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A New Approach for Solving Interval Neutrosophic Integer Programming Problems

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ABSTRACT

Linear Programming as a practical technique for solving optimization problems with linear objective functions and linear constraint plays an essential role in mathematical programming. Most of the real-world problems are included in inconsistent and astute uncertainty. That's why the optimal solution can't be found easily. The neutrosophic theory, as an extension of fuzzy set theory, is a powerful instrument to handle inconsistent, indeterminate, and incomplete information. This paper presents an applied approach for solving interval neutrosophic integer programming problems. By using the proposed approach, we can handle both incomplete and indeterminate data. In this respect, using a ranking function, we present a technique to convert the interval neutrosophic Integer Programming problem into a crisp model and then solve it by standard methods.

1. Introduction

The fuzzy set (FS) theory which is proposed by Zadeh [18] is a practical approach that is widely used to capture linguistic uncertainty in optimization problems such that it assigns to each element a degree of membership. Sometimes because of uncertainty determining the degree of membership isn't possible. For this reason, Zadeh [19] proposed Interval Fuzzy Sets (IFSs) to express the uncertainty in the membership function. An interval-valued fuzzy set is a fuzzy set in which the membership degree is assumed to belong to an interval. Attanasov in [2] has presented an extension of classical fuzzy sets, that is, the so-called intuitionistic fuzzy sets (IFSs), such it assigns the degrees of membership (truth-membership) and non-membership (falsity-membership) to each element. In this respect, Atanasov [3] by extending the membership (truth-membership) and non-membership (falsity-membership) functions to the interval numbers in proposed the interval-valued intuitionistic fuzzy set (IVIFS). Smarandache [14, 16] introduced neutrosophy, which is the study of neutralities as an extension of dialectics.

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Mohamed et al. [5] by introducing a new score function, proposed a novel method for neutrosophic integer programming problems. Neutrosophic is the derivative of neutrosophy, and it includes neutrosophic set, neutrosophic probability, neutrosophic statistics, and neutrosophic logic. Neutrosophic theory means neutrosophy applied in many fields of sciences to solve problems related to indeterminacy. Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle incomplete and indeterminate information [15]. Neutrosophic sets are characterized by three independent degrees, namely truth-membership degree (T), indeterminacy-membership degree(I), and falsity-membership degree (F). The decision-makers in the neutrosophic set want to increase the degree of truth-membership and decrease indeterminacy and falsity membership degrees. Wang et al. in [17] proposed interval neutrosophic sets (INSs) where the degrees of truth, indeterminacy, and falsity memberships were extended to a subinterval of $[0,1]$.

This paper presents a new model and method for solving Interval Neutrosophic Integer Programming (INIP) problems. Using a score function, we convert INIP into crisp problems. Integer programming problems can be defined as linear programming problems with integer restrictions on decision variables. When some but not all decision variables are restricted to be an integer, this problem is called a mixed-integer problem, and when all decision variables are integers, it's a pure integer program. Integer programming plays an essential role in supporting managerial decisions. In integer programming problems, the decision-maker may not be able to specify the objective function and/or constraints functions precisely. The rest of this paper is organized as follows: Section 2 presents some definitions of neutrosophic sets, single value neutrosophic sets, and interval neutrosophic sets. Section 3 describes the formulation of INIP. In Section 4, we propose our method for solving INIP problems. In Section 5, a numerical example is presented. Finally, the conclusions are discussed in section 6.

2. Preliminaries

This section briefly reviews some necessary backgrounds and preliminaries of Neutrosophic sets, single-valued Neutrosophic sets, and interval neutrosophic sets.

Definition 1. [4,6] A Neutrosophic Set (NS) N in a domain X (finite universe of objectives) can be represented by $T_N : X \to \left]0^-,1^+\right[$, $I_N : X \to \left]0^-,1^+\right[$ and $F_N : X \to \left]0^-,1^+\right[$ such that $0^- \le T_N(x) + I_N(x) + F_N(x) \le 3^+ \ \forall x \in X$. Where $T_N(x)$, $I_N(x)$ and $F_N(x)$ denote the truth, indeterminacy, and falsity membership functions, respectively.

