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A New Approach for Solving Intuitionistic Fuzzy Data Envelopment Analysis Problems

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ABSTRACT

Data envelopment analysis (DEA) is a mathematical technique based on linear programming applied to evaluate the efficiency of decision-making units dealing with multiple inputs and outputs. In classical DEA models, it is assumed that input and output data are accurate though real-world applications require considering inaccurate and ambiguous data. Moreover, linguistic forms may be intuitionistic in nature rather than fuzzy. We propose a novel approach to solve DEA models characterized by intuitionistic fuzzy data. The model is transformed into a linear programming problem with an intuitionistic fuzzy objective function, and an alphabetical technique is applied to solve it. The proposed approach is easy to implement, involving a lower number of calculations than more computationally demanding techniques introduced in the literature. It also provides a set of ranking results that are significantly correlated with those derived from the implementation of more complex techniques. Its applicability would allow to extend the analysis of intricate evaluation scenarios common to the standard DEA literature into intuitionistic fuzzy environments.

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric mathematical technique based on linear programming applied to evaluate the performance of decision-making units (DMUs) dealing with multiple inputs and outputs. DEA focus on the DMUs, calculates the weights of the inputs and the corresponding outputs separately and obtains the efficiency of each unit using the weighted ratio of total inputs to outputs [3].

In classical DEA models, it is generally assumed that the inputs and outputs of the units are specified using exact numerical values. However, in actual DEA applications, accurate input and output values may not be available, especially when DMUs must deal with lost, estimated, and qualitative data. The incorporation of fuzzy numbers, applied to express inaccurate information and uncertain data, into traditional DEA models led to

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the development of the fuzzy DEA literature (Peykani et al., [11]). This branch of the DEA literature has expanded considerably in the latter years, incorporating the main advances developed within the fuzzy mathematical domain (Ebrahimnejad and Amani [5], Hatami-Marbini et al., [8], Hosseinzadeh Lotfi et al., [9], Kao and Liu [10], Saati et al., [13], Wang and China [15], Wang et al., [16], and Van and Lee [17]).

One of the main features of fuzzy set theory is that when operating with fuzzy sets or numbers, the sum of the membership and non-membership values of an element is equal to one. However, in real-life situations, researchers may have to operate with unreliable information. This means that the sum of the membership and non-membership values of an element is not necessarily equal to one. The use of fuzzy data in these situations is therefore not appropriate. As a result, some of the inputs and outputs analyzed through DEA may be intuitionistic in nature rather than fuzzy. This constraint motivates the framework analyzed in the current paper, which introduces an intuitive and easily applicable DEA model designed to evaluate the relative efficiency of DMUs characterized by intuitionistic fuzzy input and output data.

Researchers have already proposed formal models to evaluate the efficiency of DMUs described via intuitionistic fuzzy variables. For instance, Puri and Yadavar [12] introduced different models to find the optimistic and pessimistic efficiencies of DMUs within an intuitionistic fuzzy environment. They developed an algorithmic technique that ranks units when optimistic and pessimistic evaluations are performed separately, using the hyperfunction technique. They also considered the case where optimistic and pessimistic evaluations are performed simultaneously as a combined approach, proposing different ranking methods based on the efficiency and inefficiency levels. The overall performance was evaluated using a hybrid intuitionistic fuzzy DEA model designed to incorporate optimistic and pessimistic conditions.

In another study, Aria and Yadav [2] generalized the auxiliary variable (SBM) measurement model to an intuitionistic fuzzy environment and obtained intuitionistic fuzzy efficiencies based on the concept of alpha- and beta-shear. These authors adapted the SBM super-efficiency model to calculate the efficiency of efficient units per given values of alpha and beta. Their model allows both to rank DMUs and compare intuitionistic fuzzy functions. These authors developed a similar approach for DEA models with intuitionistic fuzzy input and output values [1], calculating both intuitionistic fuzzy performances per given alpha and beta values and targets for the intuitionistic fuzzy inputs and outputs.

In the current paper, we propose a novel intuitionistic fuzzy arithmetic approach to solve intuitionistic fuzzy DEA problems. Our model is computationally simpler than the recent techniques described and, unlike the existing approaches, focuses on a DEA deficit model within an intuitionistic fuzzy environment. We consider DMUs characterized by intuitionistic fuzzy inputs and outputs and perform arithmetic operations on the objective function of the problem to derive a fractional model with definite constraints. Then, we apply Wang's alphabetical method to obtain the intuitionistic fuzzy efficiency of each DMU and, finally, use the accuracy function approach presented by Ebrahimnejad and Verdegai [7] to compute the resulting efficiencies and classify the DMUs.

