FUZZY OPTIMIZATION AND MODELLING 1(2) (2020) 1-7

E-ISNN: 2676-7007

Contents lists available at FOMJ

Fuzzy Optimization and Modelling

Journal homepage: http://fomj.qaemiau.ac.ir/

Paper Type: Original Article

New insight on solving fuzzy linear fractional programming in material aspects

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ARTICLE INFO

Article history:
Received 11 October 2019
Revised 12 March 2020
Accepted 26 March 2020
Available online 10 April 2020

Keywords:
Fractional programming
Fuzzy linear programming
Triangular fuzzy numbers

ABSTRACT

Recently, Srinivasan [On solving fuzzy linear fractional programming in material aspects, Materials Today: Proceedings, https://doi.org/10.1016/j.matpr.2019.04.209] proposed a method to solve fractional linear programming problem under fuzzy environment based on ranking and decomposition methods. Srinivasan also claimed that the proposed method solved fractional linear programming problem with inequality and equality constraints. In this note, we point out that the paper entitled above suffers from certain mathematical mistakes for solving these problems. Hence, the mentioned method and example are not valid. Further the exact method is stated and solved the problem.

1. Introduction

Linear Fractional Programming problem (LFPP) in fuzzy environment is used in operation research and mainly is used in decision-making process in which the objective function is a fraction of two linear functions. This problem is formulated as follows:

$$\begin{aligned} & Max(Min)\ Z = \frac{c^T x + \alpha}{d^T x + \beta} \\ & \quad s.t. \\ & \qquad Ax(\leq, =, \geq) b, \\ & \qquad x \geq 0. \end{aligned} \tag{1}$$

where $c, d \in \mathbb{R}^n, b \in \Re^m, A \in \mathbb{R}^{m\times n}, \alpha, \beta \in \mathbb{R}$.

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This model is widely interested for mathematician because the problem has many applications in real life problem. Moreover, in practical applications the data's are uncertainty and irregular. Therefore, a new set theory is introduced and it is called fuzzy set theory; first introduced by Zadeh [22]. Li and Chen [11] developed a method for solving the FLFP problem with fuzzy numbers in the objective function using a fuzzy programming approach. Based on ranking function to compare fuzzy numbers, De and Deb [9] considered a fuzzy linear fractional programming problem using sign distance ranking method where all the terms are triangular fuzzy number. Youness et al. [21] imported an approach for solving a bi-level multi-objective fractional integer programming problem consists of fuzzy numbers in the right-hand side of the constraints. Pop and Minasian [14] proposed a method for solving fully falsified linear fractional programming problems where all the parameters and variables are triangular fuzzy numbers. In [18, 19], they considered the same problem of [14] for solving fully fuzzy linear fractional programming problem. Chinnadurai and Muthukumar [2] proposed solution procedure for solving fuzzy linear fractional programming problem by using fuzzy mathematical programming approach. Very recently, a number of papers have exhibited their interest to solve the FLFP problems [2-6, 12]. Das et al. [5] proposed an algorithm for solving fuzzy linear fractional programming (FLFP) problem with the help of multi-objective LFP problem.

Recently, Srinivasan [17] studied LFPP under fuzzy environment by assuming that the parameters are fuzzy numbers with uncertain co-efficient in the objective function and proposed an algorithm based on ranking and decomposition methods.

Definition 1: [17] A convenient method for comparing of the fuzzy numbers is by use of ranking functions [13]. So we define a ranking function $\Re : F(\mathbb{R}) \to \mathbb{R}$, which maps for each fuzzy number into the real line. The centroid of centroid ranking of triangular fuzzy number is $\Re(A) = \frac{(2a+14b+2d)}{6}\frac{7w}{6}$, here $w=1$.