Definition 2. [1,8] A single-valued neutrosophic set (SVNS) N in a domain X (finite universe of objectives) can be denoted as $N = \{x, T_N(x), I_N(x), F_N(x); x \in X\}$, where $T_N : X \to [0,1]$, $I_N : X \to [0,1]$ and $F_N : X \to [0,1]$ are three maps in X that satisfy the condition $0 \le T_N(x) + I_N(x) + F_N(x) \le 3 \ \ \forall x \in X$. The numbers $T_N(x)$, $I_N(x)$ and $F_N(x)$, are respectively the degrees of truth, indeterminacy and falsity membership of element x to N .

Definition 3. [6,9] The addition and subtraction operations between two SVNNs such as $N = [(a^l, a^m, a^u); \alpha_N, \delta_N, \beta_N]$ and $M = [(b^l, b^m, b^u); \alpha_M, \delta_M, \beta_M]$ could be defined as:

$$N + M = [(a^l + b^l, a^m + b^m, a^u + b^u); \alpha_N \wedge \alpha_M, \delta_N \vee \delta_M, \beta_N \vee \beta_M], \tag{1}$$

$$N - M = [(a^l - b^l, a^m - b^m, a^u - b^u); \alpha_N \wedge \alpha_M, \delta_N \vee \delta_M, \beta_N \vee \beta_M], \tag{2}$$

Furthermore, the normal and scalar multiplications are defined as:

$$NM = [(a^l b^l, a^m b^m, a^u b^u); \alpha_N \wedge \alpha_M, \delta_N \vee \delta_M, \beta_N \vee \beta_M], \quad (3)$$

$$kN = \begin{cases} [(ka^l, ka^m, ka^u); \alpha_N, \delta_N, \beta_N], & k \geq 0, \\ [(ka^u, ka^m, ka^l); \alpha_N, \delta_N, \beta_N], & k \leq 0. \end{cases} \quad (4)$$

Definition 4. [9,10] Let N and M be two NNs. The ranking orders of these two numbers will be as:

- If $R(N) > R(M)$ then N is bigger than M,
- If $R(N) < R(M)$ then N is smaller than M,
- If $R(N) = R(M)$ then N is equal to M.

Definition 5. [13,11] Let X be a space of discourse, an interval neutrosophic set (INS) N through X taking the form $N = \{x, T_N(x), I_N(x), F_N(x);\ x \in X\}$ where $T_N(x), I_N(x), F_N(x) \subseteq [0,1]$ and $0 \leq SupT_N(x) + SupI_N(x) + SupF_N(x) \leq 3$ for all $x \in X$. $T_N(x)$, $I_N(x)$ and $F_N(x)$ represent truth membership, indeterminacy membership, and falsity membership of x to N, respectively.

Remark 1: An INS $N = [(a^l, a^m, a^u); [\alpha_N^l, \alpha_N^u], [\delta_N^l, \delta_N^u], [\beta_N^l, \beta_N^u]]$ will be reduced to the NS if $\alpha_N^l = \alpha_N^u$, $\delta_N^l = \delta_N^u$ and $\beta_N^l = \beta_N^u$.

Definition 6. [7,12] Let $N = [(a^l, a^m, a^u); [\alpha_N^l, \alpha_N^u], [\delta_N^l, \delta_N^u], [\beta_N^l, \beta_N^u]]$ and $M = [(b^l, b^m, b^u); [\alpha_M^l, \alpha_M^u], [\delta_M^l, \delta_M^u], [\beta_M^l, \beta_M^u]]$ are two INNs. The addition and subtraction operations for these two INNs are defined as follows:

$$\begin{aligned} N + M &= [(a^l + b^l, a^m + b^m, a^u + b^u); [\alpha_N^l + \alpha_M^l - \alpha_N^l \alpha_M^l, \alpha_N^u + \alpha_M^u - \alpha_N^u \alpha_M^u], [\delta_N^l \delta_M^l, \delta_N^u \delta_M^u], [\beta_N^l \beta_M^l, \beta_N^u \beta_M^u]], \\ N - M &= [(a^l - b^l, a^m - b^m, a^u - b^u); [\alpha_N^l + \alpha_M^l - \alpha_N^l \alpha_M^l, \alpha_N^u + \alpha_M^u - \alpha_N^u \alpha_M^u], \\ &\qquad [\delta_N^l \delta_M^l, \delta_N^u \delta_M^u], [\beta_N^l \beta_M^l, \beta_N^u \beta_M^u]]. \end{aligned} \quad (5)$$

3. Interval Neutrosophic Integer Programming

An integer programming problem with neutrosophic factors is presented as bellows:

$$\begin{aligned} & Max\ Z = \sum_{j=1}^{n} \tilde{c}_j x_j, \\ & s.t.\ \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, \quad i=1,2,\dots,m, \\ & \quad x_j \geq 0, \\ & \quad j=1,2,\dots,n. \end{aligned} \quad (6)$$

where x_j is an integer variable and $\tilde{c}_j, \tilde{a}_{ij}$, and $\tilde{b}_i$ represented the neutrosophic numbers.