In addition to the novelty of the approach, its main contribution relies on its computational simplicity and applicability. That is, the solution technique defined allows for an easily computable structure that fosters its inclusion within more complex hybrid techniques requiring the use of fuzzy variables. This is the case, for instance, when considering the relationship existing between multi-objective optimization and DEA models (Di Caprio et al. [4], Tavana et al. [14]). The implementation of overly complex solution methods may undermine the design of hybrid techniques, which are essential to solve a wide variety of real-life problems, while the computational simplicity of the current model incentivizes their development.

The paper proceeds as follows. The next section introduces the main definitions required to build the intuitionistic fuzzy DEA model presented in Section 3. Section 4 illustrates numerically its applicability. Section 5 summarizes the main results obtained and suggests potential extensions.

2. Intuitionistic fuzzy sets

We summarize below the basic concepts of intuitionistic fuzzy set theory, such as intuitionistic fuzzy numbers, intuitionistic fuzzy arithmetic, and the ranking of intuitionistic fuzzy numbers [7].

Definition 1. Suppose X is a reference set. The intuitionistic fuzzy set $\tilde{A}^I$ on X is defined by an orderly set of $\tilde{A}^I = \left\{ \left\langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \right\rangle ; x \in X \right\}$ in which the functions $\mu_{\tilde{A}^I} : X \to [0,1]$ and $\nu_{\tilde{A}^I} : X \to [0,1]$ indicate the degree of membership and non-membership of the element x in X, respectively, and for each $x \in X$, the condition $0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1$ is satisfied.

Definition 2. The intuitionistic fuzzy set $\tilde{A}^I$ defined on the set of real numbers is called an intuitionistic triangular fuzzy number whenever the membership and non-membership functions are defined as follows:

$$
\begin{aligned}
\mu_{\tilde{A}^I}(x) &= \begin{cases} \dfrac{x - a^L}{a^M - a^L}, & a^L \le x \le a^M \\ \dfrac{a^U - x}{a^U - a^M}, & a^M \le x \le a^U \end{cases} \\
\nu_{\tilde{A}^I}(x) &= \begin{cases} \dfrac{a^M - x}{a^M - a^{L'}}, & a^{L'} \le x \le a^M \\ \dfrac{x - a^M}{a^{U'} - a^M}, & a^M \le x \le a^{U'} \end{cases}
\end{aligned} \tag{1}
$$

where $a^{L'} \le a^L \le a^M \le a^U \le a^{U'}$. Intuitionistic triangular fuzzy numbers are represented via $\tilde{A}_I = \left(a^{L'}, a^L, a^M, a^U, a^{U'} \right)$.

Definition 3. An intuitionistic triangular fuzzy number $\tilde{A}_I = \left(a^{L'}, a^L, a^M, a^U, a^{U'} \right)$ is called an intuitionistic (positive) triangular fuzzy number whenever $a^{L'} \ge 0$) $a^{L'} > 0$ (.

Definition 4. Assume that $\tilde{A}_I = \left(a^{L'}, a^L, a^M, a^U, a^{U'} \right)$ and $\tilde{B}_I = \left(b^{L'}, b^L, b^M, b^U, b^{U'} \right)$ are intuitionistic triangular fuzzy numbers. The main intuitionistic fuzzy arithmetic operations are defined as follows:

I. $\tilde{A}_I + \tilde{B}_I = \left(a^{L'} + b^{L'}, a^L + b^L, a^M + b^M, a^U + b^U, a^{U'} + b^{U'} \right)$

II. $\tilde{A}_I - \tilde{B}_I = \left(a^{L'} - b^{U'}, a^L - b^U, a^M - b^M, a^U - b^L, a^{U'} - b^{L'} \right)$

III. $a^{L'}, b^{L'} \ge 0,\ \tilde{A}_I . \tilde{B}_I = \left(a^{L'} b^{L'}, a^L b^L, a^M b^M, a^U b^U, a^{U'} b^{U'} \right)$

IV. $b^{L'} > 0,\ \dfrac{\tilde{A}_I}{\tilde{B}_I} = \left(\dfrac{a^{L'}}{b^{U'}}, \dfrac{a^L}{b^U}, \dfrac{a^M}{b^M}, \dfrac{a^U}{b^L}, \dfrac{a^{U'}}{b^{L'}} \right)$

V. $k\tilde{A}_I = \begin{cases} \left(ka^{L'}, ka^L, ka^M, ka^U, ka^{U'} \right), & k \ge 0, \\ \left(ka^{U'}, ka^U, ka^M, ka^L, ka^{L'} \right), & k < 0, \end{cases}$

Definition 5. Let $\tilde{A}_I = \left(a^{L'}, a^L, a^M, a^U, a^{U'} \right)$ be an intuitionistic triangular fuzzy number. Its accuracy function is defined as follows:

$$H\left(\tilde{A}_I\right) = \frac{a^{L'} + a^L + 4a^M + a^U + a^{U'}}{8}$$

According to Definition 5, the comparison of two intuitionistic triangular fuzzy numbers $\tilde{A}_I$ and $\tilde{B}_I$ in terms of their accuracy function is performed as follows:

$$\tilde{A}_I \geq \tilde{B}_I \Leftrightarrow H\left(\tilde{A}_I\right) \geq H\left(\tilde{B}_I\right).$$

3. The intuitionistic fuzzy DEA model

In this section, we incorporate intuitionistic fuzzy inputs and outputs within a standard DEA model and suggest a simple and direct solution technique.

