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Attractiveness and Progress in Integer-Valued Data Envelopment Analysis

Maliheh Shahkooeei [a], Farzad Rezai Balf [b,*]

[a] *Department of Mathematics, Sari Branch, Islamic Azad University, Sari, Iran.*
[b] *Department of Mathematics, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran.*

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ABSTRACT

Data envelopment analysis (DEA) is a non-parametric technique to measure and evaluating the relative efficiencies of the set of homogenous decision making units (DMUs) with multiple inputs and multiple outputs. Traditional DEA models assume that inputs and outputs to be continuous and real-valued data. In many occasions, inputs or outputs can only take integer values. Therefore, DEA models cannot be used for determining efficiency score of such DMUs. The current paper applies the modified classic DEA models to obtain attractiveness and progress in integer-valued technologies. For this aim, in the first phase, the efficiency score of all DMUs are measured and the efficient and inefficient units are determined. Then, in the second phase, we remove the main efficient frontier that corresponds to the efficient units, and then create a new efficient frontier as the second layer efficient frontier of the remaining units (units inefficient). With repeat this process, we find the next layers until there is no any unit left. Finally, the attractiveness and progress of each unit is calculated from one efficient level relative to the other efficient level.

1. Introduction

Data envelopment analysis (DEA) was first proposed by Charnes, Cooper and Rhodes [3] which is a technique for evaluating the relative efficiency of a homogeneous group of decision making units (DMUs) with multiple outputs and multiple inputs and then it had been extended by Banker et al. [2]. In DEA and in many application problems, data are in the form of integer [4], fuzzy [7, 11, 12], ordinal and interval [1], uncertainly [10] and so on. In conventional DEA, real-valued inputs and outputs are assumed, and make production possibility set (PPS). The frontier of this PPS is called as efficiency frontier. All DMUs on this frontier are efficient otherwise; it is inefficient [5]. It is trivial that adding or removing the set of inefficient DMUs does not

* Correspondig author
E-mail address: fr.balf@qaemiau.ac.ir (Farzad Rezai Balf)

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change the performance of existing DMUs and the efficient frontier. Inefficiency scores only change if the efficient frontier changes.

However, a DMU (efficient or inefficient) may be more attractive than some other DMUs, and it may also seem unattractive compared to some other more attractive DMUs. The relative attractiveness of a DMU can be defined by the distance function it relative to an efficient frontier constructed from other DMUs which worse performance. In other words, a set of DMUs can be divided into different levels of efficient frontiers. If we remove the efficient (main) frontier, the remaining (inefficient) DMUs will form a new second level efficient frontier. Then, by removing this new efficient second level frontier, an efficient third level frontier is formed and so on until do not any the DMU remains. Each of these efficient frontiers provides an assessment context for measuring relative attractiveness, for example, a second-level efficient frontier acts as an evaluation context [13,14], for measuring the relative attractiveness of first-level DMUs. In contrast relative attractiveness, we have the concept of relative improvement, and that is achieved when better-performing DMUs are selected as the efficient frontier or evaluation area.

With respect to the shape of the efficient frontier affects the attractiveness or relative development of DMUs at different levels of the efficient frontier. Therefore, when DMUs are at a certain level, they are called to as equivalent units. Thus, it is important such that the attractiveness index or the progress index allows us to measure equal performance based on the same specific evaluation context.

In evaluating efficiencies of DMUs, when some inputs or outputs take integer values, the fundamental axioms in order to construct the classic DEA models are violated. The targets obtained by using a traditional DEA model may lead to a reference point that is out of the PPS. Lozano and Villa [8] were first addressed the integer-valued DEA. They developed a new DEA PPS and a model with integer-value for computing the integer-efficiency score and the integer targets. Following them, Kuosmanen and Kazemi Matin [6] developed a new axioms foundation for the integer-valued DEA models and also they showed that the proposed PPS by Lozano and Villa [9] is consistent with the proposed set of axioms. They also proposed a modification of the classic Farrell efficiency measure, and derive a mixed integer linear programming (MILP) formulation for computing it. In the next section, we present theses axioms.