In order to model these kinds of problems, we present the truth, indeterminacy, and falsity membership functions for an interval neutrosophic number (INN) N as follows (Figure 1):

$$T_N^L(x)=\begin{cases}\dfrac{x-a^l+h_N(a^l-x)}{a^m-a^l}, & a^l\le x\le a^m,\\[2ex] \dfrac{a^u-x+h_N(x-a^u)}{a^u-a^m}, & a^m\le x\le a^u,\\[2ex] 0, & \text{otherwise},\end{cases} \tag{7}$$

$$T_N^U(x)=\begin{cases}\dfrac{x-a^l+h_N(a^m-x)}{a^m-a^l}, & a^l\le x\le a^m,\\[2ex] \dfrac{a^u-x+h_N(x-a^m)}{a^u-a^m}, & a^m\le x\le a^u,\\[2ex] h_N, & \text{otherwise,,}\end{cases} \tag{8}$$

where $T_N(x)=[T_N^L(x),T_N^U(x)]$,

$$I_N^L(x)=\begin{cases}\dfrac{a^m-x+h_N(x-a^m)}{a^m-a^l}, & \delta a^l+(1-\delta)a^m\le x\le a^m,\\[2ex] \dfrac{x-a^m+h_N(a^m-x)}{a^u-a^m}, & a^m\le x\le(1-\delta)a^m+\delta a^u,\\[2ex] \delta, & \text{otherwise},\end{cases} \tag{9}$$

$$I_N^U(x)=\begin{cases}\dfrac{a^m-x+h_N(x-a^l)}{a^m-a^l}, & \delta a^l+(1-\delta)a^m\le x\le a^m,\\[2ex] \dfrac{x-a^m+h_N(a^u-x)}{a^u-a^m}, & a^m\le x\le(1-\delta)a^m+\delta a^u,\\[2ex] 1-\delta, & \text{otherwise},\end{cases}$$

(10)

where $I_N(x)=[I_N^L(x),I_N^U(x)]$,

$$F_N^L(x)=\begin{cases}\dfrac{a^m-x+h_N(x-a^m)}{a^m-a^l}, & a^l\le x\le a^m,\\[2ex] \dfrac{x-a^m+h_N(a^m-x)}{a^u-a^m}, & a^m\le x\le a^u,\\[2ex] 1-h_N, & \text{otherwise},\end{cases} \tag{11}$$

$$F_N^U(x)=\begin{cases}\dfrac{a^m-x+h_N(x-a^l)}{a^m-a^l}, & a^l\le x\le a^m,\\[2ex] \dfrac{x-a^m+h_N(a^u-x)}{a^u-a^m}, & a^m\le x\le a^u,\\[2ex] 1, & \text{otherwise},\end{cases} \tag{12}$$

where $F_N(x)=[F_N^L(x),F_N^U(x)]$ and $h_N=T_N^U(x)-T_N^L(x)$ such that $\delta\in(0,1)$ and $h_N\le\delta$.

The maximum value of the objective function for truth membership and the minimum values of the objective function for indeterminacy and falsity memberships can obtain as follows:

$f_{\max}=\max\{f(x_i^*)\}$ and $f_{\min}=\min\{f(x_i^*)\}$ for $(i=1,\dots,n)$ also $f_{\max}^F=f_{\max}^T-P(f_{\max}^T-f_{\min}^T)$, $f_{\min}^F=f_{\min}^T$ and $f_{\max}^I=f_{\max}^I$, $f_{\min}^I=f_{\min}^I-K(f_{\max}^T-f_{\min}^T)$ where P and K are real numbers in (0,1).