Denote by $\tilde{X}_j^I = (\tilde{x}_{1j}^I, \ldots, \tilde{x}_{ij}^I, \ldots, \tilde{x}_{mj}^I)^T$ the intuitionistic fuzzy input of the jth DMU and by $\tilde{Y}_j^I = (\tilde{y}_{1j}^I, \ldots, \tilde{y}_{rj}^I, \ldots, \tilde{y}_{sj}^I)^T$ its intuitionistic fuzzy output, with $j = 1, \ldots, n$. The intuitionistic fuzzy efficiency of DMU_j is defined as follows:

$$\tilde{E}_j^I = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}^I}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}^I} \tag{2}$$

Let $\tilde{x}_{ij}^I = \left(x_{ij}^{L'}, x_{ij}^L, x_{ij}^M, x_{ij}^U, x_{ij}^{U'}\right)$ and $\tilde{y}_{rj}^I = \left(y_{rj}^{L'}, y_{rj}^L, y_{rj}^M, y_{rj}^U, y_{rj}^{U'}\right)$ be intuitionistic triangular fuzzy inputs and outputs, respectively. According to Definition 4, the intuitionistic fuzzy efficiency can be calculated as follows:

$$\tilde{E}_j^I = \left(\frac{\sum_{r=1}^{s} u_r y_{rj}^{L'}}{\sum_{i=1}^{m} v_i x_{ij}^{U'}}, \frac{\sum_{r=1}^{s} u_r y_{rj}^{L}}{\sum_{i=1}^{m} v_i x_{ij}^{U}}, \frac{\sum_{r=1}^{s} u_r y_{rj}^{M}}{\sum_{i=1}^{m} v_i x_{ij}^{M}}, \frac{\sum_{r=1}^{s} u_r y_{rj}^{U}}{\sum_{i=1}^{m} v_i x_{ij}^{L}}, \frac{\sum_{r=1}^{s} u_r y_{rj}^{U'}}{\sum_{i=1}^{m} v_i x_{ij}^{L'}} \right) \tag{3}$$

The intuitionistic fuzzy DEA model designed to measure the intuitionistic fuzzy performance of DMU_p is defined as follows:

$$\begin{aligned} \max\ & \tilde{E}_p^I = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rp}^I}{\sum_{i=1}^{m} v_i \tilde{x}_{ip}^I} \\ \text{s.t.}\ & \tilde{E}_j^I = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}^I}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}^I} \leq 1, \qquad j = 1, 2, \ldots, n, \\ & u_r \geq 0, v_i \geq 0,\ r = 1, \ldots, s, i = 1, \ldots, m, \end{aligned} \tag{4}$$

Incorporating the definition given in Equation (3) into Model (4) we obtain:

$$\begin{aligned}
&\max \tilde{E}_p^I = \left(\frac{\sum_{r=1}^{s} u_r y_{rp}^{L'}}{\sum_{i=1}^{m} v_i x_{ip}^{U'}}, \frac{\sum_{r=1}^{s} u_r y_{rp}^{L}}{\sum_{i=1}^{m} v_i x_{ip}^{U}}, \frac{\sum_{r=1}^{s} u_r y_{rp}^{M}}{\sum_{i=1}^{m} v_i x_{ip}^{M}}, \frac{\sum_{r=1}^{s} u_r y_{rp}^{U}}{\sum_{i=1}^{m} v_i x_{ip}^{L}}, \frac{\sum_{r=1}^{s} u_r y_{rp}^{U'}}{\sum_{i=1}^{m} v_i x_{ip}^{L'}} \right) \\
&\text{s.t.} \quad \tilde{E}_j^I = \left(\frac{\sum_{r=1}^{s} u_r y_{rj}^{L'}}{\sum_{i=1}^{m} v_i x_{ij}^{U'}}, \frac{\sum_{r=1}^{s} u_r y_{rj}^{L}}{\sum_{i=1}^{m} v_i x_{ij}^{U}}, \frac{\sum_{r=1}^{s} u_r y_{rj}^{M}}{\sum_{i=1}^{m} v_i x_{ij}^{M}}, \frac{\sum_{r=1}^{s} u_r y_{rj}^{U}}{\sum_{i=1}^{m} v_i x_{ij}^{L}}, \frac{\sum_{r=1}^{s} u_r y_{rj}^{U'}}{\sum_{i=1}^{m} v_i x_{ij}^{L'}} \right) \le 1, \quad j = 1,2,\dots,n, \\
&\qquad u_r \ge 0, v_i \ge 0, \ r = 1,\dots,s, i = 1,\dots,m,
\end{aligned} \tag{5}$$

