**International Journal of Mathematical Modelling & Computations-Word Style Guide‎ ‎for Authors**

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**Abstract.** ‎This guide is for authors who are preparing papers for the ‎*International Journal of Mathematical Modelling* & *Computations*‎ ‎(IJM‌‌‌2C) using the Word document preparation system‎, ‎which is available via the‎ ‎journal homepage on the IJM‌‌‌2C website‎. ‎

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# 1. Introduction

All manuscript submissions from authors to *International‎ ‎Journal of Mathematical Modelling* & *Computations* (IJM2C) should be made online via the Manuscript Central site‎

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# 2. Model formulation

In this study the dynamical system of ordinary differential equations is formulated to show the dynamics of human population in the presence of Human Immunodeficiency

The descriptions of compartments are as follows: (i) Susceptible compartment. It is denoted by. These are humans who are free of HIV infection but are capable of becoming infected future in infectious environment (ii) Primary compartment. It is denoted by.This compartment includes all humans who infected with HIV for the first time and that do not know their HIV status but transmit the disease to others with effective contact (iii) Secondary compartment. It is denoted by. This compartment includes all humans who know that they are infected with virus. They join either ART or Herbal medicine user(iv) Herbs user compartment. It is denoted by. This compartment includes of infectious humans that uses only herbal medicine as a treatment. They join both treatment compartment at some rate (v) ART user compartment. It is denoted by. This compartment includes of infectious humans that uses only ART medicine as a treatment. They join both treatment compartment at some rate(vi) Treatment compartment. It is denoted by. This compartment includes all HIV infected population who use both ART and Herbs as a treatment. (vii) Drug resistant compartment. This compartment includes portion of individuals from treatment class that are resistant to both ART and Herbs medicine. (viii) AIDS compartment. It is denoted by. This compartment includes who are at last stage or advanced stage of HIV.

Now, a mathematical model of Human Immunodeficiency virus (HIV) is formulated based on the stated assumptions on the human population as listed below:

1. Deterministic dynamical system in the presence of Human Immunodeficiency virus (HIV) classifies human population under observation into eight compartments as SPIAJTRV model at any time.
2. Susceptible humans are recruited to the compartment at some constant rate.
3. Susceptible humans can be infected if they make effective contact with primary infected population whose status of HIV is not known yet and join primary infected compartment at a constant rate.
4. Primary infected humans transfer into secondary compartment at a constant rate.
5. Secondary infected humans transfer into herbs compartment at a constant rate and transfer into ART compartment at a constant rate.
6. Herbs compartment humans transfer into treatment compartment at a constant rate.
7. ART user human compartment transfer into treatment compartment at the rate
8. Humans in treatment compartment transfer into resistant compartment at the constant rate of.
9. Resistant compartment individuals transfer to AIDS compartment at the rate of .
10. All categories of human’s compartments face the same natural mortality with a rate.
11. All AIDS humans suffer disease induced death at a constant rate.All parameters used in the dynamical system are positive.

Table 1. Notations and description of model variables.

|  |  |
| --- | --- |
| Variable | Description |
|  | Population size of susceptible humans |
|  | Population size of primary infected humans |
|  | Population size of secondary infected population |
|  | Population size of Herbs user humans |
|  | Population size of ART user humans |
|  | Population size of both ART and Herbs user |
|  | Population size of resistant to both treatment |
|  | Population size of AIDS humans |

Table 2. Model parameters notations and description.

|  |  |
| --- | --- |
| Parameter | Description |
|  | Recruitment rate of susceptible human population. With this constant rate new humans will born and enter into susceptible compartment |
|  | Transmission rate of primary infected humans. With this rate primary infected humans transfer into |
|  | Rate of humans transferring from compartment to |
|  | Rate of humans transferring from compartment to |
|  | Rate of humans transferring from compartment to |
|  | Rate of humans transferring from compartment to |
|  | Rate of humans transferring from compartment to |
|  | Rate of humans transferring from compartment to |
|  | Rate of humans transferring from compartment to |
|  | Natural death rate. With this rate humans in all compartments die naturally |
|  | Disease induced death rate of AIDS humans |

Now considering basic assumptions and description of both model variables and parameters given the schematic diagram of the formulated deterministic dynamical system is described in the Figure 1.