The need to deal with attractiveness and progress in DEA naturally occurs when one want to choice the best DMU among of the classified DMUs. The integrality of some variables make problem for using the conventional DEA models, therefore, we use modified models to obtain attractiveness and progress in integer-valued technology.

The rest of the paper is organized as follows. In Section 2, we present the integer-valued DEA. In Section 3, we give the context-dependent DEA. Then we use the modified context-dependent DEA models to integer valued technology in Section 4. A numerical example will be shown in Section 5. Finally, Section 6 gives our conclusions.

2. Integer-valued DEA

Lozano and Villa [8] were first addressed the integer-valued DEA. They modified the PPS of the proposed mixed integer linear programming model in order to evaluate the efficiencies of DMUs and computed integer targets. Kuosmanen and Kazemi Matin [6] with spreading article completed the subjects said by Lozano and Villa [9]. They modified the fundamental axioms of DEA for integer-valued technology as follows. With due regard to envelopment axiom:

(1) Envelopment: $(x_j, y_j) \in T, \ \forall j \in J = \{1, ..., n\}$,

Imposing an additional axiom:

(2) Integrality: $(x, y) \in T \Rightarrow (x, y) \in Z_{+}^{m+s}$

This additional condition contradicts axioms free disposability, convexity and constant returns to scale that respectively, modifies as follow:

(3) Natural disposability: $(x,y)\in T$ and $(u,v)\in Z_{+}^{m+s}$, $y\geq v\Rightarrow(x+u,y-v)\in T$,

(4) Additivity: $(x,y),(x',y')\in T\Rightarrow(x+x',y+y')\in T$,

(5) Natural divisibility: $(x,y)\in T$ and $\exists\lambda\in[0,1]$: $(\lambda x,\lambda y)\in Z_{+}^{m+s}\Rightarrow(\lambda x,\lambda y)\in T$.

They showed that the PPS proposed by Lozano and Villa [9] is consistent with the proposed set of axioms. Then, they characterized a PPS that satisfies in the minimum extrapolation principle subject to the properties (1)-(5). Note that this PPS consists of separately points. A sequential application of the axioms can generate new feasible points that are not achievable by applying the axioms just once. In this section, an appropriate integer constrained production possibility set together with the envelopment form model is introduced. Although, we have assumed CRS [3], the definitions and the model can be trivially adapted to VRS [2], or any other convex technology.

Assume that we have a set of n DMUs with m inputs and s outputs. We divide the input set $I=I^{I}\cup I^{NI}$ and the output set $O=O^{I}\cup O^{NI}$, where subsets I^{I} and O^{I} are subject to the integer condition and subsets I^{NI} and O^{NI} are real-valued. With respect to that it was said formerly in above, the PPS can be as follows:

$$T=\{(\hat{x},\hat{y})\,\Big|\,\sum_{j=1}^{n}\lambda_j x_{ij}\leq\hat{x}_i\ ,\ \sum_{j=1}^{n}\lambda_j y_{rj}\geq\hat{y}_r\ ,\ \hat{x}_i,\hat{y}_r\in Z_{+},\ \forall i\in I^{I},\forall r\in O^{I}\ \}$$

They proposed the following MILP problem in order to input efficiency scores relative to the general T reference technology:

$$\begin{aligned}
\min\quad & \theta_o-\varepsilon\Big(\sum_{i=1}^{m}s_i^{-}+\sum_{i=1}^{p}s_i^{I}+\sum_{r=1}^{s}s_r^{+}\Big) & \\
s.t.\quad & \sum_{j=1}^{n}\lambda_j y_{rj}=y_{ro}+s_r^{+}, & \forall r\in O,\\
& \sum_{j=1}^{n}\lambda_j x_{ij}=\theta_o x_{io}-s_i^{-}, & \forall i\in I^{NI},\\
& \sum_{j=1}^{n}\lambda_j x_{ij}=\hat{x}_i-s_i^{-}, & \forall i\in I^{I},\\
& \theta_o x_{io}-s_i^{I}=\hat{x}_i, & \forall i\in I^{I},\\
& \hat{x}_i\in Z_{+}\ , & \forall i\in I^{I},\\
& \lambda_j\geq 0, & \forall j,\\
& s_r^{+}\geq 0, s_i^{-}\geq 0, s_i^{I}\geq 0, & \forall r\in O,\forall i\in I,\ \forall i\in I^{I},
\end{aligned}\tag{1}$$