Note that the intuitionistic fuzzy constraint of Model (5) will be lower than or equal to one whenever its last component is lower than or equal to one. Therefore, Model (5) can be rewritten as follows:

$$\begin{aligned}
&\max \ \tilde{E}_p^I = \left(\frac{\sum_{r=1}^{s} u_r y_{rp}^{L'}}{\sum_{i=1}^{m} v_i x_{ip}^{U'}}, \frac{\sum_{r=1}^{s} u_r y_{rp}^{L}}{\sum_{i=1}^{m} v_i x_{ip}^{U}}, \frac{\sum_{r=1}^{s} u_r y_{rp}^{M}}{\sum_{i=1}^{m} v_i x_{ip}^{M}}, \frac{\sum_{r=1}^{s} u_r y_{rp}^{U}}{\sum_{i=1}^{m} v_i x_{ip}^{L}}, \frac{\sum_{r=1}^{s} u_r y_{rp}^{U'}}{\sum_{i=1}^{m} v_i x_{ip}^{L'}} \right) \\
&\text{s.t.} \quad \frac{\sum_{r=1}^{s} u_r y_{rj}^{U'}}{\sum_{i=1}^{m} v_i x_{ij}^{L'}} \le 1, \qquad j = 1,2,\dots,n, \\
&\qquad u_r \ge 0, v_i \ge 0, \ r = 1,\dots,s, i = 1,\dots,m,
\end{aligned} \tag{6}$$

Clearly, the objective function of Model (6) is given by an intuitionistic fuzzy variable, but all fuzziness has been eliminated from its constraints. Therefore, Model (6) can be interpreted as a multi-objective linear programming problem and solved using an alphabetical approach. The subsequent procedure is described below.

First, Model (7) is introduced to calculate the lower bound efficiency of the objective function of Model (6):

$$\begin{aligned}
&\max E_p^{L'} = \frac{\sum_{r=1}^{s} u_r y_{rp}^{L'}}{\sum_{i=1}^{m} v_i x_{ip}^{U'}} \\
&s.t. \\
&\frac{\sum_{r=1}^{s} u_r y_{rj}^{U'}}{\sum_{i=1}^{m} v_i x_{ij}^{L'}} \le 1 \quad , \quad j = 1,2,\dots,n \\
&u_r, v_i \ge \varepsilon, \quad i = 1,2,\dots,m \quad , \quad r = 1,2,\dots,s
\end{aligned} \tag{7}$$

Model (8) is introduced in the next stage to calculate the optimal value of E_p^L while incorporating the optimal value obtained for the lower bound of the objective function, namely, $E_p^{L'*}$:

$$\max E_p^L = \frac{\sum_{r=1}^{s} u_r y_{rp}^L}{\sum_{i=1}^{m} v_i x_{ip}^U}$$
$$s.t.$$
$$\frac{\sum_{r=1}^{s} u_r y_{rj}^{U'}}{\sum_{i=1}^{m} v_i x_{ij}^{L'}} \le 1 \quad , \quad j = 1, 2, \ldots, n \tag{8}$$
$$\frac{\sum_{r=1}^{s} u_r y_{rp}^{L'}}{\sum_{i=1}^{m} v_i x_{ip}^{U'}} = E_p^{L'*}$$
$$u_r, v_i \ge \varepsilon, \quad i = 1, 2, \ldots, m \quad , \quad r = 1, 2, \ldots, s$$

The remaining optimal values composing the resulting intuitionistic fuzzy performance are calculated as follows:

$$\max E_p^M = \frac{\sum_{r=1}^{s} u_r y_{rp}^M}{\sum_{i=1}^{m} v_i x_{ip}^M}$$
$$s.t.$$
$$\frac{\sum_{r=1}^{s} u_r y_{rj}^{U}}{\sum_{i=1}^{m} v_i x_{ij}^{L}} \le 1 \quad , \quad \text{j=1,2,...,n} \tag{9}$$
$$\frac{\sum_{r=1}^{s} u_r y_{rp}^{L'}}{\sum_{i=1}^{m} v_i x_{ip}^{U'}} = E_p^{L'*} \quad , \quad \frac{\sum_{r=1}^{s} u_r y_{rp}^{L}}{\sum_{i=1}^{m} v_i x_{ip}^{U}} = E_p^{L*}$$
$$u_r, v_i \ge \varepsilon, \quad i = 1, 2, \ldots, m \quad , \quad r = 1, 2, \ldots, s$$