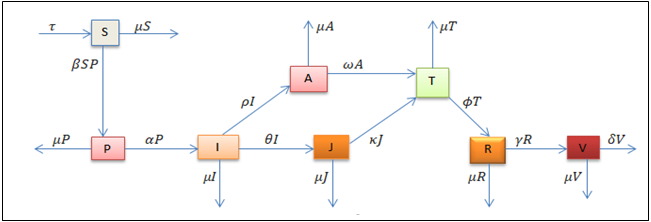


Figure 1. Schematic diagram of compartmental structure of the model.

Based on the model assumptions, the notations of variables and parameters and the schematic diagram, the model equations are formulated and are given as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |
|  |  | (2) |
|  |  | (3) |
|  |  | (4) |
|  |  | (5) |
|  |  | (6) |
|  |  | (7) |
|  |  | (8) |

The non-negative initial conditions of the model equations (1)-(8) are denoted by This system consists of seven first order non-linear ordinary differential equations.

# 3. Mathematical analysis of the model

In this section we describe the mathematical analysis of the present improved and modified model. The analysis consists of the following points (i) existence, positivity and boundedness of solutions (ii) Equilibrium points (iii) disease free equilibrium points (iv) endemic equilibrium points (v) basic reproduction number(vi) stability analysis of the disease free equilibrium points (vii) local stability of disease free equilibrium point (viii) global stability of disease free equilibrium point. These mathematical aspects of the model are presented and discussed in the following sub-sections respectively.

## 3.1 *Existence, uniqueness, positivity and boundedness of solution*

In order to say that the formulated dynamical system is biologically valid and mathematically well-posed, it is required to show that the solutions of the system of differential equations (1)-(8) exist, non-negative and bounded for all time . The details are given as the followings:

**Theorem 3.1 ([5])** Suppose the system of first order differential equation has the form,

or in compact form can be written as

Let denote the region in -dimensional space given by

If the partial derivatives are continuous and bounded in , then there exists a unique continuous vector solution in the interval where is a positive constant.

**Lemma 3.1 (Existence and uniqueness)** Solutions of the model equations (1)-(8) together with the initial conditions exist in i.e. the model variables and exist for all and will remain in .

***Proof*** …

## 3.2 *Equilibrium points*

In order to understand the dynamics of the model, it is necessary to determine equilibrium points of the solution region. An equilibrium solution is a steady state solution of the model equations (1)-(8) in the sense that if the system begins at such a state, it will remain there for all times. In other words, the population sizes remain unchanged and thus the rate of change for each population vanishes. Equilibrium points of the model are found, categorized, stability analysis is conducted and the results have been presented in the following sub-sections:

### *3.2.1 Disease free equilibrium point*

Disease free equilibrium point is a steady state solution where there is no disease in the population. Now, absence of disease implies that and also setting the right hand sides of the model equations (1)-(8) equal to zero results in giving , solution of which is the population size of the susceptible humans at the disease free equilibrium and is given by .Thus, the disease free equilibrium point of the model equations (1)-(8) is given by

### *3.2.2 Endemic equilibrium point*

The endemic equilibrium point is a steady state solution when the disease persists in the population. The endemic equilibrium point is obtained by setting rates of changes of variables with respect to time of model equations (1)-(8) to zero. That is, setting the model equations take the form as

|  |  |  |
| --- | --- | --- |
|  |  | (9) |
|  |  | (10) |
|  |  | (11) |
|  |  | (12) |
|  |  | (13) |
|  |  | (14) |
|  |  | (15) |
|  |  | (16) |

# 4. Result and discussion

In this study, a model describing the dynamics of eight compartments human population pertaining to HIV (Human Immunodeficiency Virus) with treatments are formulated and analyzed. ART only users and Herbs only users joins treatment compartment to use both alternatives for better medifications. Further, it is observed that the disease transmission decreases with decreased transmission rate value and disease persist in the population with increasing transmission rate value. The mathematical analysis has shown that if the reproduction number then the disease free equilibrium point is locally and globally asymptotically stable. Also, the disease free equilibrium point is unstable if implying that the transmission of disease increases.

# 5. Conclusion

In this study, a mathematical model of eight compartments has been formulated to show the dynamics of human populations subjected to HIV/AIDS. Moreover, the formulated model is verified as biologically meaningful and mathematically well posed. The reproduction number is directly proportional to recruitment and probability of transmission rates. From computation of reproduction number, we observed that natural death rate is indirectly proportional to the propagation of the disease. It is also observed that the equilibrium points of model equations are locally asymptotically stable. Further, the Global stability of disease free equilibrium points are described.

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