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Malmquist Productivity Index under Fuzzy Environment

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ABSTRACT

The integration of data envelopment analysis (DEA) approach and Malmquist productivity index (MPI) is one of the popular and powerful techniques in order to calculate of changes in productivity of homogeneous decision making units (DMUs) over different time periods. In this paper, an extended Malmquist productivity index will be presented that is capable to be employed in the presence of fuzzy data and linguistic variables. It should be noted that possibilistic programming (PP) as well as chance-constrained programming (CCP) approaches are applied to handle data ambiguity. The implementation of the proposed fuzzy Malmquist productivity index (FMPI) is illustrated by a numerical example under triangular fuzzy data. Finally, the results show the applicability and efficacy of the extended MPI to calculate the changes of productivity of DMUs under fuzzy environment.

1. Introduction

Data envelopment analysis (DEA) is a powerful mathematical programming approach for performance measurement of peer decision making units (DMUs) [6, 7, 11, 20, 21, 24, 28, 29, 48, 49]. One of the important issues in performance evaluation of DMU in real-world problems and applications is to identify the progress and decline of DMUs over time periods. Whether the DMU has a degree or type of functional change, including progression, regression, or stagnation over its previous period compared to other DMUs. The combination of DEA and Malmquist productivity index (MPI) can be used to calculate, identify, and evaluate trends and types of DMU changes.

The very important point to be taken into account when calculating MPI is to consider the uncertainty of data in the process of computing this indicator. It should be noted that ignoring this important point can mislead to the identification and classification of DMUs in terms of trend and type of productivity changes. Also, conventional and traditional DEA models cannot be applied in the presence of data uncertainty [2, 3, 19, 22, 23, 30, 33, 35, 36, 39, 41, 43, 44, 50, 51, 52, 60]. As a result, proposing and applying new uncertain Malmquist productivity index that is capable to be employed under fuzzy data seems to be essential.

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Therefore, the goal of this paper is to provide a new Malmquist productivity index in order to calculate the productivity changes of DMUs in the presence of fuzzy numbers and linguistic variables. To reach this goal, fuzzy chance-constrained programming (FCCP) approach is applied to handle data ambiguity and epistemic uncertainty. Notably, the FCCP approach is an applicable and effective method in fuzzy data envelopment analysis (FDEA) for dealing with the uncertainty that is caused by the absence or lack of knowledge about the exact value of model parameters in fuzzy mathematical programming (FMP) [8, 12, 15, 16, 26, 27, 32, 34, 37, 38, 40, 42, 45, 46, 58].

The rest of this paper is organized as follows. The preliminaries and formulation of traditional Malmquist productivity index will be explained in Section 2. Then, an extended Malmquist productivity index based on fuzzy chance-constrained programming will be presented in Section 3. The implementation, applicability, and efficacy of the proposed fuzzy Malmquist productivity index will be illustrated by a numerical example in Section 4. Finally, conclusions, discussions, as well as some directions for future researches will be introduced in Section 5.

2. Malmquist Productivity Index

Färe & Grosskopf [9] were the pioneer researches that combined MPI and DEA method to calculate the productivity changes. They have proposed this indicator by taking into account two periods of time and calculating technological changes and efficiency changes over these two periods. Suppose that there are n homogenous decision making units $\text{DMU}_j \ (j=1,\dots,n)$ that convert m inputs $x_{ij} \ (i=1,\dots,m)$ into s outputs $y_{rj} \ (r=1,\dots,s)$ and DMU_0 is an under evaluation DMU. By applying the envelopment form of input-oriented CCR model, $\Delta_0^t(x_0^t, y_0^t)$, $\Delta_0^{t+1}(x_0^{t+1}, y_0^{t+1})$, $\Delta_0^t(x_0^{t+1}, y_0^{t+1})$, and $\Delta_0^{t+1}(x_0^t, y_0^t)$ are estimated from Models (1) to (4), respectively:

$$\Delta_0^t\left(x_0^t, y_0^t\right) = \text{Min} \ \ \theta \tag{1}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta x_{i0}^t, \quad \forall i$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{r0}^t, \quad \forall r$$