$$\begin{aligned}
&\max E_p^{U} = \frac{\sum_{r=1}^{s} u_r y_{rp}^{U}}{\sum_{i=1}^{m} v_i x_{ip}^{L}} \\
&s.t. \\
&\frac{\sum_{r=1}^{s} u_r y_{rj}^{U'}}{\sum_{i=1}^{m} v_i x_{ij}^{L'}} \le 1 \quad , \quad j = 1,2,\ldots,n \\
&\frac{\sum_{r=1}^{s} u_r y_{rp}^{L'}}{\sum_{i=1}^{m} v_i x_{ip}^{U'}} = E_p^{L'*} \ , \ \frac{\sum_{r=1}^{s} u_r y_{ro}^{L}}{\sum_{i=1}^{m} v_i x_{io}^{U}} = E_p^{L*} \ , \ \frac{\sum_{r=1}^{s} u_r y_{rp}^{M}}{\sum_{i=1}^{m} v_i x_{ip}^{M}} = E_p^{M*} \\
&u_r, v_i \ge \varepsilon, \quad i = 1,2,\ldots,m \quad , \quad r = 1,2,\ldots,s
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\max E_p^{U'} = \frac{\sum_{r=1}^{s} u_r y_{rp}^{U'}}{\sum_{i=1}^{m} v_i x_{ip}^{L'}} \\
&s.t. \\
&\frac{\sum_{r=1}^{s} u_r y_{rj}^{U'}}{\sum_{i=1}^{m} v_i x_{ij}^{L'}} \le 1 \quad , \quad j = 1,2,\ldots,n \\
&\frac{\sum_{r=1}^{s} u_r y_{rp}^{L'}}{\sum_{i=1}^{m} v_i x_{ip}^{U'}} = E_p^{L'*} \ , \ \frac{\sum_{r=1}^{s} u_r y_{rp}^{L}}{\sum_{i=1}^{m} v_i x_{ip}^{U}} = E_p^{L*} \ , \ \frac{\sum_{r=1}^{s} u_r y_{rp}^{M}}{\sum_{i=1}^{m} v_i x_{ip}^{M}} = E_p^{M*} \ , \ \frac{\sum_{r=1}^{s} u_r y_{rp}^{U}}{\sum_{i=1}^{m} v_i x_{ip}^{L}} = E_p^{U*} \\
&u_r, v_i \ge \varepsilon, \quad i = 1,2,\ldots,m \quad , \quad r = 1,2,\ldots,s
\end{aligned} \tag{11}$$

The optimal weights obtained after solving Model (6) alphabetically are introduced in Equation (3) to obtain the intuitionistic fuzzy efficiency of the DMUs. In order to derive a complete ranking of the units, we must apply Definition 5 to compute the accuracy function of all the intuitionistic fuzzy efficiencies

$$H\left(\tilde{E}_p^{I}\right) = \frac{E_p^{L'} + E_p^{L} + 4E_p^{M} + E_p^{U} + E_p^{U'}}{8}$$

The efficiencies must be subsequently compared and ranked, providing a final evaluation of the DMUs. We illustrate numerically the implementation of this technique in the next section.

4. Numerical example

In this section, an example is given to illustrate the intuitive applicability of the proposed method. Consider the performance evaluation of 12 DMUs defined by sets of two intuitionistic fuzzy inputs and outputs. The value of the inputs and outputs describing the corresponding DMUs are listed in Table 1.

Table 1. Intuitionistic fuzzy inputs and outputs describing the DMUs

DMUs	Input 1	Input 2	Output 1	Output 2
1	(15,17,20,23,25)	(145,148,151,153,154)	(95,97,100,103,105)	(84,86,90,95,97)
2	(12,15,19,22,26)	(127,29,131,134,136)	(146,148,150,151,153)	(45,48,50,53,55)
3	(20,22,25,28,30)	(154,158,160,163,165)	(154,157,160,163,165)	(50,53,55,57,59)
4	(22,23,27,29,33)	(163,165,167,170,172)	(176,178,180,183,185)	(67,70,72,74,75)
5	(18,20,22,25,27)	(153,155,158,161,163)	(84,91,94,97,99)	(60,62,66,68,71)
6	(49,51,55,58,60)	(250,252,255,258,260)	(226,228,230,231,232)	(85,87,90,93,95)
7	(29,31,33,36,39)	(,230,233,235,236,238)	(215,217,220,223,225)	(81,83,88,92,95)
8	(27,29,31,34,36)	(200,202,206,208,210)	(148,150,152,154,155)	(76,78,80,83,84)
9	(24,27,30,32,35)	(238,241,244,246,248)	(183,187,190,194,196)	(94,97,100,102,104)
10	(44,47,50,54,55)	(260,262,268,271,273)	(244,246,250,253,255)	(95,97,100,104,105)
11	(48,51,53,55,57)	(300,302,306,308,309)	(256,258,260,261,262)	(140,142,147,149,152)
12	(32,34,38,40,41)	(280,281,284,286,287)	(243,248,250,256,258)	(117,119,120,123,125)

Models (7) to (11) are applied to obtain the set of optimal weights, which, at the same time, are used to compute the corresponding intuitionistic fuzzy efficiencies through Equation (3). The weight and efficiency values obtained for each DMU are described in Table 2.