2. Shortcomings of the existing method

In Section 3[17], Srinivasan presented an algorithm using the LU decomposition method and in Section 4 [17], an example was solved by the mentioned method. However, this method suffers from certain mathematical mistakes. The main incorrect assumptions are:

1. In the fuzzy version of the problem (1), the variables are non-negative triangular fuzzy numbers. However, the LU decomposition method doesn't guarantee that the variables are nonnegative fuzzy number.

2. The mentioned method used transformation technique to convert LFP in to LP problem. However, in the mentioned transformation method, all the variables have not changed; see for example, the problem (3) in [17].

3. In problem (4) [17], all the constraints are inequality numbers. However, in LU decomposition method the equations are $AY = B$ which is contradicts. Because, it is a linear equation, so the constraints should be equality numbers. Srinivasan [17] have not transformed the fuzzy inequality constraints of problem into fuzzy equality constraints of problem by adding slack variables/surplus variables.

Hence, all results proposed in [17] are wrong.

3. Main result

In the following, we present a correction version of real life example of Section 4 [17]. Here, in this note we assumed that the constraints are equality triangular fuzzy numbers and it is shown that the objective value are exactly satisfied.

Example 1 [17]**:** In an important in India, A wooden company is the producer of two kinds of products A and B with profit around (1, 2, 3) and around (0, 1, 2) dollar per unit, respectively. However, the costs for each one unit of the above products are around (1, 3, 5) and around (1, 1, 1) dollars, respectively. It is assume that a fixed cost of around (3, 6, 9) dollar is added to the cost function due to expected duration through the process of production. Suppose the raw material needed for manufacturing product A and B is about (4, 7, 10) units per pound and about (0, 1, 2) units per dollar, respectively, the supply for this raw material is restricted to about(4, 6, 8)) dollar. Man-hours per unit for the product A is about (1, 5, 9) hour and product B is about (2, 3, 4) hour per unit for manufacturing but total Man-hour available is about (5, 6, 7) hour daily. Determine how many products A and B should be manufactured in order to maximize the total profit.

Solution

Real life problem can be formulated to the following fuzzy linear fractional programming problem:

Let x1 and x2 to be the amount of units of A and B to be produced. Then the above problem can be formulated as:

$$
\begin{aligned}
&Maz\ \tilde{z} = \frac{(1,2,3)x_1 + (0,1,2)x_2}{(1,3,5)x_1 + (1,1,1)x_2 + (3,6,9)} \\
&s.t. \\
&(4,7,10)x_1 + (0,1,2)x_2 = (4,6,8), \\
&(1,5,9)x_1 + (2,3,4)x_2 = (5,6,7), \\
&x_1 \text{and } x_2 \text{ are free.}
\end{aligned}
$$

Using definition 1 the problem can be written as:

$$
\begin{aligned}
&Maz\ Z = \frac{7x_1 + 3.5x_2}{10.5x_1 + 3.5x_2 + 21} \\
&s.t. \\
&24.5x_1 + 3.5x_2 = 21,, \\
&17.5x_1 + 10.5x_2 = 21, \\
&x_1 \text{and } x_2 \text{ are free.}
\end{aligned}
$$

By using Charnes-Cooper transformation technique, the problem is converted into the following LP problem.

$$
\begin{aligned}
&Maz\ Z = 7y_1 + 3.5y_2 \\
&s.t. \\
&10.5y_1 + 3.5y_2 + 21t = 1, \\
&24.5y_1 + 3.5y_2 - 21t = 0, \\
&17.5y_1 + 10.5y_2 - 21t = 0, \\
&y_1, y_2 \text{ and } t\ are\ free.
\end{aligned}
$$

LU decomposition technique is used to solve the problem and the solutions are found for weights. For example $w=1$,

$$\begin{aligned} &Maz\ w = 7y_1 + 3.5y_2 \\ &s.t. \\ &10.5y_1 + 3.5y_2 + 21t = 1, \\ &24.5y_1 + 3.5y_2 - 21t = 0, \\ &17.5y_1 + 10.5y_2 - 21t = 0, \\ &y_1, y_2 \text{ and } t\ are\ free. \end{aligned}$$

