transportation. *ISA Transactions*, 107, 117–133.
30. Niroomand, S., Mahmoodirad, A., & Mosallaeipour, S. (2019). A hybrid solution approach for fuzzy multiobjective dual supplier and material selection problem of carton box production systems. *Expert Systems*, 36(1), 1–17.
31. Parvathi, R., & Malathi, C. (2012.a). Intuitionistic fuzzy linear optimization. *Notes on Intuitionistic Fuzzy Sets*, 18, 48–56.
32. Parvathi, R., & Malathi, C. (2012.b). Intuitionistic fuzzy linear programming problems. *World applied Sciences Journal*, 17, 1802–1807.
33. Popa, L. (2023). A new ranking method for trapezoidal intuitionistic fuzzy numbers and its application to multi-criteria decision making. *International Journal of Computers Communications & Control*, 18(2), 5118.
34. Salehi, M., Maleki, H. R., & Niroomand, S. (2017). A multi-objective assembly line balancing problem with worker's skill and qualification considerations in fuzzy environment. *Applied Intelligence*, 48, 2137-2156.
35. Sanei, M., Mahmoodirad, A., & Molla-Alizadeh-Zavardehi, S. (2013). An Electromagnetism-like algorithm for fixed charge solid transportation problem. *International journal of mathematical modelling & computations*, 3(4), 345- 354.
36. Santos Arteaga, F. J., Ebrahimnejad, A., & Zabihi, A. (2021). A new approach for solving intuitionistic fuzzy data envelopment analysis Problems. *Fuzzy Optimization and Modelling Journal*, 2 (2), 46-57.
37. Singh, S.K., & Yadav, S.P. (2016). A novel approach for solving fully intuitionistic fuzzy transportation problem. *International Journal of Operational Research*, 26 (4), 460-472.
38. Suresh, M., Vengataasalam, S., & Prakash, K.A. (2014). Solving intuitionistic fuzzy linear programming problems by ranking function. *Journal of Intelligent & Fuzzy Systems*, 27, 3081–3087.
39. Torabi, S.A., & Hassini, E. (2008). An interactive possibilistic programming approach for multiple objective supply chain master planning. *Fuzzy Sets and Systems*, 159, 193–214.
40. Wan, S.P., & Dong, J.Y. (2014). Possibility linear programming with trapezoidal fuzzy numbers. *Applied Mathematical Modelling*, 38(5-6), 1660-1672.
41. Zimmermann, H.J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, 1, 45–55.

Mahmoodirad, A. (2023). A Novel Approach for Solving Linear Programming Problems with Intuitionistic Fuzzy Numbers. *Fuzzy Optimization and Modelling Journal*, 4 (3), 1-15.

https://doi.org/ 10.30495/FOMJ.2023.1995970.1113

Received: 8 September 2023 Revised: 3 October 2023 Accepted: 8 October 2023