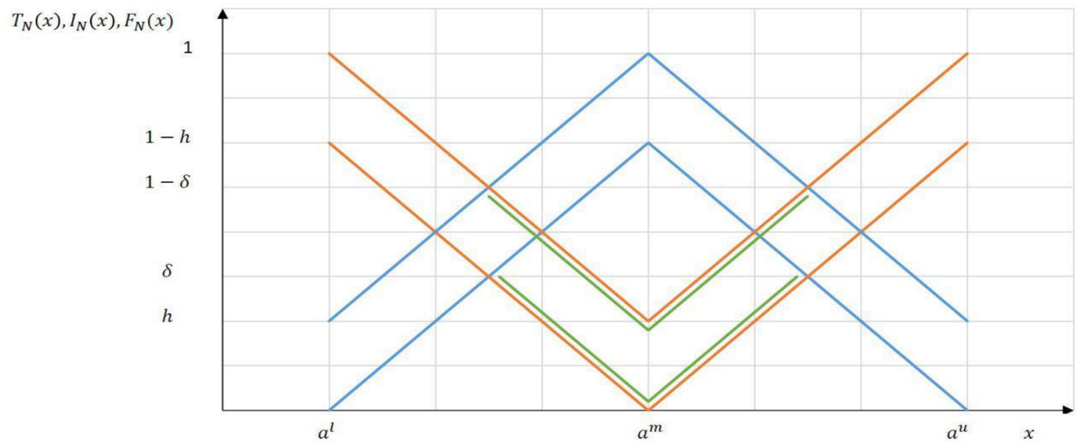

Figure 1. Truth, indeterminacy and falsity membership functions of INN.

Nevertheless, the neutrosophic optimize integer programming problem can be written as follow:

$$\begin{aligned} &\max T(x), \\ &\min I(x), \\ &\min F(x), \\ &s.t. \\ &\quad T(x) \geq F(x), \\ &\quad T(x) \geq I(x), \\ &\quad 0 \leq T(x)+I(x)+F(x) \leq 3, \\ &\quad T(x), I(x), F(x) \geq 0, \\ &\quad x \geq 0 \text{ is integer.} \end{aligned} \tag{13}$$

The problem (13) can be written to the equivalent form as follows:

$$\begin{aligned} &\max \alpha, \min \delta, \min \beta, \\ &s.t. \\ &\quad \alpha \leq T(x), \\ &\quad \delta \geq I(x), \\ &\quad \beta \geq F(x), \\ &\quad \alpha \geq \delta, \\ &\quad \alpha \geq \beta, \\ &\quad 0 \leq \alpha+\delta+\beta \leq 3, \\ &\quad x \geq 0, \ is \text{ integer.} \end{aligned} \tag{14}$$

where α represented the minimal degree of acceptation, β represented the maximal rejection degree and δ represented the maximal degree of indeterminacy.

The model (14) can be written to another type of neutrosophic optimization model where formulated as follow:

$$\begin{aligned}
&\max(\alpha-\delta-\beta),\\
&s.t.\\
&\quad \alpha \le T(x),\\
&\quad \delta \ge I(x),\\
&\quad \beta \ge F(x),\\
&\quad \alpha \ge \delta,\\
&\quad \alpha \ge \beta,\\
&\quad 0 \le \alpha+\delta+\beta \le 3,\\
&\quad \alpha,\delta,\beta \ge 0,\\
&\quad x \ge 0 \text{ is integer.}
\end{aligned} \tag{15}$$

Model (15) is equivalent with the following one:

$$\begin{aligned}
&\min(1-\alpha)+\delta+\beta,\\
&s.t.\\
&\quad \alpha \le T(x),\\
&\quad \delta \ge I(x),\\
&\quad \beta \ge F(x),\\
&\quad \alpha \ge \delta,\\
&\quad \alpha \ge \beta,\\
&\quad 0 \le \alpha+\delta+\beta \le 3,\\
&\quad \alpha,\delta,\beta \ge 0,\\
&\quad x \ge 0 \text{ is integer.}
\end{aligned} \tag{16}$$

4. The new method for solving Interval Neutrosophic Integer Programming problems

In this section we introduced a new approach to find the optimal solution for solving INIP problems.at the first by using of a score function we convert INIP problem into crisp model and then using of Branch and Bound Algorithm solve it same as the classic integer programming problem. The algorithm of the proposed method is presented as follows:

Step 1: In order to compare any two triangular INNs based on the proposed ranking function, let $N=[(a^l,a^m,a^u);[\alpha_N^l,\alpha_N^u],[\delta_N^l,\delta_N^u],[\beta_N^l,\beta_N^u]]$ be a symmetric interval neutrosophic number, where $[\alpha_N^l,\alpha_N^u],[\delta_N^l,\delta_N^u]$ and $[\beta_N^l,\beta_N^u]$ are respectively the truth, indeterminacy, and falsity membership degrees of N. Also a^l,a^m and a^u are respectively the lower, median, and upper bounds for N.