Table 2. Optimal inputs and outputs weights and intuitionistic efficiencies per DMU

DMUs	v_1	v_2	u_1	u_2	$\tilde{E}_p^I$
1	0.0003	0.0069	0.0001	0.0102	(0.8133,0.8383,0.889,0.9574,1)
2	0.0001	0.0077	0.0064	0.0001	(0.8944,0.9207,0.9549,0.9768,1)
3	0.0001	0.0065	0.0041	0.0037	(0.7591,0.7905,0.8245,0.8543,0.8921)
4	0.0013	0.006	0.0029	0.006	(0.8488,0.8851,0.9199,0.9557,0.98)
5	0.0007	0.0065	0.0031	0.0064	(0.,5976,0.6381,0.6848,0.7204,0.7559)
6	0.0004	0.0039	0.0019	0.0039	(0.733,0.7504,0.7752,0.799,0.8157)
7	0.0005	0.0042	0.0021	0.0042	(0.7769,0.797,0.8287,8598,0.8888)
8	0.0009	0.0049	0.0024	0.0049	(0.6855,0.707,0.7296,0.7642,0.7802)
9	0.0007	0.0041	0.002	0.0041	(0.7216,0.7485,0.7734,0.8006,0.8281)
10	0.0002	0.0038	0.0018	0.0037	(0.7542,0.7704,0.7974,0.836,0.85)
11	0.0003	0.0033	0.0016	0.0032	(0.8272,0.8396,08642,0.8839,0.9016)
12	0.0007	0.0035	0.0017	0.0035	(0.7962,0.8145,0.828,0.8594,0.874)

The accuracy function is then applied to define the efficiency and corresponding rank position of each DMU, both of which are presented in Table 3.

Table 3. Accuracy values and ranking positions for each DMU

DMUs	$H\left(\tilde{E}_j^I\right)$	Rank
1	0.8956	2
2	0.6562	12
3	0.8243	6
4	0.9187	1
5	0.6814	11
6	0.7749	8
7	0.8297	5
8	0.7319	10
9	0.7582	9
10	0.8	7
11	0.8636	3
12	0.832	4

We provide now some basic comparisons with respect to the rankings obtained applying some of the main competing models develop in the literature to the same numerical example. In particular, we focus on the super-efficiency and geometric approaches introduced by Puri and Yadav [12]. Table 4 compares the rankings obtained applying the proposed accuracy function with those derived from their optimistic super-efficiency, $S\tilde{E}_j^{OI}$, pessimistic super-efficiency, $S\tilde{E}_j^{PI}$, and geometric, $\tilde{E}_j^{geometric}$, models. The geometric model is defined as the square root of the product between their optimistic and pessimistic intuitionistic fuzzy DEA techniques ([12]).

Table 4. Ranking comparisons relative to the models introduced by Puri and Yadav [12]

DMUs	$H\left(\tilde{E}_j^I\right)$	$S\tilde{E}_j^{OI}$	$S\tilde{E}_j^{PI}$	$\tilde{E}_j^{geometric}$
1	2	1	3	2
2	12	2	5	5
3	6	8	10	8
4	1	4	1	1
5	11	12	11	12
6	8	10	12	10
7	5	7	6	7
8	10	11	9	11
9	9	5	8	6
10	7	9	7	9
11	3	6	2	4
12	4	3	4	3

In order to provide additional intuition, Figure 1 illustrates the corresponding rankings. The DMUs are represented in the horizontal axis, while the subsequent ranking position are described in the vertical axis. It can already be observed that there are noticeable differences between the current approach and the super-efficiency ones. Though the patterns described by the rankings bear some resemblance, the actual rankings delivered by both types of techniques differ considerably. On the other hand, the geometric model, which requires a more convoluted evaluation procedure, seems to be significantly correlated with the current method.