where $p=\left|I^{I}\right|\leq m$. Symbol ε denotes a non-Archimedean infinitesimal, variables s_i^{-} , s_r^{+} and, s_i^{I} represent the non-radial slacks. This model guaranties the integrality of obtained targets. The output oriented of this model is related as:

$$
\begin{aligned}
max \quad & \varphi_o - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{q} s_r^I + \sum_{r=1}^{s} s_r^+) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} = x_{io} - s_i^-, && \forall i \in I, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} = \varphi_o y_{ro} + s_r^+, && \forall r \in O^{NI}, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} = \hat{y}_r + s_r^+, && \forall r \in O^{I}, \\
& \varphi_o x_{io} + s_r^I = \hat{y}_r, && \forall r \in O^{I}, \\
& \hat{y}_r \in Z_+ , && \forall r \in O^{I}, \\
& \lambda_j \geq 0, && \forall j, \\
& s_r^+ \geq 0, s_i^- \geq 0, s_r^I \geq 0, && \forall r \in O, \forall i \in I, \forall r \in O^I,
\end{aligned} \tag{2}
$$

where $q = \left| O^I \right| \leq s$. This model guaranties the integrality of output targets. In Section 4 we use model (2) in order to find the evaluation contexts. Note that, DMU*o* in model (2) is efficient if and only if in optimal solution $\varphi_o^* = 1$.

3. Context-dependent DEA

Researchers of the consumer choice theory point out that consumers choice is often influenced by the evaluating context. For example, if there are three groups of goods in three price range, then consumer want to find the best goods in each of groups depends on the other at the same level. Context-dependent DEA refer to relative attractiveness and progress. If the low (high) levels consider as evaluating context, we deal with relative attractiveness (progress). Next, we study these concepts. As in beginning said, in DEA, first fundamental axioms the PPS is created then efficiency frontier is defined. All DMUs on this frontier are efficient otherwise; they are introduced as inefficient DMUs. Consider the following model:

$$
\begin{aligned}
\varphi_o^* = max \quad & \varphi_o \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, && \forall i \in I, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi_o y_{ro}, && \forall r \in O, \\
& \lambda_j \geq 0, && j = 1, \dots, n.
\end{aligned} \tag{3}
$$

That is called the output oriented CCR model [3], where, $\varphi_o^* \geq 1$. All DMUs with $\varphi_o^* = 1$ are efficient on efficiency frontier. Let $L^1 = \{ DMU_k \in J^1 \mid \varphi_k^* = 1 \}$ be the set of all efficient DMUs and $J^1 = \{1, \dots, n\}$. If we remove the efficient frontier, then the efficient DMUs of remaining units will form a second-level efficient frontier. Let $J^2 = \{ j \mid j \in J^1 - L^1 \}$. J^2, consists all of DMUs except efficient DMUs in first level. Now, consider the following model:

$$\begin{aligned}
\varphi_o^* = & \max \quad \varphi_o \\
s.t. & \sum\nolimits_{j\in J^2} \lambda_j x_{ij} \le x_{io}, \qquad \forall i \in I, \\
& \sum\nolimits_{j\in J^2} \lambda_j y_{rj} \ge \varphi_o y_{ro}, \quad \forall r \in O, \\
& \lambda_j \ge 0, \qquad\qquad j \in J^2.
\end{aligned} \tag{4}$$

The model (4) specifies the efficient DMUs in second-level, and we have, $L^2 = \{ DMU_k \in J^2 \mid \varphi_k^* = 1\}$. If we remove the second-level efficient frontier, a third-level efficient frontier is formed, and so on, until no any DMU is left. In general case we have: $L^1 = \{ DMU_k \in J^1 \mid \varphi_k^* = 1\}$, $J^{L+1} = J^l - L^l$, and therefore, we have

$$\begin{aligned}
\varphi_k^* = & \max \quad \varphi_k \\
s.t. & \sum\nolimits_{j\in J^l} \lambda_j x_{ij} \le x_{ik}, \qquad \forall i \in I, \\
& \sum\nolimits_{j\in J^l} \lambda_j y_{rj} \ge \varphi_k y_{rk}, \quad \forall r \in O, \\
& \lambda_j \ge 0, \qquad\qquad j \in J^l.
\end{aligned} \tag{5}$$