The ranking function for the interval neutrosophic number N will be defined as follows:

$$L(N)=\frac{1}{4}[a^l+a^u+2a^m]+(\bar{\alpha}_N-\bar{\delta}_N-\bar{\beta}_N), \tag{17}$$

where $\bar{\alpha}_N=\dfrac{\alpha_N^l+\alpha_N^u}{2}$, $\bar{\delta}_N=\dfrac{\delta_N^l+\delta_N^u}{2}$ and $\bar{\beta}_N=\dfrac{\beta_N^l+\beta_N^u}{2}$. Moreover, we have :

$$N \geq \tilde{0} \quad \text{if} \quad \frac{a^l + a^u + 2a^m}{4} \geq 0.$$

Step 2: Create the decision set which include the highest degree of truth-membership and the least degree of falsity and indeterminacy memberships.

Step 3: Find the optimal solution to the linear programming model with the integer restrictions relaxed.

Step 4: At the previous node let the relaxed solution be the upper bound and the rounded-down integer solution be the lower bound.

Step 5: Select the variable with the greatest fractional part for branching. Create two new constraints for this variable reflecting the partitioned integer values. The result will be a new ≤ constraint and a new≥ constraint.

Step6: Create two new nodes, one for the ≥ constraint and one for the ≤ constraint.

Step 7: Solve the relaxed linear programming model with the new constraint added at each of these nodes.

Step 8: The relaxed solution is the upper bound at each node, and the existing maximum integer solution is the lower bound.

Step 9: If the process produces a feasible integer solution with the greatest upper bound of these nodes. Integer solution (at any node) is the lower bound value of any ending node; the optimal integer solution has been reached. If feasible integer solution doses not emerge, branch from the node with the greatest upper bound.

Step 10: Return to step 5.

For a minimization model, relaxed solutions are rounded up and upper and lower bounds are reversed.

5. Numerical example

A mobile factory produces four basic units, such as Camera, Speaker, Ram, and Screen. All productions have to get through four parts. These four parts include Design, Fabrication, Probe, and Assembly. The favorable time for each unit manufactured and its profit is presented in Table 1. The minimum production amount for supplementing monthly products is presented in Table 2. The purpose of the company is producing products in this limit for maximizing the general profits.

Table 1. Departments and profits

Products	Design	Fabrication	Probe	Assembly	Unit profit
P_1	0.2	0.5	0.1	0.1	$\tilde{14}$\$
P_2	0.5	3	2	0.6	$\tilde{7}$\$
P_3	0.4	4	4	0.8	$\tilde{5}$\$
P_4	1	2	0.2	0.2	$\tilde{8}$\$

Table 2. Time capacity and minimum production level

Sector	Capacity (in hours)	Products	Minimum production level
Design	$1\tilde{3}00$	P_1	$1\tilde{0}0$
Fabrication	$3\tilde{3}40$	P_2	$2\tilde{8}0$
Probe	$1\tilde{8}00$	P_3	$1\tilde{9}4$
Assembly	$2\tilde{1}00$	P_4	$4\tilde{0}0$

The neutrosophic values for each INN in the previous tables are represented as follows:

$1\tilde{4} = \langle (12,14,16),[0.3,0.7],[0.2,0.8],[0.2,0.9] \rangle$,

$\tilde{7} = \langle (2,7,12),[0.1,0.6],[0.4,0.7],[0.6,0.8] \rangle$,

$\tilde{5} = \langle (4,5,6),[0.2,0.5],[0.4,0.9],[0.3,0.4] \rangle$,

$\tilde{8} = \langle (3,8,13),[0.2,0.5],[0.3,0.8],[0.6,0.9] \rangle$,

$1\tilde{3}00 = \langle (1000,1300,1600),[0.1,0.6],[0.2,0.7],[0.3,0.8] \rangle$,

$3\tilde{3}40 = \langle (3215,3340,3465),[0.7,0.9],[0.2,0.7],[0.4,0.9] \rangle$,

$1\tilde{8}00 = \langle (1390,1800,2210),[0.4,1],[0.2,0.6],[0.1,0.2] \rangle$,

$2\tilde{1}00 = \langle (1818,2100,2510),[0.3,0.7],[0.1,0.6],[0.4,0.8] \rangle$,

$1\tilde{0}0 = \langle (99,100,101),[0.1,0.7],[0.2,0.6],[0.3,0.4] \rangle$,

$2\tilde{8}0 = \langle (230,280,330),[0.7,0.9],[0.1,0.2],[0.2,0.5] \rangle$,

$1\tilde{9}4 = \langle (184,194,204),[0.1,0.6],[0.3,0.7],[0.1,0.7] \rangle$,

$4\tilde{0}0 = \langle (200,400,600),[0.1,0.4],[0.2,0.6],[0.4,0.8] \rangle$

Let x_1, x_2, x_3 and x_4 represent the number of produced Cameras, Speakers, Rams, and Screens, respectively. The above problem can be formulated as follows:

$$
\begin{aligned}
&Max\ \tilde{Z} \approx 1\tilde{4}x_1 + \tilde{7}x_2 + \tilde{5}x_3 + \tilde{8}x_4\\
&s.t.\\
&\quad 0.2x_1 + 0.5x_2 + 0.4x_3 + 1x_4 \tilde{\leq} 1\tilde{3}00,\\
&\quad 0.5x_1 + 3x_2 + 4x_3 + 2x_4 \tilde{\leq} 3\tilde{3}40,\\
&\quad 0.1x_1 + 2x_2 + 4x_3 + 0.2x_4 \tilde{\leq} 1\tilde{8}00,\\
&\quad 0.1x_1 + 0.6x_2 + 0.8x_3 + 0.2x_4 \tilde{\leq} 2\tilde{1}00,\\
&\quad x_1 \tilde{\geq} 1\tilde{0}0,\\
&\quad x_2 \tilde{\geq} 2\tilde{8}0,\\
&\quad x_3 \tilde{\geq} 1\tilde{9}4,\\
&\quad x_4 \tilde{\geq} 4\tilde{0}0,\\
&\quad x_1, x_2, x_3 \geq 0.
\end{aligned}
$$

By applying the proposed ranking function in Equation (17) the following crisp model can be obtained:

$$
\begin{aligned}
&Max\ Z = 13.45x_1 + 6.1x_2 + 4.4x_3 + 7.05x_4\\
&s.t.\\
&\quad 0.2x_1 + 0.5x_2 + 0.4x_3 + 1x_4 \leq 1299.35,\\
&\quad 0.5x_1 + 3x_2 + 4x_3 + 2x_4 \leq 3339.7,\\
&\quad 0.1x_1 + 2x_2 + 4x_3 + 0.2x_4 \leq 1800.15,\\
&\quad 0.1x_1 + 0.6x_2 + 0.8x_3 + 0.2x_4 \leq 2099.55,\\
&\quad x_1 \geq 99.65,\\
&\quad x_2 \geq 280.3,\\
&\quad x_3 \geq 193.45,\\
&\quad x_4 \geq 399.25,\\
&\quad x_1, x_2, x_3, x_4 \geq 0.
\end{aligned}
$$

By using the standard integer programming method, the results of the above problem are obtained as follows:

$x^* = (1841, 281, 194, 400)$, and $Z^* = 30299.$

4. Conclusion

Since many real-world problems are too complex to be defined in precise terms, indeterminacy is often involved in any engineering design process. Neutrosophic as an extension of FS and IFS is an efficient tool to cope with indeterminacy. In this research, we first proposed an INIP model and then proposed a new method for solving INIP problems based on a novel ordering approach. To increase the acceptance degree and reduce the degrees of indeterminacy and rejection, we proposed a ranking function capable of converting every triangular interval neutrosophic number to its equivalent crisp value. Subsequently, every INIP problem could be converted to the crisp model where can be solved by standard methods easily. In particular, the illustrative example explored to solve the mentioned problem based on the conventional approach.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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