$$\lambda_j \ge 0, \quad \forall j$$

$$\Delta_0^{t+1}\left(x_0^{t+1}, y_0^{t+1}\right) = \text{Min} \ \ \theta \tag{2}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le \theta x_{i0}^{t+1}, \quad \forall i$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \ge y_{r0}^{t+1}, \quad \forall r$$

$$\lambda_j \ge 0, \quad \forall j$$

$$\Delta_0^t\left(x_0^{t+1}, y_0^{t+1}\right) = \text{Min} \;\; \theta \tag{3}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{t} \le \theta x_{i0}^{t+1}, \quad \forall i$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t} \ge y_{r0}^{t+1}, \quad \forall r$$

$$\lambda_j \ge 0, \quad \forall j$$

$$\Delta_0^{t+1}\left(x_0^{t}, y_0^{t}\right) = \text{Min} \;\; \theta \tag{4}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le \theta x_{i0}^{t}, \quad \forall i$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \ge y_{r0}^{t}, \quad \forall r$$

$$\lambda_j \ge 0, \quad \forall j$$

Finally, Malmquist productivity index is calculated using Equation (5):

$$\text{MPI}_0 = \sqrt{\frac{\Delta_0^t\left(x_0^{t+1}, y_0^{t+1}\right) * \Delta_0^{t+1}\left(x_0^{t+1}, y_0^{t+1}\right)}{\Delta_0^t\left(x_0^{t}, y_0^{t}\right) * \Delta_0^{t+1}\left(x_0^{t}, y_0^{t}\right)}} \tag{5}$$

It needs to be explained that based on the value of the MPI, which can be more or equal to or less than one, the productivity change of the DMU under consideration is interpreted as follows:

- $\text{MPI}_0 > 1$, increase productivity and observe progress.
- $\text{MPI}_0 < 1$, decrease productivity and observe regress.
- $\text{MPI}_0 = 1$, no change in productivity at time $t+1$ in comparison to t.

3. Fuzzy Malmquist Productivity Index

In this section, the fuzzy Malmquist productivity index is proposed. It should be noted that for presenting fuzzy MPI, possibilistic programming (PP) as well as chance-constrained programming (CCP) approaches are employed. Now by applying fuzzy chance-constrained programming, Models (1) to (4), are rewritten to Models (6) to (9), respectively. Note that δ is confidence level for satisfying the fuzzy chance constraints. Also, the inputs and outputs have a triangular distribution $\tilde{x}(x^{(1)}, x^{(2)}, x^{(3)})$ and $\tilde{y}(y^{(1)}, y^{(2)}, y^{(3)})$ with condition of $x^{(1)} < x^{(2)} < x^{(3)}$ and $y^{(1)} < y^{(2)} < y^{(3)}$.

$$\Phi_0^t\left(x_0^t, y_0^t, \delta\right) = \text{Min} \;\; \theta \tag{6}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j \left(\left(1-\delta\right) x_{ij}^{t\,(1)} + \left(\delta\right) x_{ij}^{t\,(2)}\right) \le \theta\left(\left(\delta\right) x_{i0}^{t\,(2)} + \left(1-\delta\right) x_{i0}^{t\,(3)}\right), \;\; \forall i$$

$$\sum_{j=1}^{n} \lambda_j \left(\left(\delta\right) y_{rj}^{t\,(2)} + \left(1-\delta\right) y_{rj}^{t\,(3)}\right) \ge \left(\left(1-\delta\right) y_{r0}^{t\,(1)} + \left(\delta\right) y_{r0}^{t\,(2)}\right), \;\; \forall r$$

$$\lambda_j \ge 0, \;\; \forall j$$

$$\Phi_0^{t+1}\left(x_0^{t+1}, y_0^{t+1}, \delta\right) = \text{Min} \;\; \theta \tag{7}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j \left(\left(1-\delta\right) x_{ij}^{t+1(1)} + \left(\delta\right) x_{ij}^{t+1(2)}\right) \le \theta\left(\left(\delta\right) x_{i0}^{t+1(2)} + \left(1-\delta\right) x_{i0}^{t+1(3)}\right), \;\; \forall i$$