The model (5) classifies all DMUs to $L^l, (l = 1,...,T)$. Now, based upon these evaluation contexts, we can obtain relative attractiveness measure of DMU*o*, from a specific level L^{l_o}, with respect to the next levels L^{l_o+d}, $(d = 1,..., T - l_o)$, by the following context-dependent DEA:

$$\begin{aligned}
\rho_o^*(d) = & \max \quad \rho_o(d) \qquad\qquad d = 1,..., L - l_o \\
s.t. & \sum\nolimits_{j\in L^{l_o+d}} \lambda_j x_{ij} \le x_{io}, \qquad\qquad \forall i \in I, \\
& \sum\nolimits_{j\in L^{l_o+d}} \lambda_j y_{rj} \ge \rho_o(d) y_{ro}, \quad \forall r \in O, \\
& \lambda_j \ge 0, \qquad\qquad\qquad j \in L^{l_o+d}.
\end{aligned} \tag{6}$$

Model (6) is similar to the output oriented CCR model, which each time efficient frontier is changing. By above model (6), $\rho_o^*(d) < 1$, and $\rho_o^*(d+1) < \rho_o^*(d)$ for all $d = 1,..., T - l_o$.

Definition 1. $A_o^*(d) = \dfrac{1}{\rho_o^*(d)}$ (>1) is called d-degree attractiveness of DMU*o* from specific level L^{l_o}.

If $A_t^*(d)$ become larger, so, DMU*t* has more attractiveness, because DMU*t* makes itself more distinctive from the evaluation context L^{l_o+d}, and it is shown that DMU*t* is better. Progress is vice versa attractiveness. Relative progress of DMU*t* is computed, when the high levels are chosen as the evaluation context. To obtain the progress measure for a specific $DMUo \in L^{l_o}$, $l_o \in \{2,...,T\}$, we use the following context-dependent DEA:

$$
\begin{aligned}
P_o^*(g) = max \quad & P_o(g) && g = 1,\dots,l_o - 1 \\
s.t. \ & \sum_{j\in L^{l_o-g}} \lambda_j x_{ij} \le x_{io}, && \forall i \in I, \\
& \sum_{j\in L^{l_o-g}} \lambda_j y_{rj} \ge P_o(g) y_{ro}, && \forall r \in O, \\
& \lambda_j \ge 0, && j \in L^{l_o-g}.
\end{aligned}
\tag{7}
$$

For each levels (g), $P_o^*(g) > 1$ and $P_o^*(g) < P_o^*(g+1)$.

Definition 2. $P_o^*(g)$ is called g -degree progress of DMU*o* from specific level L^{l_o} .

When $P_o^*(g)$ be little, DMU*o* has less distance to evaluation context, therefore, a smaller value of $P_o^*(g)$ is preferred.

4. Context-dependent DEA with integer-valued data

In conventional context-dependent DEA, by removing efficient DMUs, then the efficiency frontier was removed and with repeat this process DMUs classified into different levels. Now suppose that inputs and/or outputs are integer valued, and therefore the related constraints to the PPS and also efficiency frontier will change. Hence, the following model (8) is proposed. Note that the index d must be change in term of evaluation levels:

$$
\begin{aligned}
\rho_o^{*I}(d) = max \quad & \rho_o^I(d) && d = 1,\dots,T - l_o \\
s.t. \ & \sum_{j\in L^{l_o+d}} \lambda_j x_{ij} \le x_{io}, && \forall i \in I, \\
& \sum_{j\in L^{l_o+d}} \lambda_j y_{rj} \ge \rho_o^I(d) y_{ro}, && \forall r \in O^{NI}, \\
& \sum_{j\in L^{l_o+d}} \lambda_j y_{rj} \ge \tilde{y}_r, && \forall r \in O^I, \\
& \rho_o^I(d) y_{ro} \le \tilde{y}_r, && \forall r \in O^I, \\
& \tilde{y}_r \in Z_+, && \forall r \in O^I, \\
& \lambda_j \ge 0, && j \in L^{l_o+d},
\end{aligned}
\tag{8}
$$

where the subset O^I is subject to the integrality condition and the subset O^{NI} is real-valued. The model (8) computes the integer attractiveness of DMUs.