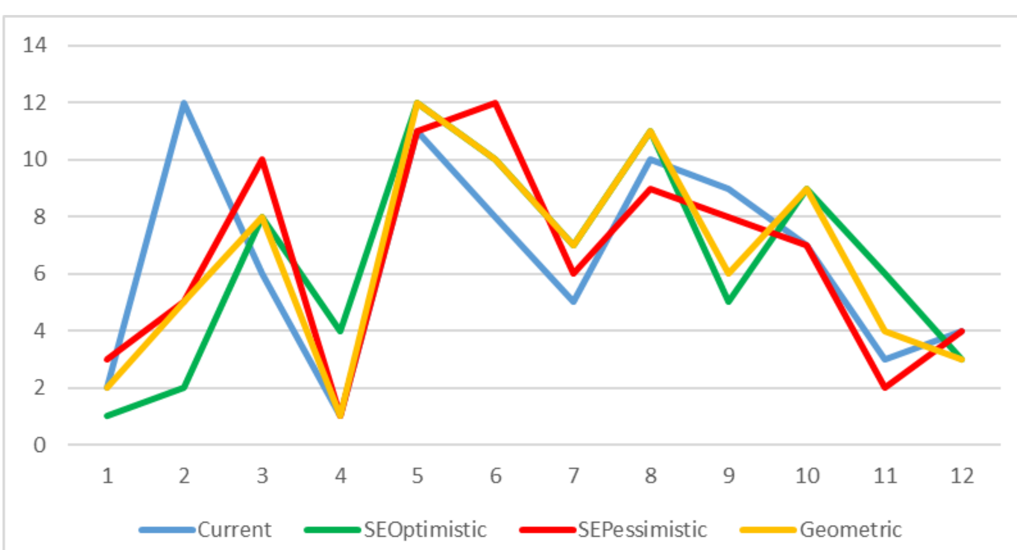

Figure 1. An illustration of the current ranking and those of Puri and Yadav [12]

The intuition derived from Figure 1 is validated in Table 5, which presents the Spearman correlations between the different rankings. The similar trends exhibited by the current accuracy function method and the geometric model are further highlighted through the scatter plot presented in Figure 2. Clearly, a perfectly correlated scenario would require both ranking techniques to define an identity function. Even though this is not the case, we observe a very similar evaluation pattern derived from both methods. That is, the method proposed, which is computationally simple and intuitive, provides a set of ranking results that are significantly correlated with those derived from the implementation of a substantially complex technique such as the geometric one.

Table 5. Ranking correlations with the models developed by Puri and Yadav [12]

Spearman's rho Correlations		Current	SEOptimistic	SEPessimistic	Geometric
Current	Correlation Coefficient	1.000	0.462	0.699*	0.727**
	Sig. (2-tailed)	.	0.131	0.011	0.007
	N	12	12	12	12
SEOptimistic	Correlation Coefficient	0.462	1.000	0.769**	0.916**
	Sig. (2-tailed)	0.131	.	0.003	0.000
	N	12	12	12	12
SEPessimistic	Correlation Coefficient	0.699*	0.769**	1.000	0.902**
	Sig. (2-tailed)	0.011	0.003	.	0.000
	N	12	12	12	12
Geometric	Correlation Coefficient	0.727**	0.916**	0.902**	1.000
	Sig. (2-tailed)	0.007	0.000	0.000	.
	N	12	12	12	12

*. Correlation is significant at the 0.05 level (2-tailed).
**. Correlation is significant at the 0.01 level (2-tailed).

We conclude by noting that differences with respect to the optimistic and pessimistic super-efficiency approaches are however obtained, a consistent result given the fact that none of the main features defining these models are incorporated into the current setting, allowing for potential developments and comparisons to be performed through future research.

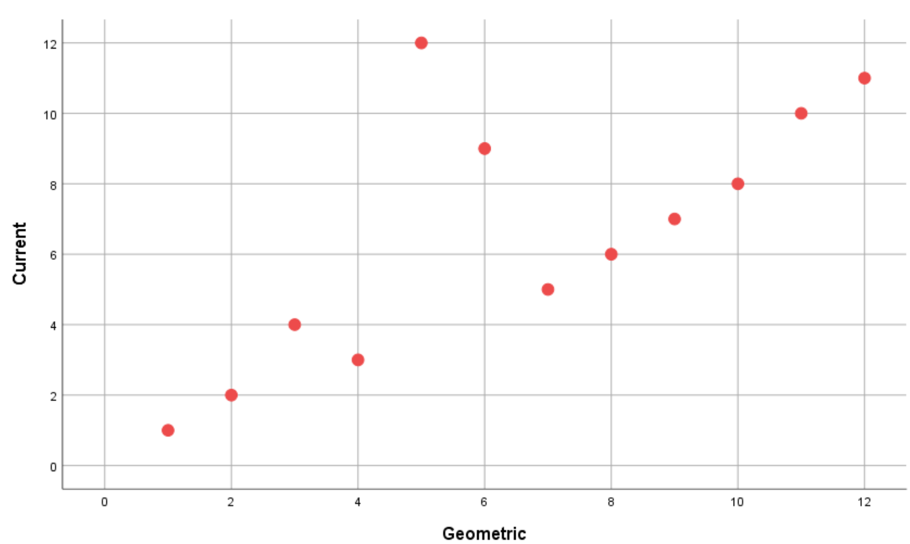

Figure 2. Scatter plot of the rankings generated by the current and geometric models

5. Conclusion

We have introduced an intuitive and easily applicable DEA model designed to account for intuitionistic fuzzy inputs and outputs. The model has been transformed into a linear programming problem with an intuitionistic fuzzy objective function, and an alphabetical technique has been applied to solve it. The proposed approach is easy to implement, involving a lower number of calculations than the standard alpha- and beta-cutting approaches. It also provides a set of ranking results that are significantly correlated with those derived from the application of more complex techniques. In this regard, future research could incorporate the effects of optimism and pessimism or multiple decision stages, allowing for a simplified analysis of scenarios common to the standard DEA literature within an intuitionistic fuzzy environment.