$$\sum_{j=1}^{n} \lambda_j \left(\left(\delta\right) y_{rj}^{t+1(2)} + \left(1-\delta\right) y_{rj}^{t+1(3)}\right) \ge \left(\left(1-\delta\right) y_{r0}^{t+1(1)} + \left(\delta\right) y_{r0}^{t+1(2)}\right), \;\; \forall r$$

$$\lambda_j \ge 0, \;\; \forall j$$

$$\Phi_0^{t}\left(x_0^{t+1}, y_0^{t+1}, \delta\right) = \text{Min} \;\; \theta \tag{8}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j \left(\left(1-\delta\right) x_{ij}^{t\,(1)} + \left(\delta\right) x_{ij}^{t\,(2)}\right) \le \theta\left(\left(\delta\right) x_{i0}^{t+1(2)} + \left(1-\delta\right) x_{i0}^{t+1(3)}\right), \;\; \forall i$$

$$\sum_{j=1}^{n} \lambda_j \left(\left(\delta\right) y_{rj}^{t\,(2)} + \left(1-\delta\right) y_{rj}^{t\,(3)}\right) \ge \left(\left(1-\delta\right) y_{r0}^{t+1(1)} + \left(\delta\right) y_{r0}^{t+1(2)}\right), \;\; \forall r$$

$$\lambda_j \ge 0, \;\; \forall j$$

$$\Phi_0^{t+1}\left(x_0^{t}, y_0^{t}, \delta\right) = \text{Min} \;\; \theta \tag{9}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} \lambda_j \left(\left(1-\delta\right) x_{ij}^{t+1(1)} + \left(\delta\right) x_{ij}^{t+1(2)}\right) \le \theta\left(\left(\delta\right) x_{i0}^{t\,(2)} + \left(1-\delta\right) x_{i0}^{t\,(3)}\right), \;\; \forall i$$

$$\sum_{j=1}^{n} \lambda_j \left(\left(\delta\right) y_{rj}^{t+1(2)} + \left(1-\delta\right) y_{rj}^{t+1(3)}\right) \ge \left(\left(1-\delta\right) y_{r0}^{t\,(1)} + \left(\delta\right) y_{r0}^{t\,(2)}\right), \;\; \forall r$$

$$\lambda_j \ge 0, \;\; \forall j$$

Finally, fuzzy Malmquist productivity index based on FCCP approach for desired confidence level is calculated using Equation (10):

$$\mathrm{FMPI}_0(\delta)=\sqrt{\frac{\Phi_0^t\left(x_0^{t+1},y_0^{t+1},\delta\right)*\Phi_0^{t+1}\left(x_0^{t+1},y_0^{t+1},\delta\right)}{\Phi_0^t\left(x_0^t,y_0^t,\delta\right)*\Phi_0^{t+1}\left(x_0^t,y_0^t,\delta\right)}} \quad (10)$$

According to the value of the FMPI, which can be more or equal to or less than one, the productivity change of the DMU under consideration for desired confidence level is interpreted as follows:

- $\mathrm{PMPI}_0(\delta)>1$, increase productivity and observe progress.
- $\mathrm{PMPI}_0(\delta)<1$, decrease productivity and observe regress.
- $\mathrm{PMPI}_0(\delta)=1$, no change in productivity at time $t+1$ in comparison to t.

4. Numerical Results

In this section, the applicability of FMPI that proposed in this research is evaluated by using a numerical example. The numerical example is related to five DMUs with one fuzzy input and output in the form of a triangular fuzzy number. Numerical data of the example for periods t and $t+1$. are presented in Tables 1 and 2, respectively:

Table 1. Data for Period t

Period t	**DMU A**	**DMU B**	**DMU C**	**DMU D**	**DMU E**
Input	(1, 2, 3)	(3, 5, 7)	(1, 3, 5)	(5, 7, 9)	(3, 4, 5)
Output	(2, 3, 4)	(2, 4, 6)	(3, 5, 7)	(5, 6, 7)	(7, 8, 9)