Definition 3. $A_o^{*I}(d) = \dfrac{1}{\rho_o^{*I}(d)}$ (≥ 1) is called d-degree integer-attractiveness of DMU*o* from specific level E^{l_o} .

In the following, we calculate the relative progress of equivalent DMUs, units that lie on a certain level of efficiency, so that their evaluation context is the efficiency frontier with better performance. Therefore, in related to relative progress, the model (7) can be modified as follows:

$$
\begin{aligned}
& P_o^{*I}(g)=\max \quad P_o^I(g) && g=1,\dots,l_o-1 \\
& s.t. \ \sum_{j\in L^{l_o-g}} \lambda_j x_{ij} \le x_{io}, && \forall i\in I, \\
& \sum_{j\in L^{l_o-g}} \lambda_j y_{rj} \ge P_o^I(g) y_{ro}, && \forall r\in O^{NI}, \\
& \sum_{j\in L^{l_o-g}} \lambda_j y_{rj} \ge \tilde{y}_r, && \forall r\in O^{I}, \\
& P_o^I(g) y_{ro} \le \tilde{y}_r, && \forall r\in O^{I}, \\
& \tilde{y}_r \in Z_+, && \forall r\in O^{I}, \\
& \lambda_j \ge 0, && j\in L^{l_o-g},
\end{aligned} \tag{9}
$$

Model (9) computes integer progress of DMUs. It is clear, $1\le P_o^{*I}(g)\le P_o^{*}(g)$, for each $g=1,\dots,l_o-1$.

Definition 4. $P_o^{*I}(g)$, is called g-degree integer-progress of DMU*o* from specific level L^{l_o}.

As it is seen a proposed approach with modified classic DEA models has been used in order to obtain attractiveness and progress with integer data. It is obvious that effect of the shape of the production frontier (efficient frontier) is important in the achievement to the attractiveness and relative development of DMUs regards to the different levels of the efficient frontiers. Therefore, when DMUs are at a certain level, that is, as equivalent units, then the attractiveness index or the progress index allows us to measure equal performance based on the same specific evaluation context.

5. Numerical example

In this section by a numerical example, we show how compute score of integer-attractiveness and relative progress. Consider 9 DMUs each of which utilizes the same single input x to produce the same two outputs, y_1 and y_2, in the amounts that are shown in Table 1.

Table 1. Input and Output of DMUs.

DMU	1	2	3	4	5	6	7	8	9
Input	1	1	1	1	1	1	1	1	1
Output1	2	6	8	2	4	7	2	4	5
Output2	7	5	2	5	4	2	3	2	1

Consider 9 DMUs are given in Table 1. We apply model (5) to obtain L^l (note that the index l must be change in term of evaluation levels). We have 3 levels, $L^1=\{1,2,3\}$ (efficient DMUs), $L^2=\{4,5,6\}$ and $L^3=\{7,8,9\}$ (inefficient DMUs). Now, we use model (8) and model (9) to obtain $\rho_o^{*I}(d)$ and $P_o^{*I}(g)$, $(o\in\{1,\dots,9\})$ respectively. The results are tabulated in table 2 (we used Lingo software).

We can see (in Table 2), $DMUs\in L^1$ have 2-degree integer-attractiveness and $DMUs\in L^2$ have 1-degree integer-attractiveness. Note that the DMUs in L^3 have no integer-attractiveness (as the DMUs in L^1 have no integer progress, see Figure 1.). The most value of 2-degree integer-attractiveness is related to DMU2 in L^1 and the most value of 1-degree integer-progress is due to DMU6 in L^2. The least value of 2-degree integer-progress is related to DMU8 in L^3 and 1-degree integer-progress is related to DMU6 with value 1, while it is impossible in real data technology. It is worthwhile to note that if we use the model (6) to obtain integer attractiveness

DMUs in L^1, we have $A_1^*(1)=1.4$, $A_2^*(1)=1.35$ and $A_3^*(1)=1.142$,therefore, DMU1 has the most 1-degree. Attractiveness, that is inconsistent with our results.