This can be written in the following form:

$$\begin{aligned} &\text{Find } w \\ &s.t. \\ &-7y_1 - 3.5y_2 + w \le 0, \\ &10.5y_1 + 3.5y_2 + 21t = 1, \\ &24.5y_1 + 3.5y_2 - 21t = 0, \\ &17.5y_1 + 10.5y_2 - 21t = 0, \\ &y_1, y_2 \text{ and } t \text{ are free.} \end{aligned}$$

Adding slack variables and the problem can be written $AY = B,$ where

$$A = \begin{bmatrix} -7 & -3.5 & 0 & 1 \\ 10.5 & 3.5 & 21 & 0 \\ 24.5 & 3.5 & -21 & 0 \\ 17.5 & 10.5 & -21 & 0 \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ t \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

These solutions are solved by LINGO software and the following values are obtained.

$$x_1 = 0.75, x_2 = 0.75, Z = 0.25.$$

This solution exactly satisfies the constraints.

Example 2: Consider the following fuzzy linear fractional programming problem:

$$\begin{aligned} &Max\,\tilde{Z} = \frac{(3,5,7)\otimes x_1 + (2,3,4)\otimes x_2}{(4,5,6)\otimes x_1 + (1,2,3)\otimes x_2 + (0,1,2)} \\ &s.t. \\ &(2,3,4)\otimes x_1 + (3,5,7)\otimes x_2 \le (11,15,19), \\ &(4,5,6)\otimes x_1 + (1,2,3)\otimes x_2 \le (8,10,12), \\ &x_1, x_2 \ge 0. \end{aligned}$$

Solution

We solve the above problem by using our corrected proposed method. The transformed linear programming problems are solved by the classical methods, which are discussed in Section 3. We obtain the fuzzy optimal value as $Z = 1.14$.

Example 3 [6]: Dali Company is the leading producer of soft drinks and low-temperature foods in Taiwan. Currently, Dali plans to develop the South-East Asian market and broaden the visibility of Dali products in the Chinese market. Notably, following the entry of Taiwan to the World Trade Organization, Dali plans to seek strategic alliance with prominent international companies and introduced international bread to lighten the embedded future impact. In the domestic soft drinks market, Dali produces tea beverages to meet demand from four distribution centers in Taichung, Chiayi, Kaohsiung, and Taipei, with production being based at three plants in Changhua, Touliu, and Hsinchu. According to the preliminary environmental information, Table.1 summarizes the potential supply available from the given three plants. The forecast demand from the four distribution centers is as shown in Table 2. The profit of the company gained by each route is presented in Table 3. Table 4 summarizes the unit shipping cost for each route for the upcoming season. The environmental coefficient and related parameters generally are imprecise numbers with triangular possibility distributions over the planning horizon due to incomplete or unobtainable information. For example, the unavailable supply of the Changhua plant is (7.2, 8, 8.8) thousand dozen bottles, the forecast demand of the Taichung distribution center is (6.2, 7, 7.8) thousand dozen bottles, profit per dozen bottles for Changhua to Taichung is (8, 10, 10.8) dollars and the transportation cost per dozen bottles for Changhua to Taichung is (8, 10, 10.8) dollars. The management of Dali is initiating a study to maximize the profit as much as possible.