Table 2. Data for Period $t+1$

Period $t+1$	**DMU A**	**DMU B**	**DMU C**	**DMU D**	**DMU E**
Input	(2, 4, 6)	(7, 8, 9)	(2, 3, 4)	(1, 2, 3)	(7, 8, 9)
Output	(3, 6, 9)	(1, 3, 5)	(3, 4, 5)	(5, 7, 9)	(4, 5, 6)

Now Models (6) to (9), are solved for different confidence levels including 0%, 25%, 50%, 75, and 100%. The results of Models (6) to (9), are presented in Tables 3 to 6, respectively:

Table 3. The Results of $\Phi^{t}(x^{t}, y^{t})$

DMUs	Confidence Levels				
	0%	25%	50%	75%	100%
DMU A	0.09524	0.18881	0.33333	0.55556	0.75000
DMU B	0.04082	0.08876	0.16667	0.28926	0.40000
DMU C	0.08571	0.17949	0.33333	0.58442	0.83333
DMU D	0.07937	0.14253	0.22917	0.34848	0.42857
DMU E	0.20000	0.35223	0.55556	0.82888	1.00000

Table 4. The Results of $\Phi^{t+1}(x^{t+1}, y^{t+1})$

DMUs	Confidence Levels				
	0%	25%	50%	75%	100%
DMU A	0.05556	0.10027	0.16875	0.27222	0.42857
DMU B	0.01235	0.02521	0.04412	0.07071	0.10714
DMU C	0.08333	0.12745	0.18750	0.26923	0.38095
DMU D	0.18519	0.29412	0.45000	0.67407	1.00000
DMU E	0.04938	0.07143	0.09926	0.13434	0.17857

Table 5. The Results of $\Phi^{t}(x^{t+1}, y^{t+1})$

DMUs	Confidence Levels				
	0%	25%	50%	75%	100%
DMU A	0.07143	0.15734	0.30000	0.53030	0.75000
DMU B	0.01587	0.03956	0.07843	0.13774	0.18750
DMU C	0.10714	0.20000	0.33333	0.52448	0.66667
DMU D	0.23810	0.46154	0.80000	1.31313	1.75000
DMU E	0.06349	0.11209	0.17647	0.26171	0.31250

Table 6. The Results of $\Phi^{t+1}(x^t, y^t)$

DMUs	Confidence Levels				
	0%	25%	50%	75%	100%
DMU A	0.07407	0.12032	0.18750	0.28519	0.42857
DMU B	0.03175	0.05656	0.09375	0.14848	0.22857
DMU C	0.06667	0.11438	0.18750	0.30000	0.47619
DMU D	0.06173	0.09083	0.12891	0.17889	0.24490
DMU E	0.15556	0.22446	0.31250	0.42549	0.57143

Finally, the results of fuzzy Malmquist productivity index under different confidence levels are introduced in Table 7 as follows:

Table 7. The Results of FMPI

DMUs	Confidence Levels				
	0%	25%	50%	75%	100%
DMU A	0.75000	0.83333	0.90000	0.95455	1.00000
DMU B	0.38889	0.44571	0.47059	0.47619	0.46875
DMU C	1.25000	1.11429	1.00000	0.89744	0.80000
DMU D	3.00000	3.23810	3.49091	3.76812	4.08333
DMU E	0.31746	0.31823	0.31765	0.31574	0.31250

As it can be seen in Table 7, DMU C is so sensitive to changing data. Therefore, if uncertainty of data is not considered, analysis of the productivity changes of this DMU can be invalid. Accordingly, the numerical results show the efficacy of the proposed FMPI.

5. Conclusions and Future Research Directions

In this study, a new Malmquist productivity index is extended that is capable to be used in the presence of fuzzy data. For presenting FMPI, possibilistic programming and chance-constrained programming are applied. Finally, for solving and showing the validation of the proposed FMPI, a numerical example was used. Note that for future researches, the Malmquist productivity index can be extended based on other uncertain programming approaches such as stochastic programming, robust optimization, Z-number theory, and interval programming [1, 4, 5, 10, 13, 14, 17, 18, 25, 31, 47, 53, 54, 55, 56, 57, 59].

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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