Table 2. Integer-attractiveness and integer-progress.

DMUs	$A_o^{*I}(1)$	$A_o^{*I}(2)$	$P_o^{*I}(1)$	$P_o^{*I}(2)$
1	1.4	2.333	-	-
2 *	1.499	2.5	-	-
3	1.142	2	-	-
4	-	1.666	1.2	-
5	-	2	1.25	-
6 *	-	1.749	1	-
7	-	-	1.333	2
8	-	-	1.25	1.5
9 *	-	-	1.4	1.6

The best unit in each levels is specified by a * in above it. Consider $DMUo \in L^{l_o}$, $l_o \in \{1,...,T\}$. To comparison DMU_o with others DMUs at the same level, L^{l_o} , we present following measure for ranking:

$$r_o = \frac{1+\sum_{d=1}^{T-l_o} A_o^{*I}(d)}{1+\sum_{g=1}^{l_o-1} P_o^{*I}(g)}$$

The results of ranking DMUs for every level of efficiency frontier are given in Table 3.

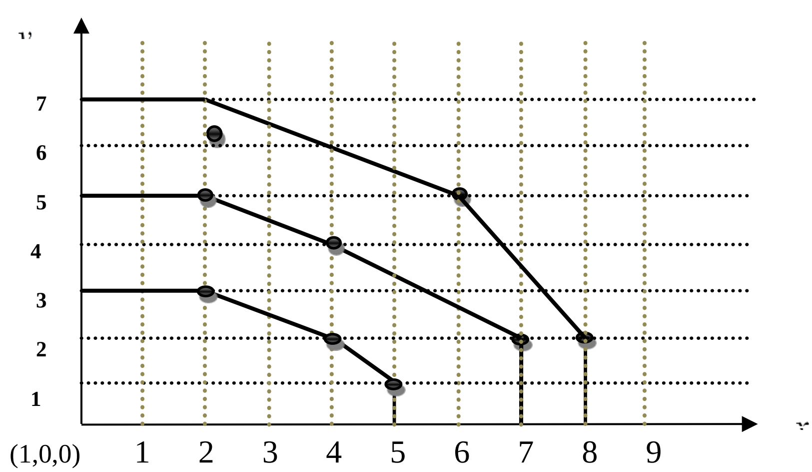

Figure 1. Illustration of the numerical example.

Table 3. The rank of DMUs with the equal performance

DMUs	$r_o(1)$	$r_o(2)$	$r_o(3)$	*Rank*
1	4.733	-	-	1
2	4.999	-	-	2
3	4.142	-	-	3
4	-	1.212	-	3
5	-	1.333	-	2
6	-	1.375	-	1
7	-	-	0.231	3
8	-	-	0.267	1
9	-	-	0.250	2

According to Table 3, it results that the first DMU has the first rank among all the first level DMUs and the third DMU has the third rank. For the second level DMUs, the third DMU has the first rank and the first DMU has the third rank. Finally, for the third level DMUs, the second DMU has the first rank and the first DMU has the third rank. Therefore, decision manager can make better decisions about DMUs decision making based on their information's of level of their performance.

6. Conclusions

Relative attractiveness and progress are two concepts of context-dependent data envelopment analysis (DEA). However, it clearly violates the standard axioms of DEA, so that the efficient frontier obtained of traditional DEA models cannot use for evaluating the relative attractiveness and progress, in presence of the integer-valued technology. This paper dealt with relative attractiveness and progress, when some inputs and/or outputs can only take integer values. Therefore, we used the modified models in order to measure integer-attractiveness and integer-progress. For this purpose, by using the integer DEA models, we determined the efficient and inefficient units. Then, we divided all the units into different levels of efficient frontiers, and we measured attractiveness and progress from one level to another level. Also, we rank the equivalent units by using the attractiveness index and the progress index which allows us to measure their equal performance based on evaluation context. For future studies, uncertainty programming approaches such as fuzzy mathematical programming together with integer programming can be applied in order to deal with another type of data uncertainty.

Conflict of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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