Table 1: Supply of the plants

Source	Changhua	Touliu	Hsinchu
Supply (Thousand dozen bottles)	(7.2, 8, 8.8)	(12, 14, 16)	(10.2, 12, 13.8)

Table 2: Demand of the destinations

Destination	Taichung	Chiayi	Kaohsiung	Taipei
Demand (thousand dozen bottles)	(6.2, 7, 7.8)	(8.9, 10, 11.1)	(6.5, 8, 9.5)	(7.8, 9, 10.2)

Table 3: Profit of the company

Source	Destination			
	Taichung	Chiayi	Kaohsiung	Taipei
Changhua	(8, 10, 10.8)	(20.4, 22, 24)	(8, 10, 10.6)	(18.8, 20, 22)
Touliu	(14, 15, 16)	(18.2, 20, 22)	(10, 12, 13)	(6, 8, 8.8)
Hsinchu	(18.4, 20, 21)	(9.6, 12, 13)	(7.8, 10, 10.8)	(14, 15, 16)

Table 4: Shipping costs

Source	Destination			
	Taichung	Chiayi	Kaohsiung	Taipei
Changhua	(1.5, 2, 2.5)	(4, 5, 6)	(1.3, 2, 2.5)	(3, 4, 5)
Touliu	(2.5, 3, 4)	(2, 3, 4)	(2.3, 3, 4)	(1.5, 2, 2.5)
Hsinchu	(3, 4, 5)	(2, 3, 4)	(1.5, 2, 2.7)	(2, 3, 4)

The real world problem can be modeled to the following FFLFP problem:

$$Max\ Z = \frac{\left\{\begin{array}{l}(8,10,10.8)x_{11}+(20.4,22,24)x_{12}+(8,10,10.6)x_{13}+(18.8,20,22)x_{14}+\\(14,15,16)x_{21}+(18.2,20,22)x_{22}+(10,12,13)x_{23}+(6,8,8.8)x_{24}+\\(18.4,20,21)x_{31}+(9.6,12,13)x_{32}+(7.8,10,10.8)x_{33}+(14,15,16)x_{34}\end{array}\right\}}{\left\{\begin{array}{l}(1.5,2,2.5)x_{11}+(4,5,6)x_{12}+(1.3,2,2.5)x_{13}+(3,4,5)x_{14}+\\(2.5,3,4)x_{21}+(2,3,4)x_{22}+(2.3,3,4)x_{23}+(1.5,2,2.5)x_{24}+\\(3,4,5)x_{31}+(2,3,4)x_{32}+(1.5,2,2.7)x_{33}+(2,3,4)x_{34}\end{array}\right\}}$$

$$\begin{array}{l} s.t. \\ x_{11}+x_{12}+x_{13}+x_{14} \le (7.2,8,8.8) \\ x_{21}+x_{22}+x_{23}+x_{24} \le (12,14,16) \\ x_{31}+x_{32}+x_{33}+x_{34} \le (10.2,12,13.8) \\ x_{11}+x_{21}+x_{31} \ge (6.2,7,7.8) \\ x_{12}+x_{22}+x_{32} \ge (8.9,10,11.1) \\ x_{13}+x_{23}+x_{33} \ge (6.5,8,9.5) \\ x_{14}+x_{24}+x_{34} \ge (7.8,9,10.2) \\ x_{ij} \ge 0, \qquad i = 1,2,3;\ j=1,2,3,4. \end{array}$$

We solve the above problem by using our proposed method. The transformed linear programming problems are solved by classical methods and we get the fuzzy optimal value of the problem as $Z = 5.25$.