We conclude by highlighting the capacity of our model to simplify the incorporation of intuitionistic fuzzy variables within complex hybrid multi-objective optimization problems. The same reasoning applies to group decision making environments dealing with intuitionistic fuzzy variables (Ebrahimnejad el al., [6]), defining a substantial set of potential extensions within both lines of research.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. Arya, A., & Yadav, S.P. (2019). Development of intuitionistic fuzzy data envelopment analysis models and intuitionistic fuzzy input–output targets. *Soft Computing*, 23, 8975-8993.
2. Arya, A., & Yadav, S.P. (2019). Development of intuitionistic fuzzy super-efficiency slack based measure with an application to health sector. *Computers & Industrial Engineering*, 115, 368-380.
3. Charnes, A., Cooper, W.W., Golany, B., Seiford, L.M., & Stutz, J. (1985). Foundations of data envelopment analysis and Pareto–Koopmans empirical production functions. *Journal of Econometrics*, 30, 91-107.
4. Di Caprio, D., Ebrahimnejad, A., Ghiyasi, M., & Santos Arteaga, F.J. (2020). Integrating fuzzy goal programming and data envelopment analysis to incorporate preferred decision-maker targets in efficiency measurement. *Decisions in Economics and Finance*, 43, 673-690.
5. Ebrahimnejad, A., & Amani, N. (2021). Fuzzy data envelopment analysis in the presence of undesirable outputs with ideal points. *Complex & Intelligent Systems*, 7, 379-400.
6. Ebrahimnejad, A., Tavana, M., & Santos-Arteaga, F.J. (2016). An integrated Data Envelopment Analysis and simulation method for group consensus ranking. *Mathematics and Computers in Simulation*, 119, 1-17.
7. Ebrahimnejad, A., & Verdegay, J.L. (2018). A new approach for solving fully intuitionistic fuzzy transportation problems. *Fuzzy Optimization and Decision Making*, 17, 447-474.
8. Hatami-Marbini, A., Ebrahimnejad, A., & Lozano, S. (2017). Fuzzy efficiency measures in data envelopment analysis using lexicographic multiobjective approach. *Computers & Industrial Engineering*, 105, 362-376.
9. Hosseinzadeh Lotfi, F., Ebrahimnejad, A., Vaez-Ghasemi, M., & Moghaddas, Z. *Data Envelopment Analysis with R*, Volume 386 of Studies in Fuzziness and Soft Computing, Springer, Cham (2020).
10. Kao, C., & Liu, S.T. (2000). Fuzzy efficiency measures in data envelopment analysis. *Fuzzy Sets and Systems*, 113, 427-437.
11. Peykani, P., Hosseinzadeh Lotfi, F., Sadjadi, S.J., Ebrahimnejad, A., & Mohammadi, E. (2021). Fuzzy chance-constrained data envelopment analysis: A structured literature review, current trends, and future directions. *Fuzzy Optimization and Decision Making*. https://doi.org/10.1007/s10700-021-09364-x
12. Puri, J., & Yadav, A.S.P. (2015). Intuitionistic fuzzy data envelopment analysis: An application to the banking sector in India, *Expert Systems with Applications*, 42, pp 4982-4998.
13. Saati, S., Memariani, A., & Jahanshahloo, G. R. (2002). Efficiency analysis and ranking of DMUs with fuzzy data. *Fuzzy Optimization and Decision Making*, 1, 255–267.
14. Tavana, M., Ebrahimnejad, A., Santos-Arteaga, F.J., Mansourzadeh, S.M., & Kazemi Matin, R. (2018). A hybrid DEA-MOLP model for public school assessment and closure decision in the city of Philadelphia. *Socio-Economic Planning Sciences*, 61, 70-89.
15. Wang, Y.M., & Chin, K.S. (2011). Fuzzy data envelopment analysis: A fuzzy expected value approach. *Expert Systems with Applications*, 38, 11678-11685.
16. Wang, Y.M., Luo, Y., & Liang, L. (2009). Fuzzy data envelopment analysis based upon fuzzy arithmetic with an application to

performance assessment of manufacturing enterprises. *Expert systems with applications*, 36, 5205-5211.

17. Wen, M., & Li, H. (2009). Fuzzy data envelopment analysis (DEA): Model and ranking method. *Journal of Computational and Applied Mathematics*, 223, 872-878.

Santos Arteaga, F. J., Ebrahimnejad, A., & Zabihi, A. (2021). A New Approach for Solving Intuitionistic Fuzzy Data Envelopment Analysis Problems. *Fuzzy Optimization and Modelling Journal*, 2(2), 46-57.

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