4. Conclusions

On the basis of the present study, it can be concluded that the fuzzy linear fractional programming problems within equality constraints cannot be solved by using linear equations of LU decomposition. Hence, the example solved by Srinivasan [17] is incorrect. It can be solved by using equality constraints or adding slack variables. Furthermore, a correct version has been given in this paper.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. Chinnadurai, V., & Muthukumar, S. (2016). Solving the linear fractional programming problem in a fuzzy environment: Numerical approach. *Applied Mathematical Modelling*, 40(11-12), 6148-6164.
2. Das, S. K. (2017). Modified method for solving fully fuzzy linear programming problem with triangular fuzzy numbers. *International Journal of Research in Industrial Engineering*, 6(4), 293-311.
3. Das, S. K., & Mandal, T. (2017). A MOLFP method for solving linear fractional programming under fuzzy environment. *International Journal of Research in Industrial Engineering*, 6(3), 202-322.
4. Das, S. K., & Mandal, T. (2017). A new model for solving fuzzy linear fractional programming problem with ranking function. *Journal of Applied Research on Industrial Engineering*, 4(2), 89-96.
5. Das, S. K., Edalatpanah, S. A., & Mandal, T. (2018). A proposed model for solving fuzzy linear fractional programming problem: Numerical Point of View. *Journal of Computational Science*, 25, 367-375.
6. Das, S. K., Mandal, T., & Behera, D. (2019). A new approach for solving fully fuzzy linear programming problem. *International Journal of Mathematics in Operational Research*, 15(3), 296-309.
7. Das, S. K., Mandal, T., & Edalatpanah, S. A. (2017). A mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers. *Applied intelligence*, 46(3), 509-519.

8. Das, S. K., Mandal, T., & Edalatpanah, S. A. (2017). A new approach for solving fully fuzzy linear fractional programming problems using the multi-objective linear programming. *RAIRO-Operations Research*, 51(1), 285-297.
9. De, P. K., & Deb, M. (2013, February). Solving fuzzy linear fractional programming problem using signed distance ranking. *In 2013 3rd IEEE International Advance Computing Conference (IACC)* (pp. 806-812). IEEE.
10. Deb, M., & De, P. K. (2015). Optimal solution of a fully fuzzy linear fractional programming problem by using graded mean integration representation method. *Applications and Applied Mathematics*, 10(1), 571-587.
11. Li, D., & Chen, S. (1996). A fuzzy programming approach to fuzzy linear fractional programming with fuzzy coefficients. *Journal of Fuzzy Mathematics*, 4, 829-834.
12. Maiti, I., Pramanik, S., Mandal, T., Das, S. K., (2020). Solving Multi-objective linear fractional programming problem based on Stanojevic's normalization technique under fuzzy environment. *International Journal of Operation Research*, 10.1504/IJOR.2020.10028794.
13. Nachammai, A., & Thangaraj, P. (2012). Solving fuzzy linear fractional programming problem using metric distance ranking. *Applied Mathematical Sciences*, 6(26), 1275-1285.
14. Pop, B., & Stancu-Minasian, I. M. (2008). A method of solving fully fuzzified linear fractional programming problems. *Journal of Applied Mathematics and Computing*, 27(1-2), 227-242.
15. Sapan, K. D., Mandal, T., & Edalatpanah, S. A. (2015). A note on "A new method for solving fully fuzzy linear fractional programming with a triangular fuzzy numbers". *Applied Mathematics and Computational Intelligence*, 4(1), 361-367.
16. Singh, S. K., & Yadav, S. P. (2016). Fuzzy programming approach for solving intuitionistic fuzzy linear fractional programming problem. *International Journal of Fuzzy Systems*, 18(2), 263-269.
17. Srinivasan, R. (2020). On solving fuzzy linear fractional programming in material aspects. Materials Today: Proceedings, 21, 155-157.
18. Stanojević, B., & Stancu-Minasian, I. M. (2009). On solving fully fuzzified linear fractional programs. *Advanced Modeling and Optimization*, 11(4), 503-523.
19. Stanojević, B., & Stancu-Minasian, I. M. (2012). Evaluating fuzzy inequalities and solving fully fuzzified linear fractional programs. *Yugoslav Journal of Operations Research*, 22(1), 41-50.
20. Upmanyu, M., & Saxena, R. R. (2015). On solving multi objective Set Covering Problem with imprecise linear fractional objectives. *RAIRO-Operations Research*, 49(3), 495-510.
21. Youness, E. A., Emam, O. E., & Hafez, M. S. (2014). Fuzzy bi-level multi-objective fractional integer programming. *Applied Mathematics & Information Sciences*, 8(6), 2857.
22. Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. *Fuzzy sets and systems*, 1(1), 3-28.